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INTERNATIONAL

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REVIEW

A TEXTBOOK

ON

PLUMBING, HEATING, AND VENTILATION

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

I

ARITHMETIC
MENSURATION
MECHANICS

WITH PRACTICAL QUESTIONS AND EXAMPLES

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PREFACE

The Instruction and Question Papers which are furnished to the students of the International Correspondence Schools become so badly worn and soiled that, when a student has completed his Course, he has worn out his Instruction Papers, and they are no longer suitable for reference or review. Since the Instruction Papers are very valuable, especially to those who have studied them, there has grown up a demand for Sets of the Instruction and Question Papers, indexed for convenient reference, and durably bound for preservation. Again, many of our students can spare but little time for study, and are, therefore, a long while passing through their Courses. These students also desire the papers in bound volumes to use for reference. Other students begin Courses, but for various reasons are unable to complete them, and feel that, having paid for their Scholarships, they ought to have the textbooks, even though they cannot finish their Courses.

The volumes for the present Course, that in Sanitary Plumbing, Heating, and Ventilation, are five in number:

The first volume in the order of study contains the Instruction Papers and Examination Questions on Arithmetic, Mensuration, and Mechanics.

The second volume in the order of study contains the Instruction Papers and Examination Questions on Plumbing and Drainage, Gas and Gas Fitting, and Electric-Light Wiring and Bell Work.

The third volume in the order of study contains the Instruction Papers and Examination Questions on Principles of Heating and Ventilation, Steam Heating, Hot-Water Heating, Furnace Heating, and Ventilation of Buildings.

Of the two remaining volumes, one contains the Drawing Plates and the instructions for drawing them; it forms a very complete course in Mechanical Drawing. The other volume contains the Tables, Rules, and Formulas conveniently arranged for reference, and the answers to the Examination Questions. Formerly, the Tables and Formulas were printed in a separate volume, making six volumes in the set instead of five, as at present; but on account of the extreme thinness of the volume, it was decided to combine it with the Key Volume under one cover.

These Instruction Papers are written from a practical standpoint, and contain only such information as the student requires in order to obtain a good working knowledge of the subjects which form the groundwork of a course in Plumbing, Heating, and Ventilation. There is no padding, and the student gets in a clear and concise form the exact information which he desires.

To keep the student always interested in his work, we do not give him half a dozen ways of doing the same thing; neither do we enter into speculative discussions of different subjects, all of which tend to confuse the student and leave him in doubt as to which one (or any one) is correct; we give him the best or most suitable rule, formula, or method that we know of, without mentioning any of the others. In short, the papers are written for practical men, and all the difficulties encountered by a student studying by himself, particularly those which are due to a definition or explanation being too technical, abstract, or not clear enough to be readily understood, have been carefully considered and overcome.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

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ARITHMETIC.

DEFINITIONS.

1. **Arithmetic** is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* hammer, *one* bolt, *one* dozen.

3. A **number** is a unit, or a collection of units, as *one*, *three* pipes, *five* tees.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* elbows is *one* elbow.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as *three tees*, *five elbows*, *ten pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers which express units of the *same kind*, as six *wall hooks* and ten *wall hooks*, two *joints* and five *joints*.

8. **Unlike numbers** are numbers which express units of *different kinds*, as ten *joints* and eight *pipe strips*, seven *dollars* and five *tubes*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1.) By words. (2.) By figures. (3.) By letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading the numbers which have been expressed by figures or letters.

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12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher or zero.*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher** or **zero,** and, when standing alone, has no value.

The other nine figures are called **digits,** and each one has a value of its own.

Any whole number is called an **integer.**

13. As there are only *ten figures* used in expressing numbers, *each figure* must express a different value at different times.

14. The value of a figure depends upon its *position* in relation to others.

15. Figures have **simple** values and **local** or **relative** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local** or **relative** value is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus. 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it, thus. 66

The original figure is still *six units*, but the second one is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions, six hundred sixty-six thousands, six hundred sixty-six.*

18. The **increased value** of each of these figures is its *local* or *relative* value. Each figure is *ten times* greater in value than the one immediately on its *right*.

19. The **cipher** (0) has no value itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred*, and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens*. To mean 4 *hundreds*, the 4 should have two figures on its right, and a cipher is therefore inserted in the place usually given to tens, to show that the number is composed of *hundreds* and *units* only, and that there are no tens. *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the cipher at the right-hand side to show that there were no units, and only *hundreds* and *tens;* thus, 450. *Four thousand and fifty* would be expressed 4050, the first cipher indicating that there are no hundreds and the second that there are no units.

NOTE.—When speaking of the figures of a number by referring to them as first figure, second figure, etc., always begin to count at the *left*. Thus, in the number 41,625, 4 is the first figure, 6 the third figure, 5 the fifth or last figure, etc.

20. In *reading* figures, it is usual to *point off* the number into groups of three figures each, beginning with the right-hand or **units** column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2 ,	1	9	8 ,	7	6	5 ,	4	3	2

In pointing off these figures, begin at the right-hand figure and count—*units*, *tens*, *hundreds*; the next group of three figures is *thousands*, therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma; we next read *millions*, *tens of millions*, *hundreds of millions*, and insert another comma; we then read *billions*, *tens of billions*, *hundreds of billions*.

The entire line of figures would be read: *Four hundred thirty-two billions, one hundred ninety-eight millions, seven hundred sixty-five thousands, four hundred thirty-two.* When we thus *read* a line of figures it is called **numeration,** and if the **numeration** be changed back to *figures*, it is called **notation.**

For instance, the writing of the figures

72,584,623

would be the **notation,** and the **numeration** would be *seventy-two millions, five hundred eighty-four thousands, six hundred twenty-three.*

21. NOTE.—It is customary to leave the *s* off the words millions, thousands, etc., in cases like the above, both in speaking and writing; hence, the above would usually be expressed, seventy-two million, five hundred eighty-four thousand, six hundred twenty-three.

22. The four fundamental processes of Arithmetic are **addition, subtraction, multiplication,** and **division.** They are called fundamental processes, because all operations in Arithmetic are based upon them.

ADDITION.

23. **Addition** is the process of finding the sum of two or more numbers. The sign of addition is +. It is read *plus*, and means *more*. Thus, 5 + 6 is read *5 plus 6*, and means that 5 and 6 are to be added.

24. The sign of equality is =. It is read *equals* or *is equal to*. Thus, 5 + 6 = 11 may be read *5 plus 6 equals 11*.

25. *Like numbers* can be added; *unlike numbers* can not be added. Thus, 6 dollars *can* be added to 7 dollars, and the sum will be 13 dollars; but 6 dollars *can not* be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12:

TABLE 1.

1 and 1 are 2	2 and 1 are 3	3 and 1 are 4	4 and 1 are 5
1 and 2 are 3	2 and 2 are 4	3 and 2 are 5	4 and 2 are 6
1 and 3 are 4	2 and 3 are 5	3 and 3 are 6	4 and 3 are 7
1 and 4 are 5	2 and 4 are 6	3 and 4 are 7	4 and 4 are 8
1 and 5 are 6	2 and 5 are 7	3 and 5 are 8	4 and 5 are 9
1 and 6 are 7	2 and 6 are 8	3 and 6 are 9	4 and 6 are 10
1 and 7 are 8	2 and 7 are 9	3 and 7 are 10	4 and 7 are 11
1 and 8 are 9	2 and 8 are 10	3 and 8 are 11	4 and 8 are 12
1 and 9 are 10	2 and 9 are 11	3 and 9 are 12	4 and 9 are 13
1 and 10 are 11	2 and 10 are 12	3 and 10 are 13	4 and 10 are 14
1 and 11 are 12	2 and 11 are 13	3 and 11 are 14	4 and 11 are 15
1 and 12 are 13	2 and 12 are 14	3 and 12 are 15	4 and 12 are 16
5 and 1 are 6	6 and 1 are 7	7 and 1 are 8	8 and 1 are 9
5 and 2 are 7	6 and 2 are 8	7 and 2 are 9	8 and 2 are 10
5 and 3 are 8	6 and 3 are 9	7 and 3 are 10	8 and 3 are 11
5 and 4 are 9	6 and 4 are 10	7 and 4 are 11	8 and 4 are 12
5 and 5 are 10	6 and 5 are 11	7 and 5 are 12	8 and 5 are 13
5 and 6 are 11	6 and 6 are 12	7 and 6 are 13	8 and 6 are 14
5 and 7 are 12	6 and 7 are 13	7 and 7 are 14	8 and 7 are 15
5 and 8 are 13	6 and 8 are 14	7 and 8 are 15	8 and 8 are 16
5 and 9 are 14	6 and 9 are 15	7 and 9 are 16	8 and 9 are 17
5 and 10 are 15	6 and 10 are 16	7 and 10 are 17	8 and 10 are 18
5 and 11 are 16	6 and 11 are 17	7 and 11 are 18	8 and 11 are 19
5 and 12 are 17	6 and 12 are 18	7 and 12 are 19	8 and 12 are 20
9 and 1 are 10	10 and 1 are 11	11 and 1 are 12	12 and 1 are 13
9 and 2 are 11	10 and 2 are 12	11 and 2 are 13	12 and 2 are 14
9 and 3 are 12	10 and 3 are 13	11 and 3 are 14	12 and 3 are 15
9 and 4 are 13	10 and 4 are 14	11 and 4 are 15	12 and 4 are 16
9 and 5 are 14	10 and 5 are 15	11 and 5 are 16	12 and 5 are 17
9 and 6 are 15	10 and 6 are 16	11 and 6 are 17	12 and 6 are 18
9 and 7 are 16	10 and 7 are 17	11 and 7 are 18	12 and 7 are 19
9 and 8 are 17	10 and 8 are 18	11 and 8 are 19	12 and 8 are 20
9 and 9 are 18	10 and 9 are 19	11 and 9 are 20	12 and 9 are 21
9 and 10 are 19	10 and 10 are 20	11 and 10 are 21	12 and 10 are 22
9 and 11 are 20	10 and 11 are 21	11 and 11 are 22	12 and 11 are 23
9 and 12 are 21	10 and 12 are 22	11 and 12 are 23	12 and 12 are 24

This table should be carefully committed to memory. Since 0 has no value, the sum of any number and 0 is the number itself; thus, 17 and 0 are 17.

27. For *addition*, place the numbers to be added directly under each other, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure*

of the *number above it*, thus bringing the unit figures of all the numbers to be added in the same vertical line. Proceed as in the following examples:

28. EXAMPLE.—What is the sum of 131, 222, 21, 2, and 413?

SOLUTION.—

131
222
21
2
413
sum 789 Ans.

EXPLANATION.—After placing the numbers in proper order, begin at the bottom of the right-hand or *units* column, and add, mentally repeating the different sums. Thus, three and two are five and one are six and two are eight and one are nine, the sum of the numbers in *units* column. Place the 9 directly beneath as the first or *units* figure in the sum.

The sum of the numbers in the next or *tens* column equals 8 *tens*, which is the second or *tens* figure in the sum.

The sum of the numbers in the next or *hundreds* column equals 7 *hundreds*, which is the third or *hundreds* figure in the sum.

The sum or answer is 789.

29. EXAMPLE.—What is the sum of 425, 36, 9,215, 4, and 907?

SOLUTION.—

425
36
9215
4
907
27
60
1500
9000
sum 10587 Ans.

EXPLANATION.—The sum of the numbers in the first or units column is seven and four are eleven and five are sixteen and six are twenty-two and five are twenty-seven, or 27 units; i. e., two tens and seven units. Write 27 as shown.

The sum of the numbers in the second or tens column is six tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third or hundreds column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth or thousands column, 9, which represents 9,000. Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

NOTE.—It frequently happens, when adding a long column of figures, that the sum of two numbers, one of which does not occur in the addition table, is required. Thus, in the first column above, the sum of 16 and 6 was required. We know from the table that $6 + 6 = 12$; hence, the first figure of the sum is 2. Now, the sum of any number less than 20 and of any number less than 10 must be less than 30, since $20 + 10 = 30$; therefore, the sum is 22. Consequently, in cases of this kind, add the first figure of the larger number to the smaller number, and, if the result is greater than 9, increase the second figure of the larger number by 1. Thus, $44 + 7 = ?$ $4 + 7 = 11$; hence, $44 + 7 = 51$.

30. The addition may also be performed as follows:

425
36
9215
4
907

sum 10587 Ans.

EXPLANATION.—The sum of the numbers in units column is 27 units, or 2 tens and 7 units. Write the 7 units as the first or right-hand figure in the sum. Reserve the two tens and add them to the figures in tens column. The sum of the figures in the tens column plus the 2 tens reserved and carried from the units column is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third or hundreds figure in the sum and carry the 1 to the next column. $1 + 9 = 10$, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below:

SOLUTION.—

	890
	82
	90
	393
	281
	80
	770
	83
	492
	80
	383
	84
	191
sum	3899 Ans.

EXPLANATION.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write down the 9 and carry 1 to the next column. The sum of the digits in the second column + 1 is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 reserved is 38.

The entire sum is 3,899. Ans.

32. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column, and add the remaining figure or figures to the next column.*

33. Proof.—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of

(*a*) 104 + 203 + 613 + 214.
(*b*) 1,875 + 3,143 + 5,826 + 10,832.
(*c*) 4,865 + 2,145 + 8,173 + 40,084.
(*d*) 14,204 + 8,173 + 1,065 + 10,042.

Ans. (*a*) 1,134. (*b*) 21,676. (*c*) 55,267. (*d*) 33,484.

(*e*) 10,832 + 4,145 + 3,133 + 5,872.
(*f*) 214 + 1,231 + 141 + 5,000.
(*g*) 123 + 104 + 425 + 126 + 327.
(*h*) 6,354 + 2,145 + 2,042 + 1,111 + 3,333.

Ans. (*e*) 23,982. (*f*) 6,586. (*g*) 1,105. (*h*) 14,985.

1. A building has 104 fittings on the fourth floor, 203 on the third, 613 on the second, and 214 on the first. Required, the total number of fittings in the building. Ans. 1,134 fittings.

2. In a building there are 6 water tanks, containing 1,875, 3,143, 5,826, 10,832, 106, and 94 gallons, respectively. How many gallons of water are contained in the six tanks? Ans. 21,876 gal.

SUBTRACTION.

35. In Arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference** or **remainder.**

36. The sign of subtraction is − . It is read **minus,** and means *less.* Thus, 12 − 7 is read *12 minus 7,* and means that 7 is to be taken from 12.

37. EXAMPLE.—From 7,568 take 3,425.

SOLUTION.—

minuend 7568
subtrahend 3425
remainder 4143 Ans.

EXPLANATION.—Begin at the right-hand or *units* column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the *minuend* than in the *subtrahend,* and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

EXAMPLE.—From 8,453 take 844.

SOLUTION.—

minuend	8453
subtrahend	844
remainder	7609 Ans.

EXPLANATION.—Begin to subtract at the right-hand *units* column. We can not take 4 from 3, and must, therefore, borrow 1 from 5 in *tens* column and annex it to the 3 *units* column. The 1 *ten* = 10 *units*, which added to the 3 in *units* column = 13 *units*. 4 from 13 = 9, the first or *units* figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second or *tens* figure. We can not take 8 from 4, and must, therefore, borrow 1 from 8 in *thousands* column. Since 1 *thousand* = 10 *hundreds*, 10 *hundreds* + 4 *hundreds* = 14 *hundreds*, and 8 from 14 = 6, the third or *hundreds* figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder or answer.

The operation of borrowing is performed by mentally placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend	10000
subtrahend	8763
remainder	1237 Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains, from which to subtract 6; 6 from 9 = 3. For the

same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. Rule.—*Place the subtrahend (or smaller number) under the minuend or larger number, in the same manner as for addition, and proceed as in Arts.* ***37, 38,*** *and* ***39.***

41. Proof.—*To prove an example in subtraction, add the remainder to the subtrahend. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8763
remainder	1237
minuend	10000 Ans.

EXAMPLES FOR PRACTICE.

42. From

(*a*) 94,278 take 62,574.	Ans.	(*a*) 31,704.
(*b*) 53,714 take 25,824.		(*b*) 27,890.
(*c*) 71,832 take 58,109.		(*c*) 13,723.
(*d*) 20,804 take 10,408.		(*d*) 10,396.
(*e*) 310,465 take 102,141.		(*e*) 208,324.
(*f*) (81,043 + 1,041) take 14,831.		(*f*) 67,253.
(*g*) (20,482 + 18,216) take 21,214.		(*g*) 17,484.
(*h*) (2,040 + 1,213 + 542) take 3,791.		(*h*) 4.

1. If out of 1,037 pounds of pipe fittings 259 pounds remained after completing a piece of work, how many pounds were used? Ans. 778 lb.

2. If out of 3,849 feet of pipe taken for a piece of work about a building, 1,034 feet were used on the third floor, 864 feet on the second floor, 297 feet in the cellar, and 1,600 feet underground, how many feet remained? Ans. 54 ft.

3. A pot contains 15 pounds of solder, out of which 7 joints are wiped, each joint requiring 1 pound of solder; a bar of solder weighing 5 pounds is then added, and 7 more joints are wiped. How much solder remains, assuming none to be lost? Ans. 6 lb.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. The sign of multiplication is $\times$. It is read *times* or *multiplied by*. Thus, 9×6 is read *9 times 6*, or *9 multiplied by 6*.

46. It matters not in what order the numbers to be multiplied together are placed. Thus, 6×9 is the same as 9×6.

47. In the following table, the product of any two numbers (neither of which exceeds twelve) may be found·

TABLE 2.

1 times 1 is 1	2 times 1 are 2	3 times 1 are 3
1 times 2 are 2	2 times 2 are 4	3 times 2 are 6
1 times 3 are 3	2 times 3 are 6	3 times 3 are 9
1 times 4 are 4	2 times 4 are 8	3 times 4 are 12
1 times 5 are 5	2 times 5 are 10	3 times 5 are 15
1 times 6 are 6	2 times 6 are 12	3 times 6 are 18
1 times 7 are 7	2 times 7 are 14	3 times 7 are 21
1 times 8 are 8	2 times 8 are 16	3 times 8 are 24
1 times 9 are 9	2 times 9 are 18	3 times 9 are 27
1 times 10 are 10	2 times 10 are 20	3 times 10 are 30
1 times 11 are 11	2 times 11 are 22	3 times 11 are 33
1 times 12 are 12	2 times 12 are 24	3 times 12 are 36
4 times 1 are 4	5 times 1 are 5	6 times 1 are 6
4 times 2 are 8	5 times 2 are 10	6 times 2 are 12
4 times 3 are 12	5 times 3 are 15	6 times 3 are 18
4 times 4 are 16	5 times 4 are 20	6 times 4 are 24
4 times 5 are 20	5 times 5 are 25	6 times 5 are 30
4 times 6 are 24	5 times 6 are 30	6 times 6 are 36
4 times 7 are 28	5 times 7 are 35	6 times 7 are 42
4 times 8 are 32	5 times 8 are 40	6 times 8 are 48
4 times 9 are 36	5 times 9 are 45	6 times 9 are 54
4 times 10 are 40	5 times 10 are 50	6 times 10 are 60
4 times 11 are 44	5 times 11 are 55	6 times 11 are 66
4 times 12 are 48	5 times 12 are 60	6 times 12 are 72
7 times 1 are 7	8 times 1 are 8	9 times 1 are 9
7 times 2 are 14	8 times 2 are 16	9 times 2 are 18
7 times 3 are 21	8 times 3 are 24	9 times 3 are 27
7 times 4 are 28	8 times 4 are 32	9 times 4 are 36
7 times 5 are 35	8 times 5 are 40	9 times 5 are 45
7 times 6 are 42	8 times 6 are 48	9 times 6 are 54
7 times 7 are 49	8 times 7 are 56	9 times 7 are 63
7 times 8 are 56	8 times 8 are 64	9 times 8 are 72
7 times 9 are 63	8 times 9 are 72	9 times 9 are 81
7 times 10 are 70	8 times 10 are 80	9 times 10 are 90
7 times 11 are 77	8 times 11 are 88	9 times 11 are 99
7 times 12 are 84	8 times 12 are 96	9 times 12 are 108
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

This table should be carefully committed to memory.
Since 0 has no value, the product of 0 and any number is 0.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

multiplicand	425
multiplier	5
product	2125 Ans.

EXPLANATION.—For convenience, the *multiplier* is generally written *under* the *right-hand figure* of the *multiplicand.* On looking in the multiplication table, we see that 5×5 are 25. Multiplying the *first figure* at the *right* of the *multiplicand*, or 5, by the *multiplier* 5, it is seen that 5 times 5 units are 25 units, or 2 tens and 5 units. Write the *5 units* in *units* place in the *product*, and *reserve* the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5×2 are 10. Multiplying the *second figure* of the multiplicand by the multiplier 5, we see that 5 times 2 tens are 10 tens, and 10 tens plus the 2 tens reserved, are 12 tens, or 1 hundred plus 2 tens. Write the *2 tens* in *tens* place, and reserve the 1 hundred to add to the product of hundreds. Again, we see by the multiplication table that 5×4 are 20. Multiplying the *third* or *last figure* of the multiplicand by the multiplier 5, we see that 5 times 4 hundreds are 20 hundreds, and 20 hundreds plus the 1 hundred reserved, are 21 hundreds, or 2 thousands plus 1 hundred, which we write in *thousands* and *hundreds* places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

	425
	425
	425
	425
	425
sum	2125 Ans.

EXAMPLES FOR PRACTICE.

49. Find the product of

(*a*) $61{,}483 \times 6$.
(*b*) $12{,}375 \times 5$.
(*c*) $10{,}426 \times 7$.
(*d*) $10{,}835 \times 3$.

Ans. (*a*) 368,898. (*b*) 61,875. (*c*) 72,982. (*d*) 32,505.

(*e*) 98,376 × 4.
(*f*) 10,873 × 8.
(*g*) 71,543 × 9.
(*h*) 218,734 × 2.

Ans. (*e*) 393,504. (*f*) 86,984. (*g*) 643,887. (*h*) 437,468.

1. In a building there are 7 lines of soil pipe, each measuring 125 feet and containing 29 fittings. (*a*) If 210 fittings are allowed in the work, and 4 are split in calking, how many fittings remain? (*b*) Required, the number of feet of pipe, allowing 1 foot for every fitting.

Ans. (*a*) 3 fittings. (*b*) 672 ft.

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50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

SOLUTION.—

multiplicand	475
multiplier	234
	1900
	1425
	950
product	111150 Ans.

EXPLANATION.—For convenience, the multiplier is generally written *under* the multiplicand, placing units under units, tens under tens, etc.

We *can not* multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the first partial product; 3 times 475 = 1,425, the second partial product, the right-hand *figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the third partial product, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply the entire multiplicand by it; since the result will be zero, place a cipher under the cipher in the multiplier. Thus,

(*a*)	(*b*)	(*c*)	(*d*)
0	2	15	708
×0	×0	× 0	× 0
0 Ans.	0 Ans.	0 Ans.	0 Ans.

(*e*)	(*f*)	(*g*)
3114	4008	31264
203	305	1002
9342	20040	62528
62280	120240	3126400
632142 Ans.	1222440 Ans.	31326528 Ans.

In examples (*e*), (*f*), and (*g*), we multiply by 0 as directed above; then multiply by the next figure of the multiplier and place the first figure of the product alongside the 0, as shown.

EXAMPLES FOR PRACTICE.

54. Find the product of

(*a*) 3,842 × 26.
(*b*) 3,716 × 45.
(*c*) 1,817 × 124.
(*d*) 675 × 38.

Ans. (*a*) 99,892. (*b*) 167,220. (*c*) 225,308. (*d*) 25,650.

(*e*)	1,875 × 33.	Ans.	(*e*)	61,875.
(*f*)	4,836 × 47.		(*f*)	227,292.
(*g*)	5,682 × 543.		(*g*)	3,085,326.
(*h*)	3,257 × 246.		(*h*)	801,222.
(*i*)	2,875 × 302.		(*i*)	868,250.
(*j*)	17,819 × 1,004.		(*j*)	17,890,276.
(*k*)	38,674 × 205.		(*k*)	7,928,170.
(*l*)	18,304 × 100.		(*l*)	1,830,400.
(*m*)	7,834 × 10.		(*m*)	78,340.
(*n*)	87,543 × 1,000.		(*n*)	87,543,000.
(*o*)	48,763 × 100.		(*o*)	4,876,300.

1. How many lead tacks will be required to support 157 lines of -inch lead pipe, allowing 129 tacks to each line? Ans. 20,253 tacks.

2. How many 1-inch No. 12 screws will be required, allowing 258 crews to each line? Ans. 40,506 screws.

3. A cubic foot of water weighs 1,000 ounces. What is the total veight in ounces of the water in 3 tanks, containing 35, 60, and 43 cubic eet, respectively, and the combined weight of the tanks and their inings, if the tanks and linings weigh 64,394 ounces? Ans. 202,394 oz.

DIVISION.

55. **Division** is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which shows how many times the *divisor is contained in the dividend* is called the **quotient.**

56. The sign of division is ÷. It is read *divided by.* $54 \div 9$ is read *54 divided by 9.* Another way to write *54 divided by 9* is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$.

In both of these cases 54 is the dividend and 9 is the divisor.

Division is the *reverse* of **multiplication.**

57. **To divide when the divisor consists of but one figure,** proceed as in the following example:

EXAMPLE.—What is the quotient of 875 ÷ 7?

SOLUTION.—

divisor dividend quotient

7)875(125 Ans.
7
17
14
35
35
remainder 0

EXPLANATION.—7 is contained in 8 *hundreds* 1 *hundred* times. Place the one as the first or *left-hand figure* of the quotient. Multiply the divisor 7 by the 1 *hundred* of the quotient, and place the product 7 *hundreds* under the 8 *hundreds* in the dividend, and subtract. Beside the remainder 1, bring down the next or *tens* figure of the quotient, in this case 7, making 17 *tens;* 7 is contained in 17, 2 times. Write the 2 as the *second figure* of the quotient. Multiply the divisor 7 by the 2 in the quotient, and subtract the product from 17. Beside the remainder 3, bring down the next or *units* figure of the dividend, in this case 5, making 35 *units.* 7 is contained in 35, 5 times, which is placed in the quotient. Multiplying the divisor by the last figure of the quotient, 5 times 7 = 35, which subtracted from 35, under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

dividend
divisor 7) 8 1 7 3 5
quotient 1 2 5 Ans.

The mental operation is as follows: 7 is contained in 8, once and 1 remainder; imagine 1 to be placed before 7, making 17; 7 is contained in 17, 2 times and 3 over; imagine 3 to be placed before 5, making 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient 125.

The small figures are placed in the example given to better illustrate the explanation; they are never written when actually performing division in this way.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

SOLUTION.—

divisor *dividend* *quotient*

63) 2 7 0 2 8 2 6 (4 2 9 0 2 Ans.

2 5 2

1 8 2

1 2 6

5 6 8

5 6 7

1 2 6

1 2 6

0

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial, we must find how many times 63 is contained in 270; 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first or left-hand figure in the quotient. Multiply the divisor 63 by 4, and subtract the product 252 from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product 189 is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor, 63, by 2, and subtracting the product 126 from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568; 6 is contained in 56 about 9 times. Multiply the divisor 63 by 9 and subtract the product 567 from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times without a remainder. Therefore, 42,902 is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right of the dividend, with a line between, for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, annexing to the remainder the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be a remainder at last, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor and add the remainder, if there be any, to the product. The result will be the dividend.*

Thus,

divisor dividend quotient

63) 4235 (67 $\frac{14}{63}$ Ans.
378
455
441
14

Proof,

remainder 14
quotient 67
divisor 63
201
402
4221
remainder 14
dividend 4235

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*) 126,498 by 58.
(*b*) 3,207,594 by 767.
(*c*) 11,408,202 by 234.
(*d*) 2,100,315 by 581.
(*e*) 969,936 by 4,008.
(*f*) 7,481,888 by 1,021.
(*g*) 1,525,915 by 5,003.
(*h*) 1,646,301 by 381.

Ans. (*a*) 2,181. (*b*) 4,182. (*c*) 48,753. (*d*) 3,615. (*e*) 242. (*f*) 7,328. (*g*) 305. (*h*) 4,321.

1. A building contains 6 lines of upright cast-iron soil pipes, each line being 105 feet high, and each pipe being 5 feet long. (*a*) How many pipes are required for the necessary work? (*b*) How many pounds of lead are required for calking each joint if 504 pounds are used for the whole? (*c*) How many feet of twisted oakum or spun yarn are required for each joint, if 4 bales, each containing 126 feet, be used altogether?

Ans. (*a*) 126 pipes. (*b*) 4 lb. (*c*) 4 ft.

2. How long a time can a reservoir containing 9,432,605 gallons of water supply a town whose daily water consumption is 269,503 gallons, the reservoir not being replenished? Ans. 35 days.

CANCELATION.

63. **Cancelation** is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers which, when *multiplied* together, *equal the given number*. Thus, 5 and 3 are factors of 15, since $5 \times 3 = 15$. Likewise, 8 and 7 are the factors of 56, since $8 \times 7 = 56$.

65. A **prime number** is one which can not be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc., are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, $2 \times 2 \times 3 \times 5$.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to each other, so also are 22, 25, and 21 — all composite numbers.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

Canceling a factor in both dividend and divisor is the same as *dividing them both by the same number*, and this, evidently, *does not change the quotient.*

Write the numbers forming the *dividend* above a horizontal line, and those forming the *divisor* below it; then cancel the equal factors.

68. EXAMPLE.—Divide $4 \times 45 \times 60$ by 9×24.

SOLUTION.—Placing the dividend over the divisor, and canceling

$$\frac{\overset{5}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{\underset{6}{\cancel{9}} \times \underset{1}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the four and write the 1 over it; also, cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =.$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also, cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{1}{\cancel{24}}} =.$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also, cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times 50}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{21}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together, and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times 50}{\underset{1}{\cancel{9}} \times \underset{1}{\cancel{21}}} = \frac{1 \times 5 \times 10}{1 \times 1} = 50. \quad \text{Ans.}$$

It is usual to omit the 1's when canceling them, instead of writing them as above.

69. Rule.—I. *Cancel the common factors from both the dividend and divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide

(*a*) $14 \times 18 \times 16 \times 40$ by $7 \times 8 \times 6 \times 5 \times 3$.
(*b*) $3 \times 65 \times 50 \times 100 \times 60$ by $30 \times 60 \times 13 \times 10$.
(*c*) $8 \times 4 \times 3 \times 9 \times 11$ by $11 \times 9 \times 4 \times 3 \times 8$.
(*d*) $164 \times 321 \times 6 \times 7 \times 4$ by $82 \times 321 \times 7$.
(*e*) $50 \times 100 \times 200 \times 72$ by $1{,}000 \times 144 \times 100$.
(*f*) $48 \times 63 \times 55 \times 49$ by $7 \times 21 \times 11 \times 48$.
(*g*) $110 \times 150 \times 84 \times 32$ by $11 \times 15 \times 100 \times 64$.
(*h*) $115 \times 120 \times 400 \times 1{,}000$ by $23 \times 1{,}000 \times 60 \times 800$.

Ans. (*a*) 32. (*b*) 250. (*c*) 1. (*d*) 48. (*e*) 5. (*f*) 105. (*g*) 42. (*h*) 5.

FRACTIONS.

71. A **fraction** is a part of a whole number: *One-half, one-third, two-fifths* are fractions.

72. *Two* numbers are required to express a fraction; one is called the **numerator,** and the other the **denominator.**

73. The numerator is placed above the denominator, with a line between them; as, $\frac{2}{3}$. 3 is the *denominator*, and shows into how many *equal parts* the *unit* or *one* is divided. The *numerator* 2 shows how many of these equal parts are taken or considered. The denominator also indicates the names of the parts.

$\frac{1}{2}$ is read one-half.
$\frac{3}{4}$ is read three-fourths.
$\frac{3}{8}$ is read three-eighths.
$\frac{5}{16}$ is read five-sixteenths.
$\frac{29}{47}$ is read twenty-nine-forty-sevenths.

74. In the expression "$\frac{3}{4}$ of an apple," the denominator 4 shows that the apple is to be (or has been) cut into 4 *equal parts*, and the numerator 3 shows that *three of these parts*, or *fourths*, are taken or considered.

If each of the parts, or fourths, of the apple were cut in *two equal pieces*, there would then be twice as many pieces as before, or $4 \times 2 = 8$ pieces in all; one of these pieces would be called one-eighth, and would be expressed in figures as $\frac{1}{8}$. Three of these pieces would be called three-eighths, and written $\frac{3}{8}$. The words three-fourths, three-eighths, five-sixteenths, etc., are abbreviations of three one-fourths, three one-eighths, five one-sixteenths, etc. It is evident that the *larger the denominator*, the greater is the number of parts into which anything is divided; consequently, the parts themselves are smaller, and the value of the fraction is less for the same number of parts taken. In other words, $\frac{7}{9}$, for example, is smaller than $\frac{7}{8}$, because if an object be divided into 9 parts, the parts are smaller than if the same object had been divided into 8 parts; and, since $\frac{1}{9}$ is smaller than $\frac{1}{8}$

it is clear that 7 one-ninths is a smaller amount than 7 one-eighths. Hence, also, $\frac{7}{9}$ is less than $\frac{7}{8}$.

75. The **value** of a fraction is the numerator divided by the denominator; as, $\frac{4}{2} = 2$, $\frac{6}{2} = 3$.

76. The line between the numerator and denominator means *divided by*, or $\div$.

$\frac{3}{4}$ is equivalent to $3 \div 4$.

$\frac{5}{8}$ is equivalent to $5 \div 8$.

77. The numerator and denominator of a fraction, when considered together, are called the **terms** of a fraction.

78. The *value* of a fraction whose numerator and denominator are equal is 1.

$\frac{4}{4}$, or four-fourths, $= 1$.

$\frac{8}{8}$, or eight-eighths, $= 1$.

$\frac{64}{64}$, or sixty-four-sixty-fourths, $= 1$.

79. A **proper fraction** is a fraction whose numerator is *less* than its denominator. Its value is *less* than 1; as, $\frac{3}{4}$, $\frac{5}{8}$, $\frac{1}{16}$.

80. An **improper fraction** is a fraction whose numerator *equals* or is *greater* than the denominator. Its value is 1 or *more* than 1; as, $\frac{4}{4}$, $\frac{9}{8}$, $\frac{42}{32}$.

81. A **mixed number** is a whole number and a fraction united. $4\frac{2}{3}$ is a mixed number, and is equivalent to $4 + \frac{2}{3}$. It is read *four and two-thirds.*

REDUCTION OF FRACTIONS.

82. **Reduction of fractions** is the process of changing their form without changing their value.

83. A *fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The value is not changed, since $\frac{3}{4} = \frac{6}{8}$. For, suppose that an object, say an apple, is divided into 8 equal parts. If

these parts be arranged into 4 piles, each containing $\frac{1}{8}$ it is evident that each pile will be composed of th amount of the entire apple as would have been the ca the apple been originally cut into 4 equal parts. I one of these piles (containing 2 parts) be removed will be 3 piles left, each containing 2 equal parts, or parts in all, i. e., six-eighths. But, since one pile, quarter, was removed, there are three-quarters left. $\frac{3}{4} = \frac{6}{8}$. The same course of reasoning may be app any similar case. Therefore, multiplying both ter fraction by the same number does not alter its value.

84. To reduce a fraction to an equal fr having a given denominator:

EXAMPLE.—Reduce $\frac{7}{8}$ to an equal fraction having 96 for a nator.

SOLUTION.—Both the numerator and the denominator must b plied by the same number in order not to change the value of tion. The denominator must be multiplied by some numbe will, in this case, make the product 96; this number is eviden $8 = 12$, since $8 \times 12 = 96$. Hence, $\frac{7}{8} \times \frac{12}{12} = \frac{84}{96}$. Ans.

85. Rule.—*Divide the given denominator by the d nator of the given fraction, and multiply both terms fraction by the result.*

EXAMPLE.—Reduce $\frac{3}{4}$ to 100ths.

SOLUTION.— $100 \div 4 = 25$; hence, $\frac{3}{4} \times \frac{25}{25} = \frac{75}{100}$. Ans.

86. *A fraction is reduced to lower terms by dividi terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ viding both terms by 2.

$$\frac{8}{10} \div \frac{2}{2} = \frac{4}{5}.$$

That $\frac{8}{10} = \frac{4}{5}$ is readily seen from the explanation gi Art. **83;** for, multiplying both terms of the fraction $\frac{4}{5} \times \frac{2}{2} = \frac{8}{10}$, and, if $\frac{4}{5} = \frac{8}{10}$, $\frac{8}{10}$ must equal $\frac{4}{5}$. Hence, *di both terms of a fraction by the same number does not a value.*

87. A fraction is reduced to its *lowest terms* when merator and denominator can not both be divided by th

number without a remainder; as, $\frac{3}{4}$, $\frac{8}{5}$, $\frac{11}{24}$, $\frac{8}{15}$. In other words, the numerator and denominator are prime to each other.

EXAMPLES FOR PRACTICE.

88. Reduce the following:

(*a*) $\frac{7}{16}$ to 128ths.
(*b*) $\frac{84}{132}$ to its lowest terms.
(*c*) $\frac{64}{1000}$ to its lowest terms.
(*d*) $\frac{5}{7}$ to 49ths.
(*e*) $\frac{13}{16}$ to 10,000ths.

Ans. (*a*) $\frac{56}{128}$. (*b*) $\frac{7}{11}$. (*c*) $\frac{8}{125}$. (*d*) $\frac{35}{49}$. (*e*) $\frac{8125}{10000}$.

89. To reduce a whole number or mixed number to an improper fraction:

EXAMPLE.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.—$8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

90. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result. If it is desired to reduce a whole number to a fraction, multiply the whole number by the denominator of the given fraction, and write the result over the denominator.*

EXAMPLES FOR PRACTICE.

91. Reduce to improper fractions:

(*a*) $4\frac{1}{8}$.
(*b*) $5\frac{4}{9}$.
(*c*) $10\frac{2}{10}$.
(*d*) $37\frac{3}{4}$.
(*e*) $50\frac{4}{5}$.
(*f*) Reduce 7 to a fraction whose denominator is 16.

Ans. (*a*) $\frac{33}{8}$. (*b*) $\frac{49}{9}$. (*c*) $\frac{102}{10}$. (*d*) $\frac{151}{4}$. (*e*) $\frac{254}{5}$. (*f*) $\frac{112}{16}$.

92. To reduce an improper fraction to a whole or mixed number:

EXAMPLE.—Reduce $\frac{21}{4}$ to a mixed number.

SOLUTION.—4 is contained in 21, 5 times and 1 remaining (see Art. 75); as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

93. Rule.—*Divide the* **numerator** *by the* **denominator**; *the quotient will be* **the whole number**; *the* **remainder**, if there be any, will be *the* **numerator** *of the fractional part* of which *the* **denominator** *is the same as the* **denominator** *of the improper fraction.*

EXAMPLES FOR PRACTICE.

94. Reduce to whole or mixed numbers:

(*a*) $\frac{1445}{5}$.
(*b*) $\frac{1855}{8}$.
(*c*) $\frac{1021}{8}$.
(*d*) $\frac{1492}{8}$.
(*e*) $\frac{784}{196}$.
(*f*) $\frac{1225}{245}$.

Ans. (*a*) $289\frac{1}{5}$. (*b*) $231\frac{7}{8}$. (*c*) $127\frac{5}{8}$. (*d*) $186\frac{1}{2}$. (*e*) 4. (*f*) 5.

95. A **common denominator** of *two* or *more fractions* is a number which will contain (i. e., which may be divided by) the denominator of each of the given fractions without a remainder. The **least common denominator** is the least number that will contain each denominator of the given fractions without a remainder.

96. To find the least common denominator:

EXAMPLE.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

SOLUTION.—We first place the denominators in a row, separated by commas.

2)	4,	3,	9,	16
2)	2,	3,	9,	8
3)	1,	3,	9,	4
	1,	1,	3,	4

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. **Ans.**

EXPLANATION.—Divide the numbers by some prime number that will divide at least two of them without a remainder (if possible), bringing down to the row below those denominators which will not contain the divisor without a remainder. Dividing each of the numbers by 2, the second row becomes 2, 3, 9, 8, since 2 will not divide 3 and 9 without a remainder. Dividing again by 2, the result is 1, 3, 9, 4.

Dividing the third row by 3, the result is 1, 1, 3, 4. Since the remaining numbers are prime to each other, we cease dividing further. The product of all the divisors and of the numbers prime to each other, is $2\times2\times3\times3\times4=144$, which is the required least common denominator.

97. EXAMPLE.—Find the least common denominator of $\frac{1}{9}$, $\frac{5}{12}$, $\frac{7}{18}$.

SOLUTION.—

$$\begin{array}{r} 3\,)\,9,\ 12,\ 18 \\ \hline 3\,)\,3,\ 4,\ 6 \\ \hline 2\,)\,1,\ 4,\ 2 \\ \hline 1,\ 2,\ 1 \end{array}$$

$3\times3\times2\times2=36$. Ans.

98. To reduce two or more fractions to fractions having a common denominator:

EXAMPLE.—Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{2}$ to fractions having a common denominator.

SOLUTION.—The common denominator is any number which will contain 3, 4, and 2. The *least* common denominator is 12, because it is the smallest number which can be divided by 3, 4, and 2 without a remainder. $\frac{2}{3}=\frac{8}{12}$, $\frac{3}{4}=\frac{9}{12}$, $\frac{1}{2}=\frac{6}{12}$.

Reducing $\frac{2}{3}$ (see Art. **84**), 3 is contained in 12, 4 times. By multiplying both numerator and denominator of $\frac{2}{3}$ by 4, we find

$\frac{2\times4}{3\times4}=\frac{8}{12}$. In the same way we find $\frac{3}{4}=\frac{9}{12}$ and $\frac{1}{2}=\frac{6}{12}$.

99. Rule. — *Divide the common denominator by the denominator of the given fraction, and multiply both terms of the fraction by the quotient.*

EXAMPLES FOR PRACTICE.

100. Reduce to fractions having a common denominator:

(*a*) $\frac{3}{4}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{9}{16}$, $\frac{3}{4}$, $\frac{7}{32}$.
(*c*) $\frac{7}{8}$, $\frac{7}{88}$, $\frac{10}{11}$.
(*d*) $\frac{3}{5}$, $\frac{5}{8}$, $\frac{11}{40}$.
(*e*) $\frac{4}{10}$, $\frac{6}{40}$, $\frac{9}{20}$.
(*f*) $\frac{7}{16}$, $\frac{17}{80}$, $\frac{21}{80}$.

Ans.
(*a*) $\frac{6}{8}$, $\frac{5}{8}$, $\frac{7}{8}$.
(*b*) $\frac{9}{32}$, $\frac{24}{32}$, $\frac{7}{32}$.
(*c*) $\frac{77}{88}$, $\frac{7}{88}$, $\frac{80}{88}$.
(*d*) $\frac{24}{40}$, $\frac{25}{40}$, $\frac{11}{40}$.
(*e*) $\frac{16}{40}$, $\frac{6}{40}$, $\frac{18}{40}$.
(*f*) $\frac{14}{80}$, $\frac{17}{80}$, $\frac{21}{80}$.

ADDITION OF FRACTIONS.

101. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{1}{4}$ to $\frac{1}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

Suppose we divide an apple into 4 equal parts, and then divide 2 of these parts into two equal parts. It is evident that we shall have 2 one-fourths and 4 one-eighths. Now, if we add these parts, the result is $2 + 4 = 6$ something. But what is this something? It is not fourths, for six fourths are $1\frac{1}{2}$, and we had only 1 apple to begin with; neither is it eighths, for six eighths are $\frac{3}{4}$, which is less than 1 apple. By reducing the quarters to eighths, we have $\frac{2}{4} = \frac{4}{8}$, and adding the other 4 eighths, $4 + 4 = 8$ eighths. This result is correct, since $\frac{8}{8} = 1$. Or, we can, in this case, reduce the eighths to quarters. Thus, $\frac{4}{8} = \frac{2}{4}$; whence, adding $2 + 2 = 4$ quarters, a correct result since $\frac{4}{4} = 1$.

Before adding, fractions should be reduced to a common denominator, preferably the *least* common denominator.

102. EXAMPLE.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

SOLUTION.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\tfrac{1}{2} = \tfrac{4}{8},\ \tfrac{3}{4} = \tfrac{6}{8},\ \tfrac{5}{8} = \tfrac{5}{8}.$$

EXPLANATION.—As the denominator tells or indicates the *names* of the parts, the numerators only are added in order to obtain the total number of parts indicated by the denominator. Thus, 4 one-eighths plus 6 one-eighths plus 5 one-eighths =

$$\tfrac{4}{8} + \tfrac{6}{8} + \tfrac{5}{8} = \tfrac{4+6+5}{8} = \tfrac{15}{8} = 1\tfrac{7}{8}. \quad \text{Ans.}$$

103. EXAMPLE.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

SOLUTION.—The least common denominator in this case is 16.

$$\begin{aligned} 12\tfrac{3}{4} &= 12\tfrac{12}{16} \\ 14\tfrac{5}{8} &= 14\tfrac{10}{16} \\ 7\tfrac{5}{16} &= 7\tfrac{5}{16} \\ \hline \textit{sum} &= 33 + \tfrac{27}{16} = 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}. \quad \text{Ans.} \end{aligned}$$

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

EXAMPLE.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{2}$?

SOLUTION.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$ $3\frac{1}{2} = 3\frac{16}{32}$.

$$\begin{array}{r} 17 \\ 13\frac{6}{32} \\ \frac{9}{32} \\ 3\frac{16}{32} \\ \hline \textit{sum}\ 33\frac{31}{32} \end{array}$$ Ans.

104. Rule.—I. *Reduce the given fractions to fractions having the least common denominator, and write the sum of the numerators over the common denominator.*

II. *When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number, and add the whole number with the other whole numbers.*

EXAMPLES FOR PRACTICE.

105. Find the sum of

(*a*) $\frac{4}{5}$, $\frac{7}{24}$, $\frac{5}{8}$.
(*b*) $\frac{3}{5}$, $\frac{5}{16}$, $\frac{24}{48}$.
(*c*) $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.
(*d*) $\frac{5}{6}$, $\frac{11}{12}$, $\frac{13}{18}$.
(*e*) $\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.
(*f*) $\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.
(*g*) $\frac{4}{11}$, $\frac{7}{22}$, $\frac{14}{22}$.
(*h*) $\frac{3}{7}$, $\frac{14}{49}$, $\frac{3}{7}$.

Ans. (*a*) $1\frac{7}{12}$. (*b*) $1\frac{3}{16}$. (*c*) $1\frac{3}{16}$. (*d*) $1\frac{11}{12}$. (*e*) $1\frac{23}{66}$. (*f*) $1\frac{5}{9}$. (*g*) $1\frac{7}{22}$. (*h*) 1.

1. A piece of lead pipe work is made up of the following lengths: $39\frac{3}{4}$, $46\frac{1}{4}$, $3\frac{1}{8}$, $9\frac{13}{16}$, and $12\frac{1}{8}$ inches, respectively; also, there are 6 wipe joints, the solder on each weighing $1\frac{1}{16}$ pounds, $1\frac{31}{32}$ pounds, $\frac{9}{16}$ of a pound, $1\frac{1}{2}$ pounds, $1\frac{15}{32}$ pounds, and $\frac{31}{64}$ of a pound, respectively. (*a*) How many inches of pipe are required for the piece of work? (*b*) How many pounds of solder?

Ans. (*a*) $110\frac{47}{64}$ in. (*b*) $6\frac{31}{64}$ lb.

2. What is the capacity of a tank that will hold $9\frac{1}{2}$, $37\frac{1}{16}$, $41\frac{3}{32}$, $65\frac{5}{16}$, and $103\frac{5}{64}$ gallons? Ans. $255\frac{29}{64}$ gal.

SUBTRACTION OF FRACTIONS.

106. *Fractions can not be subtracted without first reducing them to a common denominator.* This can be shown in the same manner as in the case of addition of fractions.

EXAMPLE.—Subtract $\frac{3}{4}$ from $1\frac{1}{2}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{4} = \frac{12}{16}. \quad 1\frac{1}{2} - \frac{6}{16} = \frac{18-6}{16} = \frac{12}{16} \quad \text{Ans.}$$

107. EXAMPLE.—From 7 take $\frac{3}{8}$.

SOLUTION.—$1 = \frac{8}{8}$; therefore, since $7 = 6 + 1$, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$. $6\frac{8}{8} - \frac{3}{8} = 6\frac{5}{8}$. Ans.

108. EXAMPLE.—What is the difference between $17\frac{7}{16}$ and $9\frac{11}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{7}{16} = 17\frac{14}{32}$.

minuend	$17\frac{14}{32}$
subtrahend	$9\frac{11}{32}$
difference	$8\frac{3}{32}$ Ans.

109. EXAMPLE.—From $9\frac{1}{4}$ take $4\frac{7}{16}$.

SOLUTION.—The common denominator of the fractions is 16. $9\frac{1}{4} = 9\frac{4}{16}$.

minuend	$9\frac{4}{16}$ or	$8\frac{20}{16}$
subtrahend	$4\frac{7}{16}$	$4\frac{7}{16}$
difference	$4\frac{13}{16}$	$4\frac{13}{16}$ Ans.

EXPLANATION.—As the fraction in the subtrahend is greater than the fraction in the minuend, it can not be subtracted; therefore, *borrow* 1, or $\frac{16}{16}$, from the 9 in the minuend and add it to the $\frac{4}{16}$; $\frac{4}{16} + \frac{16}{16} = \frac{20}{16}$. $\frac{7}{16}$ from $\frac{20}{16} = \frac{13}{16}$. Since 1 was borrowed from 9, 8 remains; 4 from $8 = 4$; $4 + \frac{13}{16} = 4\frac{13}{16}$.

110. EXAMPLE.—From 9 take $8\frac{5}{16}$.

SOLUTION.—

minuend	9 or	$8\frac{16}{16}$
subtrahend	$8\frac{5}{16}$	$8\frac{5}{16}$
difference	$\frac{11}{16}$	$\frac{11}{16}$ Ans.

EXPLANATION.—As there is no fraction in the minuend from which to take the fraction in the subtrahend, borrow 1, or $\frac{16}{16}$, from 9. $\frac{5}{16}$ from $\frac{16}{16} = \frac{11}{16}$. Since 1 was borrowed from 9, only 8 is left. 8 from $8 = 0$.

111. Rule.—I. *Reduce the fractions to fractions having a common denominator. Subtract one numerator from the other and place the remainder over the common denominator.*

II. *When there are mixed numbers, subtract the fractions*

and whole numbers separately, and place the remainders side by side.

III. *When the fraction in the subtrahend is greater than the fraction in the minuend, borrow 1 from the whole number in the minuend and add it to the fraction in the minuend, from which subtract the fraction in the subtrahend.*

IV. *When the minuend is a whole number, borrow 1; reduce it to a fraction whose denominator is the same as the denominator of the fraction in the subtrahend, and place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

112. Subtract

(*a*) $\frac{10}{24}$ from $1\frac{1}{3}$.
(*b*) $\frac{7}{14}$ from $1\frac{7}{35}$.
(*c*) $\frac{4}{50}$ from $\frac{5}{10}$.
(*d*) $\frac{1}{35}$ from $\frac{4}{70}$.
(*e*) $\frac{1}{16}$ from $\frac{5}{48}$.
(*f*) $13\frac{1}{4}$ from $30\frac{1}{2}$.
(*g*) $12\frac{1}{8}$ from 27.
(*h*) $5\frac{1}{4}$ from 30.

Ans. (*a*) $\frac{1}{2}$. (*b*) $\frac{5}{35}$. (*c*) $\frac{1}{10}$. (*d*) $\frac{3}{14}$. (*e*) $\frac{1}{2}$. (*f*) $17\frac{1}{2}$. (*g*) $14\frac{7}{8}$. (*h*) $24\frac{3}{4}$.

1. A workman is given the following with which to do a certain piece of work: $9\frac{7}{8}$ feet of $\frac{3}{4}$-inch lead pipe, $6\frac{5}{32}$ feet of 1-inch pipe, $2\frac{1}{16}$ feet of $1\frac{1}{4}$-inch pipe, and $17\frac{1}{2}$ feet of $\frac{1}{2}$-inch pipe. There remained $1\frac{1}{16}$ feet of $\frac{3}{4}$-inch pipe, $\frac{7}{8}$ of a foot of 1-inch pipe, $\frac{1}{8}$ of a foot of $1\frac{1}{4}$-inch pipe, and $3\frac{5}{32}$ feet of $\frac{1}{2}$-inch pipe. How much of each kind did he require to do the work?

Ans. $8\frac{13}{16}$ ft. of $\frac{3}{4}$-inch pipe.
$5\frac{9}{32}$ ft. of 1-inch pipe.
$1\frac{13}{16}$ ft. of $1\frac{1}{4}$-inch pipe.
$14\frac{11}{32}$ ft. of $\frac{1}{2}$-inch pipe.

MULTIPLICATION OF FRACTIONS.

113. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

114. *Multiplying the numerator or dividing the denominator multiplies the fraction.*

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3}{4} \times 4 = \frac{12}{4} = 3$, Ans.

or $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

The word "of" when placed between two fractio
between a fraction and a whole number, means the
as ×, or times. Thus,

$\frac{3}{4}$ of $4 = \frac{3}{4} \times 4 = 3$.
$\frac{1}{8}$ of $\frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}$.

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3}{8} \times 2 = \frac{6}{8} = \frac{3}{4}$. Ans.

or $2 \times \frac{3}{8} = \frac{3}{8} \div 2 = \frac{3}{4}$. Ans.

115. EXAMPLE.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

SOLUTION.— $\frac{4}{16} \times \frac{7}{8} = \frac{4 \times 7}{16 \times 8} = \frac{28}{128} = \frac{7}{32}$. Ans.

or, by cancelation, $\frac{\cancel{4} \times 7}{\cancel{16} \times 8} = \frac{7}{4 \times 8} = \frac{7}{32}$. Ans.
4

116. EXAMPLE.—What is $\frac{3}{8}$ of $\frac{4}{3}$ of $\frac{16}{32}$?

SOLUTION.— $\frac{\cancel{4} \times 3 \times \cancel{16}}{8 \times \cancel{4} \times \cancel{32}} = \frac{3}{8 \times 2} = \frac{3}{16}$. Ans.
2

117. EXAMPLE.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

SOLUTION.—$9\frac{3}{4} = \frac{39}{4}$; $5\frac{5}{8} = \frac{45}{8}$.

$\frac{39}{4} \times \frac{45}{8} = \frac{39}{4} \times \frac{45}{8} = \frac{1755}{32} = 54\frac{27}{32}$. Ans.

118. EXAMPLE.—Multiply $15\frac{7}{8}$ by 3.

SOLUTION.—
$$\begin{array}{r} 15\frac{7}{8} \\ \times\ 3 \\ \hline 47\frac{5}{8} \end{array} \quad \text{or} \quad \begin{array}{r} 15\frac{7}{8} \\ \times\ 3 \\ \hline \end{array}$$
$45 + \frac{21}{8} = 45 + 2\frac{5}{8} = 47\frac{5}{8}$. An

119. Rule.—I. *Divide the product of the numera*
by the product of the denominators. All factors common
the numerators and denominators should first be cast out
cancelation.

II. *To multiply one mixed number by another, red*
them both to improper fractions.

III. *To multiply a mixed number by a whole number, fi*
multiply the fractional part by the multiplier, and if

product is an improper fraction, reduce it to a mixed number, and add the whole number part to the product of the multiplier and whole number.

EXAMPLES FOR PRACTICE.

120. Find the product of

(*a*) $7 \times \frac{3}{15}$.	Ans.	(*a*) $1\frac{8}{15}$.
(*b*) $14 \times \frac{5}{16}$.		(*b*) $4\frac{3}{8}$.
(*c*) $\frac{21}{32} \times \frac{5}{14}$.		(*c*) $\frac{15}{64}$.
(*d*) $\frac{19}{37} \times 4$.		(*d*) $2\frac{19}{37}$.
(*e*) $\frac{13}{15} \times 7$.		(*e*) $7\frac{7}{15}$.
(*f*) $17\frac{8}{21} \times 7$.		(*f*) 125.
(*g*) $\frac{105}{224} \times 32$.		(*g*) 15.
(*h*) $\frac{15}{16} \times 14$.		(*h*) $7\frac{1}{2}$.

1. There were 170 brass tubes, each measuring $2\frac{5}{17}$ inches, cut for a piece of work. How many inches of tubing were required for the work? Ans. 390 in.

2. A pipe discharges water with a uniform velocity at the rate of $73\frac{3}{8}$ gallons per hour. Working $10\frac{1}{2}$ hours per day, how many gallons will be discharged in $9\frac{1}{4}$ days? Ans. $7{,}299\frac{63}{100}$ gal.

DIVISION OF FRACTIONS.

121. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

122. *Dividing the numerator, or multiplying the denominator, divides the fraction.*

EXAMPLE.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing* the *numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \quad \text{Ans.}$$

When *multiplying* the *denominator*, we have

$$\frac{6}{8} \div 3 = \frac{6}{8 \times 3} = \frac{6}{24} = \frac{1}{4}. \quad \text{Ans.}$$

EXAMPLE.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}$. Ans.

EXAMPLE.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\frac{14}{32} \div 7 = \frac{14 \div 7}{32} = \frac{2}{32} = \frac{1}{16}$. Ans.

123. To **invert** a fraction is to *turn it upside down*; that is, make the numerator and denominator change places. Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

124. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{9}{16}$ is contained in $\frac{9}{16}$ **3 times**, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now *invert* the *divisor* $\frac{3}{16}$, and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \underset{1}{\cancel{3}}} = 3. \quad \text{Ans.}$$

This brings the same quotient as in the first case.

125. EXAMPLE.—Divide $\frac{3}{8}$ by $\frac{1}{2}$.

SOLUTION.—We can not divide $\frac{3}{8}$ by $\frac{1}{2}$, as in the first case above, for the denominators are *not* the same, therefore, we must solve as in the second case.

$$\frac{3}{8} \div \frac{1}{2} = \frac{3}{8} \times \frac{2}{1} = \frac{3 \times \overset{1}{\cancel{2}}}{\underset{4}{\cancel{8}} \times 1} = \frac{3}{4} \text{ or } 1\tfrac{1}{2}. \quad \text{Ans.}$$

126. EXAMPLE.—Divide 5 by $1\frac{9}{16}$.

SOLUTION.—$1\frac{9}{16}$ inverted becomes $\frac{16}{25}$.

$$5 \times \frac{16}{25} = \frac{\overset{1}{\cancel{5}} \times \overset{4}{\cancel{16}}}{\cancel{25}} = 3\tfrac{1}{5}. \quad \text{Ans.}$$

127. EXAMPLE.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

SOLUTION.— $3\frac{3}{4} = \frac{15}{4}$; $7\frac{7}{16} = \frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16} \times \frac{4}{15} = \frac{119 \times \overset{1}{\cancel{4}}}{\underset{4}{\cancel{16}} \times 15} = \frac{119}{60} = 1\tfrac{59}{60}. \quad \text{Ans.}$$

128. **Rule.**—*Invert the divisor, and proceed as in multiplication.*

129. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\frac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\frac{3 \times 7}{\frac{8+4}{16}}$ means that 3×7 is to be divided by the value of $\frac{8+4}{16}$.

$\frac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4} \div \frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$, or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line* is to be divided by *all below it*.

EXAMPLES FOR PRACTICE.

130. Divide

(*a*) 15 by $6\frac{3}{4}$.
(*b*) 30 by $\frac{3}{8}$.
(*c*) 172 by $\frac{4}{5}$.
(*d*) $\frac{14}{18}$ by $1\frac{7}{16}$.
(*e*) $\frac{192}{6}$ by $14\frac{3}{8}$.
(*f*) $\frac{1449}{27}$ by $17\frac{1}{9}$.
(*g*) $\frac{14}{18}$ by $\frac{145}{72}$.
(*h*) $\frac{128}{18}$ by $72\frac{1}{4}$.

Ans. (*a*) $2\frac{1}{3}$. (*b*) 40. (*c*) 215. (*d*) $\frac{112}{207}$. (*e*) $1\frac{15}{23}$. (*f*) $\frac{71}{243}$. (*g*) $\frac{56}{145}$. (*h*) $\frac{64}{657}$.

1. A tank containing $1{,}832\frac{2}{3}$ gallons of water supplies a family whose daily consumption is $521\frac{1}{3}$ gallons. How many days' supply does the tank contain? Ans. $3\frac{241}{1563}$ days' supply.

2. A steam pipe when cold measures 135 feet, and when heated measures $135\frac{1}{7}$ feet. What part of its original length does it expand? Ans. $\frac{1}{945}$.

---

131. Whenever an expression like one of the three following is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\frac{\frac{3}{4}}{9} = \frac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4} \times 4}{9 \times 4} = \frac{3}{9 \times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is 1, $\frac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \frac{5}{\frac{3 \times 9}{4}}$; and $\frac{5}{\frac{3 \times 9}{4}} \times 4 = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\frac{\frac{1}{4} \times 310 \times 27\frac{1}{2} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4} \times 310 \times \frac{55}{2} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{10}{\cancel{310}} \times 27 \times \overset{3}{\cancel{72}} \times \cancel{2} \times \overset{3}{\cancel{6}}}{\underset{4}{\underset{2}{\cancel{40}}} \times \cancel{9} \times \cancel{31} \times \underset{2}{\cancel{4}} \times \cancel{12}} =$$

$\frac{27}{2} = 13\frac{1}{2}$.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

DECIMALS.

132. **Decimals** are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

133. The denominator, which is always ten or a multiple of ten, as 10, 100, 1,000, etc., is not expressed as it would be in common fractions, by writing it under the numerator, with a line between them; as, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$. The denominator is always understood, the numerator consisting of the figures on the right of the *unit* figure. In order to distinguish

the unit figure, a period (.), called the **decimal point,** is placed between the unit figure and the next figure on the right. The decimal point may be regarded in two ways: first, as indicating that the number on the right is the numerator of a fraction whose denominator is 10, 100, 1,000, etc.; and, second, as a part of the Arabic system of notation, each figure on the right being 10 times as large as the next succeeding figure, and 10 times as small as the next preceding figure, serving merely to point out the unit figure.

134. The *reading* of a decimal number depends upon the number of decimal places in it, or the number of figures to the *right* of the unit figure.

The first figure to the right of the unit figure expresses *tenths.*

The second figure to the right of the unit figure expresses *hundredths.*

The third figure to the right of the unit figure expresses *thousandths.*

The fourth figure to the right of the unit figure expresses *ten-thousandths.*

The fifth figure to the right of the unit figure expresses *hundred-thousandths.*

The sixth figure to the right of the unit figure expresses *millionths.*

Thus:

.3 = $\frac{3}{10}$ = 3 tenths.

.03 = $\frac{3}{100}$ = 3 hundredths.

.003 = $\frac{3}{1000}$ = 3 thousandths.

.0003 = $\frac{3}{10000}$ = 3 ten-thousandths.

.00003 = $\frac{3}{100000}$ = 3 hundred-thousandths.

.000003 = $\frac{3}{1000000}$ = 3 millionths.

The first figure to the right of the unit figure is called the *first* **decimal place;** the second figure, the *second decimal place,* etc. We see in the above that the *number of decimal places in a decimal equals the number of ciphers to the right of the figure 1 in the denominator of its equivalent fraction.* This fact kept in mind will be of much assistance in reading and writing decimals.

Whatever may be written to the *left* of a *decimal p*[illegible] a whole number. The decimal point *merely separates* [illegible] *fraction on the right from the whole number on the left.*

When a whole number and decimal are written togeth[illegible] the expression is a *mixed number.* Thus, 8.12 and 17.25 [illegible] mixed numbers.

The relation of decimals and whole numbers to each ot[illegible] is clearly shown by the following table:

hundreds of millions.	tens of millions.	millions.	hundreds of thousands.	tens of thousands.	thousands.	hundreds.	tens.	units.	decimal point.	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred-thousandths.	millionths.	ten-millionths.	hundred-millionths.
9	8	7	6	5	4	3	2	1	.	2	3	4	5	6	7	8	9

The figures to the *left* of the decimal point represent *wh[illegible] numbers;* those to the *right* are *decimals.*

In both the decimals and whole numbers, the ***units*** place is made the starting point of notation and numeration. Both whole numbers and decimals *decrease* on the scale of *ten* to the *right*, and *increase* on the scale of *ten* to the *left*. The *first* figure to the *left* of units is *tens*, and the first figure to the *right* of units is *tenths*. The *second* figure to the *left* of units is *hundreds*, and the *second* figure to the *right* is *hundredths*. The *third* figure to the *left* is *thousands*, and the *third* to the *right* is *thousandths*, and so on, the *whole* numbers on the *left* and the *decimals* on the *right*. The figures equally distant from units place correspond in name, the *decimals* having the ending *ths* to distinguish them from *whole* numbers. The following is the numeration of the number in the above table: Nine hundred eighty-seven million, six hundred fifty-four thousand, three hundred twenty-one, and twenty-three million, four hundred fifty-six thousand, seven hundred eighty-nine hundred millionths.

The decimals increase to the *left*, on the scale of *ten*, the same as whole numbers; for, beginning at say 4-thousandths, in the table, the next figure to the left is *hundredths*, which is ten times as great, and the next *tenths*, or ten times the *hundredths*, and so on through both decimals and whole numbers.

135. *Annexing or taking away a cipher at the right of a decimal does not affect its value.*

.5 is $\frac{5}{10}$; .50 is $\frac{50}{100}$, but $\frac{5}{10} = \frac{50}{100}$; therefore, .5 = .50.

136. *Inserting a cipher between a decimal and the decimal point divides the decimal by 10.*

$$.5 = \tfrac{5}{10};\ \tfrac{5}{10} \div 10 = \tfrac{5}{100} = .05.$$

137. *Taking away a cipher from the left of a decimal multiplies the decimal by 10.*

$$.05 = \tfrac{5}{100};\ \tfrac{5}{100} \times 10 = \tfrac{5}{10} = .5.$$

138. In some cases it is convenient to express a mixed decimal fraction in the form of a common (improper) fraction. To do so it is only necessary to write the entire number, omitting the decimal point, as the numerator of the fraction, and the denominator of the decimal part as the denominator of the fraction. Thus, $127.483 = \frac{127483}{1000}$; for, $127.483 = 127\frac{483}{1000} = \frac{127000+483}{1000} = \frac{127483}{1000}$.

ADDITION OF DECIMALS.

139. Addition of decimals is similar in all respects to addition of whole numbers—units are placed under units, tens under tens, etc.; this, of course, brings the decimal points in line, directly under one another. Hence, in placing the numbers to be added, it is only necessary to take care that the *decimal points are in line*. In adding whole numbers, the right-hand figures are always in line; but in adding decimals, the right-hand figures will not be in line unless each decimal contains the same number of figures.

whole numbers	*decimals*	*mixed numbers*
342	.342	342.032
4234	.4234	4234.5
26	.26	26.6782
3	.03	3.06
sum 4605 Ans.	*sum* 1.0554 Ans.	*sum* 4606.2702 Ans.

140. A decimal, as .342, ought really to be expressed 0.342, but it is quite customary to omit the cipher on the left of the decimal point, though many authors use it.

EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—

242.
.36
118.725
1.005
6.
100.1
sum 468.190 Ans.

141. Rule.—*Place the numbers to be added so that the decimal points will be directly under each other. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

142. Find the sum of

(*a*) .2143, .105, 2.3042, and 1.1417.
(*b*) 783.5, 21.473, .2101, and .7816.
(*c*) 21.781, 138.72, 41.8738, .72, and 1.413.
(*d*) .3724, 104.15, 21.417, and 100.042.
(*e*) 200.172, 14.105, 12.1465, .705, and 7.2.
(*f*) 1,427.16, .244, .32, .032, and 10.0041.
(*g*) 2,473.1, 41.65, .7243, 104.067, and 21.073.
(*h*) 4,107.2, .00375, 21.716, 410.072, and .0345.

Ans. (*a*) 3.7652. (*b*) 805.9647. (*c*) 204.5078. (*d*) 225.9814. (*e*) 234.3285. (*f*) 1,437.7601. (*g*) 2,640.6143. (*h*) 4,539.02625.

1. A piece of plumbing work consists of pipes of the following dimensions and sizes: 1 piece of 1-inch pipe, 37.452 feet long; 3 pieces of ¾-inch pipe, each 29.316 feet long; 9 pieces of ⅝-inch pipe, each 17.30 feet long, and 2 pieces of ½-inch pipe, each 39.25 feet long. Find the total length of the pipe used in the work. Ans. 359.681 ft.

2. The following is a list of the prices of the several parts of a certain piece of plumbing: Cast-iron drain, soil, waste and vent pipes, and

fittings, $53.50; lead waste and vent branches, $18.25; lead and oakum, $2.40; brass ferrules, $3.80; brass floor plates, $2.00; pipe hooks, 75c.; roof flashings, 80c.; main drain trap, $5.75; roof leader traps and connections, $27.30; siphon jet closets, complete, $90.25; baths, complete, $131.35; wash basins, complete, $87.65; sinks, $9.80; lead water piping, $142.13; brass hot water piping, $99.42; kitchen boiler and stand, $27.37; kitchen range, with water back, $57.83; stops, valves, bibbs, etc., $79.61; labor (plumber and helper), $138; freight and cartage, $39.84; sundries, $5. What is the total cost of the job? Ans. $1,022.80.

SUBTRACTION OF DECIMALS.

143. As in subtraction of whole numbers, units are placed under units, tens under tens, etc., bringing the decimal points under each other, as in addition of decimals.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—

minuend	.3063
subtrahend	.132
difference	.1743 Ans.

144. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—

minuend	7.895
subtrahend	.725
difference	7.170 or 7.17 Ans.

145. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—

minuend	11.000
subtrahend	.625
difference	10.375 Ans.

146. Rule.—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract, as in whole numbers, and place the decimal point in the remainder, directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them, and subtract as before.

EXAMPLES FOR PRACTICE.

147. From

(*a*) 407.385 take 235.0004.	Ans.	(*a*) 172.3846.
(*b*) 22.718 take 1.7042.		(*b*) 21.0138.
(*c*) 1,368.17 take 13.6817.		(*c*) 1,354.4883.
(*d*) 70.00017 take 7.000017.		(*d*) 63.000153.
(*e*) 630.630 take .6304.		(*e*) 629.9996.
(*f*) 421.73 take 217.162.		(*f*) 204.568.
(*g*) 1.000014 take .00001.		(*g*) 1.000004.
(*h*) .783652 take .542314.		(*h*) .241338.

1. One piece of sheet iron is .0808 of an inch thick, and another piece is .46 of an inch thick. Find the difference between their thicknesses. Ans. .3792 of an inch.

2. A cubic inch of lead weighs .41 of a pound, and one-half of a cubic inch of cast iron weighs .1315 of a pound. What is the difference in the weight of a cubic inch of the metals? Ans. .147 of a pound.

3. One-fourth of a cubic inch of steel weighs .07 of a pound. 1 cubic inch of pure water weighs .03617 of a pound. Find the difference in the weights of 1 cubic inch of each. Ans. .24383 of a pound.

MULTIPLICATION OF DECIMALS.

148. In multiplication of decimals, we do not place the decimal points directly under each other, as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

multiplicand	.825
multiplier	13
	2475
	825
product	10.725 Ans.

In this example there are three decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

149. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.— *multiplicand* 426
multiplier .005
product 2.130 or 2.13 Ans.

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

150. It is *not* necessary to multiply by the ciphers on the *left* of a decimal; they merely determine the number of decimal places. Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with, and do not change the value.

151. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.— *multiplicand* 1.205
multiplier 1.15
6025
1205
1205
product 1.38575 Ans.

In this example there are 3 decimal places in the multiplicand and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

152. EXAMPLE.—Multiply .232 by .001.

SOLUTION.— *multiplicand* .232
multiplier .001
product .000232 Ans.

In this example we multiply the multiplicand by the digit in the multiplier, which makes 232 in the product, but since there are 3 decimal places in each, the multiplier and the

multiplicand, we must prefix 3 ciphers to the 232, to make $3 + 3$, or 6, decimal places in the product.

153. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

154. Find the product of

(*a*) $.000492 \times 4.1418$.	Ans.	(*a*) .0020377656.
(*b*) $4{,}003.2 \times 1.2$.		(*b*) 4,803.84.
(*c*) 78.6531×1.03.		(*c*) 81.012693.
(*d*) $.3685 \times .042$.		(*d*) .015477.
(*e*) $178{,}352 \times .01$.		(*e*) 1,783.52.
(*f*) $.00045 \times .0045$.		(*f*) .000002025.
(*g*) $.714 \times .00002$.		(*g*) .00001428.
(*h*) $.00004 \times .008$.		(*h*) .00000032.

1. A cubic inch of water weighs .03617 of a pound. What is the weight of a body of water whose volume is 1,500 cubic inches?

Ans. 54.255 lb.

2. A piece of lead pipe weighs 2.135 pounds per foot run. What is the weight of 3,172.017 feet? Ans. 6,772.256295 lb.

3. Wrought iron expands .00000686 of its length for every degree rise in temperature. If a 3-inch wrought-iron steam pipe has its temperature raised 150 degrees, what proportion of its length will it expand? Ans. .001029 of its length.

DIVISION OF DECIMALS.

155. In division of decimals we pay no attention to the decimal point until after the division has been performed. The *number of decimal places in the dividend must equal or be made to equal by annexing ciphers the number of decimal places in the divisor. Divide exactly as in whole numbers. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal*

places in the quotient as there are units in the remainder thus found.

EXAMPLE.—Divide .625 by 25.

divisor dividend quotient

SOLUTION.— 25) .625 (.025 Ans.

50

125

125

remainder 0

In this example there are no decimal places in the divisor, and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25, to make the 3 decimal places.

156. EXAMPLE.—Divide 6.035 by .05.

divisor dividend quotient

SOLUTION.— .05) 6.035 (120.7 Ans.

5

10

10

35

35

remainder 0

In this example we divide by 5, as if the cipher were not before it. There is one more decimal place in the dividend than in the divisor; therefore, one decimal place is pointed off in the quotient.

157. EXAMPLE.—Divide .125 by .005.

divisor dividend quotient

SOLUTION.— .005) .125 (25 Ans.

10

25

25

remainder 0

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places, and is a whole number.

158. EXAMPLE.—Divide 326 by .25.

divisor dividend quotient

SOLUTION.— .25) 326.00 (1304 **Ans.**

25

76

75

100

100

remainder 0

In this problem two ciphers were annexed to the dividend to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

159. EXAMPLE.—Divide .0025 by 1.25.

SOLUTION.— 1.25) .00250 (.002 **Ans.**

250

remainder 0

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, or 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, or 125. It is clearly evident that the dividend 25 will not contain the divisor 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only four decimal places in the dividend; but one cipher was annexed, thereby making $4+1$, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off $5-2$, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

160. Rule.—I. *Place the divisor to the left of the dividend, and proceed as in division of whole numbers; in the*

quotient, point off as many decimal places as the number of decimal places in the dividend exceed those in the divisor, prefixing ciphers to the quotient, if necessary.

II. *If in dividing one number by another there be a remainder, the remainder can be placed over the divisor, as a fractional part of the quotient, but it is generally better to annex ciphers to the remainder, and continue dividing until there are 3 or 4 decimal places in the quotient, and then if there still be a remainder, terminate the quotient by the plus sign* (+), *which shows that it can be carried further.*

161. EXAMPLE.—What is the quotient of 199 divided by 15?

SOLUTION.—

$$
\begin{array}{rl}
 & 15)199(13+\frac{4}{15}\ \text{Ans.} \\
 & \underline{15} \\
 & 49 \\
 & 45 \\
 & \underline{} \\
\textit{remainder} & 4
\end{array}
$$

Or,

$$
\begin{array}{rl}
 & 15)199.000(13.266+\ \text{Ans.} \\
 & \underline{15} \\
 & 49 \\
 & 45 \\
 & \underline{} \\
 & 40 \\
 & 30 \\
 & \underline{} \\
 & 100 \\
 & 90 \\
 & \underline{} \\
 & 100 \\
 & 90 \\
 & \underline{} \\
\textit{remainder} & 10
\end{array}
$$

$$13\tfrac{4}{15} = 13.266+$$
$$\tfrac{4}{15} = .266+$$

162. It frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried, and carry the work one place further. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that

the number is slightly greater than as indicated. In t
last example, had it been desired to obtain the answer c
rect to four decimal places, the work would have been c
ried to five places, obtaining 13.26666, and the answer wou
have been given as 13.2667—. This remark applies to a
other calculation involving decimals, when it is desired
omit some of the figures in the decimal. Thus, if it is d
sired to retain three decimal places in the number .24712
it would be expressed as .247 +; if it was desired to reta
five decimal places, it would be expressed as .24713—. Bo
the + and — signs are frequently omitted; they are seldo
used outside of Arithmetic, except in exact calculations, wh
it is desired to call particular attention to the fact that t
result obtained is not *quite* exact.

EXAMPLES FOR PRACTICE.

163. Divide

(*a*) 101.6688 by 2.36.
(*b*) 187.12264 by 123.107.
(*c*) .08 by .008.
(*d*) .0003 by 3.75.
(*e*) .0144 by .024.
(*f*) .00375 by 1.25.
(*g*) .004 by 400.
(*h*) .4 by .008.

Ans. (*a*) 43.08. (*b*) 1.52. (*c*) 10. (*d*) .00008. (*e*) .6. (*f*) .003. (*g*) .00001. (*h*) 50.

1. A cubic inch of brass weighs .315 of a pound; 396 steam valv
made of this metal weigh 1,274.724 pounds. (*a*) Find the number
cubic inches of metal in each valve. (*b*) What is the weight of 15
such valves?

Ans. (*a*) 10.219 cu. i
(*b*) 48.285 lb.

2. A tank contains 65,003.2 cubic inches of water, each cubic inc
weighing .03617 of a pound. The wood work about the tank weig
132 pounds, and the lead lining weighs 341.25 pounds. Find their tot
weight in pounds.

Ans. 2,824.4157 l

TO REDUCE A FRACTION TO A DECIMAL.

164. EXAMPLE.— $\frac{3}{4}$ equals what decimal?

SOLUTION.— $\begin{array}{r} 4\,)\,3.0\,0 \\ \hline .7\,5 \end{array}$, or $\frac{3}{4} = .75$. Ans.

EXAMPLE.—What decimal is equivalent to $\frac{7}{8}$?

SOLUTION.—

$$\begin{array}{r} 8\,)\,7.000\,(.875 \\ 64 \\ \hline 60 \\ 56 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

or $\frac{7}{8} = .875$. Ans.

165. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

166. Reduce the following common fractions to decimals:

(*a*) $\frac{15}{32}$.
(*b*) $\frac{7}{8}$.
(*c*) $\frac{21}{32}$.
(*d*) $\frac{51}{64}$.
(*e*) $\frac{4}{25}$.
(*f*) $\frac{5}{8}$.
(*g*) $\frac{10}{200}$.
(*h*) $\frac{4}{1000}$.

Ans. (*a*) .46875. (*b*) .875. (*c*) .65625. (*d*) .796875. (*e*) .16. (*f*) .625. (*g*) .05. (*h*) .004.

---

167. To reduce inches to decimal parts of a foot:

EXAMPLE.—What decimal part of a foot is 9 inches?

SOLUTION.—Since there are 12 inches in one foot, 1 inch is $\frac{1}{12}$ of a foot, and 9 inches is $9 \times \frac{1}{12}$ or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule, shows what decimal part of a foot 9 inches is.

$$\begin{array}{r} 12\,)\,9.00\,(.75 \text{ of a foot. Ans.} \\ 84 \\ \hline 60 \\ 60 \\ \hline 0 \end{array}$$

168. Rule.—I. *To reduce inches to decimal parts of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one and it is desired to terminate the division at some point, say the*

fourth decimal place, carry the division one place further; if the fifth figure is 5 or greater, increase the fourth by 1. Omit the signs + and −.

EXAMPLES FOR PRACTICE.

169. Reduce to the decimal part of a foot:

(*a*) 3 in.
(*b*) $4\frac{1}{2}$ in.
(*c*) 5 in.
(*d*) $6\frac{3}{4}$ in.
(*e*) 11 in.

Ans. (*a*) .25. (*b*) .375. (*c*) .4167. (*d*) .5521. (*e*) .9167.

TO REDUCE A DECIMAL TO A FRACTI

170. EXAMPLE.—Reduce .125 to a fraction.

SOLUTION.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

EXAMPLE.—Reduce .875 to a fraction.

SOLUTION.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

171. Rule.—*Under the figures of the decimal place as many ciphers at its right as there are decimal places decimal, and reduce the resulting fraction to its lowest ter dividing both numerator and denominator by the same nu*

EXAMPLES FOR PRACTICE.

172. Reduce the following to common fractions:

(*a*) .125.
(*b*) .625.
(*c*) .3125.
(*d*) .04.
(*e*) .06.
(*f*) .75.
(*g*) .15625.
(*h*) .875.

Ans. (*a*) $\frac{1}{8}$. (*b*) $\frac{5}{8}$. (*c*) $\frac{5}{16}$. (*d*) $\frac{1}{25}$. (*e*) $\frac{3}{50}$. (*f*) $\frac{3}{4}$. (*g*) $\frac{5}{32}$. (*h*) $\frac{7}{8}$.

1. (*a*) A boiler shell is 12.75 feet long, .375 of an inch thick, a a diameter of 44.125 inches. (*b*) Another boiler tube is 12.8 fee .1875 of an inch thick, and has a diameter of 3.125 inches. Req their respective dimensions in feet and inches and fractions of a

Ans. (*a*) 12 ft. 9 in. $\frac{3}{8}$ in. thick. $44\frac{1}{8}$ in. dia
(*b*) 12 ft. $9\frac{3}{8}$ in. $\frac{3}{16}$ in. thick $3\frac{1}{8}$ in. diam

173. To express a decimal approximately as a fraction having a given denominator:

174. EXAMPLE.—Express .5827 in 64ths.

SOLUTION.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

EXAMPLE.—Express .3917 in 12ths.

SOLUTION.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

175. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

176. Express

(*a*) .625 in 8ths.
(*b*) .3125 in 16ths.
(*c*) .15625 in 32ds.
(*d*) .77 in 64ths.
(*e*) .81 in 48ths.
(*f*) .923 in 96ths.

Ans. (*a*) $\frac{5}{8}$. (*b*) $\frac{5}{16}$. (*c*) $\frac{5}{32}$. (*d*) $\frac{49}{64}$. (*e*) $\frac{39}{48}$. (*f*) $\frac{89}{96}$.

177. The sign for dollars is $. It is read *dollars*. $25 is read *25 dollars*.

Since there are 100 cents in a dollar, one cent is 1-one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*.

Thus, $25.16 is read *twenty-five dollars and sixteen cents*; $25.168 is read *twenty-five dollars, sixteen cents and eight mills.*

178. The **vinculum** ——, **parenthesis** ( ), **bracket** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8 - 3}$, or $13 \times (8 - 3)$, shows that 3 is to be taken from 8 before multiplying by 13.

$$13 \times (8 - 3) = 13 \times 5 = 65. \quad \text{Ans.}$$
$$13 \times \overline{8 - 3} = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we
$13 \times 8 - 3 = 104 - 3 = 101$. Ans.

179. In any series of numbers connected by the s
$-$, $\times$, and $\div$, the operations indicated by the signs
performed in order from left to right, *except* that no
or subtraction may be performed if a sign of multip
or division *follows* the number on the *right* of a
addition or subtraction, until the indicated multiplic
division has been performed. In all cases the sign o
plication takes the precedence, the reason being tha
two or more numbers or expressions are connected
sign of multiplication, the numbers thus connected
garded as factors of the product indicated, and not
rate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left t
$4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

180. EXAMPLE.—What is the value of the following exp
$1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} = ?$

SOLUTION.—$1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we can
tract 22 from 268 because the sign of multiplication *follows* 22
multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 +$
$22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to hav
ded $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result
have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 25$
$+ 25 = 282$. Ans. In other words, it is necessary
form *all* of the multiplication or division included be
the signs $+$ and $-$, or $-$ and $+$, before adding or subtra
Also, had the expression been written $1{,}296 \div 12$
$- 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary t
multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the s
multiplication takes the precedence, and the final
would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; 268
267; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multipli
and division signs occurs, the indicated operations mu
be performed in order, from left to right—the multipli
must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times$

Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

181. Find the values of the following expressions:

(*a*)	$(8 + 5 - 1) + 4$.	Ans. (*a*)	8.
(*b*)	$5 \times 24 - 32$.	(*b*)	88.
(*c*)	$5 \times 24 + 15$.	(*c*)	8.
(*d*)	$144 - 5 \times 24$.	(*d*)	24.
(*e*)	$(1{,}691 - 540 + 559) + 3 \times 57$.	(*e*)	10.
(*f*)	$2{,}080 + 120 - 80 \times 4 - 1{,}670$.	(*f*)	210.
(*g*)	$\overline{(90 + 60 + 25)} \times 5 - 29$.	(*g*)	1.
(*h*)	$\overline{90 + 60 + 25} \times 5$.	(*h*)	1.2.

ARITHMETIC.

(CONTINUED.)

PERCENTAGE.

182. **Percentage** is the process of calculating by *hundredths.*

183. The *term* **per cent.** is an abbreviation of the Latin words *per centum*, which mean *by the hundred.* A certain per cent. of a number is the number of hundredths of that number which is indicated by the number of units in the per cent. Thus, 6 per cent. of 125 is $125 \times \frac{6}{100} = 7.5$; 25 per cent. of 80 is $80 \times \frac{25}{100} = 20$; 43 per cent. of 432 pounds is $432 \times \frac{43}{100} = 185.76$ pounds.

184. The **sign** of per cent. is %, and is read *per cent.* Thus, 6% is read *six per cent.;* $12\frac{1}{2}\%$ is read *twelve and one-half per cent.*, etc.

When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

Per Cent.	Decimal.	Fraction.	Per Cent.	Decimal.	Fraction.
1%.....	.01	$\frac{1}{100}$	150 %....	1.50	$\frac{150}{100}$ or $1\frac{1}{2}$
2%.....	.02	$\frac{2}{100}$ or $\frac{1}{50}$	500 %....	5.00	$\frac{500}{100}$ or 5
5%.....	.05	$\frac{5}{100}$ or $\frac{1}{20}$	$\frac{1}{4}\%$....	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
10%.....	.10	$\frac{10}{100}$ or $\frac{1}{10}$	$\frac{1}{2}\%$....	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
25%.....	.25	$\frac{25}{100}$ or $\frac{1}{4}$	$1\frac{1}{2}\%$....	.015	$\frac{1\frac{1}{2}}{100}$ or $\frac{3}{200}$
50%.....	.50	$\frac{50}{100}$ or $\frac{1}{2}$	$8\frac{1}{3}\%$....	.08$\frac{1}{3}$	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
75%.....	.75	$\frac{75}{100}$ or $\frac{3}{4}$	$12\frac{1}{2}\%$....	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
100%.....	1.00	$\frac{100}{100}$ or 1	$16\frac{2}{3}\%$....	.16$\frac{2}{3}$	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
125%.....	1.25	$\frac{125}{100}$ or $1\frac{1}{4}$	$62\frac{1}{2}\%$....	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$

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185. The names of the different elements used in percentage are: the *base*, the *rate per cent.*, the *percentage*, the *amount*, and the *difference*.

186. The **base** is the number on which the per cent. is computed.

187. The **rate** is the number of hundredths of the base to be taken.

188. The **percentage** is the part, or number of *hundredths*, of the base indicated by the rate; or, the percentage is the result obtained by multiplying the base by the rate.

Thus, when it is stated that 7% of $25 is $1.75, $25 is the base, 7% is the rate, and $1.75 is the percentage.

189. The **amount** is the sum of the base and percentage.

190. The **difference** is the remainder obtained by subtracting the percentage from the base.

Thus, if a man has $180, and he earns 6% more, he will have altogether, $\$180 + \$180 \times .06$, or $\$180 + \$10.80 = \$190.80$. Here $180 is the base; 6%, the rate; $10.80, the percentage, and $190.80, the *amount*.

Again, if an engine of 125 horsepower uses 16% of it in overcoming friction and other resistances, the amount left for obtaining useful work is $125 - 125 \times .16 = 125 - 20 = 105$ horsepower. Here 125 is the base; 16%, the rate; 20, the percentage, and 105, the *difference*.

191. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate expressed decimally.*

EXAMPLE.—Out of a lot of 300 bushels of apples, 76% were sold. How many bushels were sold?

SOLUTION.—76%, the rate, expressed decimally, is .76; the base is 300; hence, the number of bushels sold, or the percentage, is by the above rule,

$$300 \times .76 = 228 \text{ bushels.} \quad \text{Ans.}$$

Expressing the rule as a

Formula, *percentage* = *base* × *rate*.

192. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then, 1%, or $\frac{1}{100}$, of the number, is $12 \div 6$, or 2. Consequently, if $2 = 1\%$, or $\frac{1}{100}$, 100%, or $\frac{100}{100}$, $= 2 \times 100 = 200$. But, since the same result may be arrived at by dividing 12 by .06, for $12 \div .06 = 200$, it follows that

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate, expressed decimally.*

Formula, *base* = *percentage* ÷ *rate.*

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If I sold 228 bushels, how many bushels did I buy?

SOLUTION.—Here 228 is the percentage, and 76%, or .76, is the rate; hence, applying the rule,

$228 \div .76 = 300$ bushels. Ans.

193. When the base and percentage are given, to find the rate, the rate may be found, expressed decimally, by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is $200 \times .01 = 2$. Now, if 1% is 2, 12 is evidently as many per cent. as the number of times that 2 is contained in 12, or $12 \div 2 = 6\%$. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since $12 \div 200 = .06 = 6\%$. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate, expressed decimally.*

Formula, *rate* = *percentage* ÷ *base.*

EXAMPLE.—Bought 300 bushels of apples and sold 228 bushels. What per cent. of the total number of bushels was sold?

SOLUTION.—Here 300 is the base and 228 is the percentage; hence, applying rule, rate $= 228 \div 300 = .76 = 76\%$. Ans.

EXAMPLE.—What per cent. of 875 is 25?

SOLUTION.—Here 875 is the base and 25 is the percentage; hence, applying rule, $25 \div 875 = .02\frac{4}{7} = 2\frac{4}{7}\%$. Ans.

PROOF.—$875 \times .02\frac{4}{7} = 25$.

EXAMPLES FOR PRACTICE.

194. What per cent. of

(*a*)	360 is 90?	Ans.	(*a*) 25%.
(*b*)	900 is 360?		(*b*) 40%.
(*c*)	125 is 25?		(*c*) 20%.
(*d*)	150 is 750?		(*d*) 500%.
(*e*)	280 is 112?		(*e*) 40%.
(*f*)	400 is 200?		(*f*) 50%.
(*g*)	47 is 94?		(*g*) 200%.
(*h*)	500 is 250?		(*h*) 50%.

195. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate, expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and 6% is the rate. The percentage is $200 \times .06 = 12$, and, according to definition, Art. **189**, the amount is $200 + 12 = 212$. But the same result may be obtained by multiplying 200 by $1 + .06$, or 1.06, since $200 \times 1.06 = 212$. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate, expressed decimally.*

Formula, *amount* = *base* $\times$ (1 + *rate*).

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and 10% is the rate, and the amount is required. Hence, applying the rule,

$725 \times 1.10 = \$797.50$. Ans.

196. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate, expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is $200 \times .06 = 12$; and, according to definition, Art. **190**, the difference $= 200 - 12 = 188$. But the same result may be obtained by multiplying 200 by $1 - .06$, or .94, since $200 \times .94 = 188$. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate, expressed decimally.*

Formula, *difference* = *base* $\times$ (1 − *rate*).

EXAMPLE.—Bought 300 bushels of apples, and sold all but 24% of them. How many bushels were sold?

SOLUTION.—Here 300 is the base, 24% is the rate, and it is desired to find the difference. Hence, applying the rule,

$$300 \times (1 - .24) = 228 \text{ bushels.} \quad \text{Ans.}$$

197. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number increased by 6% of itself. Then it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$, or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate, expressed decimally.*

Formula, *base* = *amount* $\div$ (1 + *rate*).

EXAMPLE.—The theoretical discharge of a certain pump, when running at a piston speed of 100 feet per minute, is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount; 25% is the rate, and, applying rule,

actual discharge = $278{,}910 \div 1.25 = 223{,}128$ gallons. Ans.

198. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate, expressed decimally.*

Formula, *base* = *difference* $\div$ (1 — *rate*).

EXAMPLE.—Bought a certain number of bushels of apples and sold 76% of them. If there were 72 bushels left unsold, how many bushels did I buy?

SOLUTION.—Here 72 is the difference and 76% is the rate. Applying rule,

$$72 \div (1 - .76) = 300 \text{ bushels. Ans.}$$

EXAMPLE.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence 127,344 is the difference, and 30% is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds. Ans.}$$

199. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, $3 - 2.76 = .24$ inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **193**,

$$\text{gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \text{ Ans.}$$

200. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent., loss of itself) and 3 inches is the base. Consequently, $3 - 1.2 = 1.8$ inches is the percentage. Hence, applying the rule given in Art. **193**,

$$\text{loss per cent.} = 1.8 \div 3 = .60 = 60\%. \text{ Ans.}$$

201. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and final values; divide this difference by the initial value.*

EXAMPLE.—If a man buys a house for $1,860, and some time afterwards builds a barn for 25% of the cost of the house, does he gain or lose, and how much per cent., if he sells both house and barn for $2,100?

SOLUTION.—The cost of the barn was $\$1{,}860 \times .25 = \$465$; consequently, the initial value, or cost, was $\$1{,}860 + \$465 = \$2{,}325$. Since he sold them for $2,100, he lost $\$2{,}325 - \$2{,}100 = \$225$. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss. Ans.}$$

EXAMPLES FOR PRACTICE.

202. Solve the following:

(*a*) What is 12½% of $900?	Ans.	(*a*) $112.50.
(*b*) What is ⅛% of 627?		(*b*) 5.016.
(*c*) What is 33⅓% of 54?		(*c*) 18.
(*d*) 101 is 68¾% of what number?		(*d*) $146\frac{10}{11}$.
(*e*) 784 is 83⅓% of what number?		(*e*) 940.8.
(*f*) What % of 960 is 160?		(*f*) 16⅔%.
(*g*) What % of $3,606 is $450¼?		(*g*) 12½%.
(*h*) What % of 280 is 112?		(*h*) 40%.

1. If the speed of an engine running at 126 revolutions per minute should be increased 6½%, how many revolutions per minute would it then make? Ans. 134.19 revolutions.

2. The list price and discount of the following materials are given. Find the net cost:

	List Price.	Discount.		Net Cost.
4 porcelain-lined bathtubs.....each,	$47.00	20 %		$150.40
1 porcelain washout closet.....	26.50	35 %		17.23
1 marble basin slab and basin.....	17.75	43 %		10.12
75 ft. 1-in. lead pipe, 4 lb. per ft..per lb.,	.08	30 %	Ans.	16.80
100 ft. 1¼-in. galvanized iron pipe..per ft.,	.18¼	40½%		11.01
1 Ericsson hot-air engine, complete....	225.50	15½%		190.55
125 lb. solder .....per lb.,	.18	9½%		20.36

3. If it is found that 18.5% of the coal burns to ash under a boiler, and that on a certain day 1,100 pounds of ashes were produced, how much coal was consumed on that day? Ans. 5,945.95 lb.

4. A hydraulic ram, when the valves are in perfect condition, discharges 190.4 gallons of water per hour. A little sand got under the valve and reduced the discharge 15%. How much water did it then discharge per hour? Ans. 161 84 gal.

DENOMINATE NUMBERS.

203. A **denominate number** is a concrete number, and may be either simple or compound, as 8 quarts, 5 feet, 10 inches, etc.

204. A **simple denominate number** consists of units of but one denomination, as 16 cents, 10 hours, 5 dollars, etc.

205. A **compound denominate number** consists of units of two or more denominations of a **similar kind**, as 3 yards 2 feet 1 inch ; 34 square feet 57 square inches.

206. In **whole numbers** and in **decimals** the *law* of increase and decrease is on the scale of 10, but in **compound** or **denominate numbers** the scale varies.

MEASURES.

207. A **measure** is a *standard unit*, established by *law* or *custom*, by which *quantity* of any kind is measured. The *standard unit* of **dry measure** is the Winchester bushel: of **weight**, the pound ; of **liquid measure**, the gallon, etc.

208. Measures are of six kinds :

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

209. **Measures of extension** are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE 3.

			Abbreviation.	in.	ft.	yd.	rd.	fur.	mi.
12	inches (in.)	= 1 foot . .	ft.						
3	feet	= 1 yard . .	yd.	36 =	3 =	1			
5.5	yards	= 1 rod . . .	rd.	198 =	16½ =	5.5 =	1		
40	rods	= 1 furlong	fur.	7,920 =	660 =	220 =	40 =	1	
8	furlongs	= 1 mile . .	mi.	63,360 =	5,280 =	1,760 =	320 =	8 =	1

SURVEYOR'S LINEAR MEASURE.

TABLE 4.

7.92 inches	= 1 link	li.
25 links	= 1 rod	rd.
4 rods / 100 links	= 1 chain	ch.
80 chains	= 1 mile	mi.

mi.	ch.	rd.	li.	in.
1 =	80 =	320 =	8,000 =	63,360

210. The linear unit, generally used by surveyors, is **Gunter's chain,** which is equal to 4 rods, or 66 feet.

211. An **engineer's chain,** used by civil engineers, is 100 feet long, and consists of 100 links. In computations, the links are written as so many hundredths of a chain.

SQUARE MEASURE.

TABLE 5.

144	square inches (sq. in.)	=	1	square foot	sq.ft.
9	square feet	=	1	square yard	sq.yd.
30¼	square yards	=	1	square rod	sq.rd.
160	square rods	=	1	acre	A.
640	acres	=	1	square mile	sq.mi.

sq.mi.	A.	sq.rd.	sq.yd.	sq.ft.	sq.in.
1	= 640	= 102,400	= 3,097,600	= 27,878,400	= 4,014,489,600

SURVEYOR'S SQUARE MEASURE.

TABLE 6.

625	square links	=	1	square rod	sq.rd.
16	square rods	=	1	square chain	sq.ch.
10	square chains	=	1	acre	A.
640	acres	=	1	square mile	sq.mi.
36	square miles (6 mi. square)	=	1	township	Tp.

sq.mi.	A.	sq.ch.	sq.rd.	sq.li.
1 =	640 =	6,400 =	102,400 =	64,000,000

CUBIC MEASURE.

TABLE 7.

1728	cubic inches (cu. in.)	=	1	cubic foot	cu.ft.
27	cubic feet	=	1	cubic yard	cu.yd.
128	cubic feet	=	1	cord	cd.
24¾	cubic feet	=	1	perch	P.

cu.yd.	cu.ft.	cu.in.
1 =	27 =	46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE 8.

16	ounces (oz.)	=	1	pound	lb.
100	pounds	=	1	hundredweight	cwt.
20	cwt., or 2,000 lb.	=	1	ton	T.

T.	cwt.	lb.	oz.
1 =	20 =	2,000 =	32,000

212. The ounce is divided into halves, quarters, et
Avoirdupois weight is used for weighing coarse and heav
articles. One avoirdupois pound contains 7,000 grains.

LONG TON TABLE.

TABLE 9.

16 ounces	=	1 pound
112 pounds	=	1 hundredweight
20 cwt., or 2,240 lb.	=	1 ton

213. In all the calculations throughout this and th
succeeding volumes, 2,000 pounds will be considered on
ton, unless the long ton (2,240 pounds) is especially men
tioned.

TROY WEIGHT.

TABLE 10.

24 grains (gr.)	=	1 pennyweight
20 pennyweights	=	1 ounce
12 ounces	=	1 pound

lb.	oz.	pwt.	gr.
1	= 12	= 240	= 5,760

214. Troy weight is used in weighing gold and silver
ware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE 11.

4 gills (gi.)	=	1 pint
2 pints	=	1 quart
4 quarts	=	1 gallon
31½ gallons	=	1 barrel
2 barrels, or 63 gallons	=	1 hogshead

hhd.	bbl.	gal.	qt.	pt.	gi.
1	= 2	= 63	= 252	= 504	= 2,016

DRY MEASURE.

TABLE 12.

2 pints (pt.)	=	1 quart
8 quarts	=	1 peck
4 pecks	=	1 bushel

bu.	pk.	qt.	pt.
1	= 4	= 32	= 64

MEASURE OF TIME.

TABLE 13.

60 seconds (sec.)	= 1 minute	min.
60 minutes	= 1 hour	hr.
24 hours	= 1 day	da.
7 days	= 1 week	wk.
365 days / 12 months	= 1 common year	yr.
366 days	= 1 leap year.	
100 years	= 1 century.	

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE 14.

60 seconds (″)	= 1 minute	′.
60 minutes	= 1 degree	°.
90 degrees	= 1 right angle or quadrant	∟.
360 degrees	= 1 circle	cir.

cir.
1 = 360° = 21,600′ = 1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE 15.

10 mills (m.)	= 1 cent	ct.
10 cents	= 1 dime	d.
10 dimes	= 1 dollar	$.
10 dollars	= 1 eagle	E.

E. $ d. ct. m.
1 = 10 = 100 = 1,000 = 10,000

MISCELLANEOUS TABLE.

12 things are 1 dozen.
12 dozen are 1 gross.
12 gross are 1 great gross.
2 things are 1 pair.
20 things are 1 score.
1 league is 3 miles.
1 fathom is 6 feet.
1 meter is nearly 39.37 inches.
1 hand is 4 inches.
1 palm is 3 inches.
1 span is 9 inches.
24 sheets are 1 quire.
20 quires, or 480 sheets, are 1 ream.
1 bushel contains 2,150.4 cubic in.
1 U. S. standard gallon (also called a wine gallon) contains 231 cubic in.
1 U. S. standard gallon of water weighs 8.355 pounds, nearly.
1 cubic foot of water contains 7.481 U. S. standard gallons, nearly.
1 British imperial gallon weighs 10 pounds.

It will be of great advantage to the student to carefully memorize all of the above tables.

REDUCTION OF DENOMINATE NUMBERS.

215. Reduction of denominate numbers is the process of changing their denomination without changing their value. They may be changed from a higher to a lower denomination or from a lower to a higher—either is reduction. As,

2 hours = 120 minutes.
32 ounces = 2 pounds.

216. Principle.—Denominate numbers are changed to *lower* denominations by *multiplying*, and to *higher* denominations by *dividing*.

To reduce denominate numbers to lower denominations:

217. EXAMPLE.—Reduce 5 yd. 2 ft. 7 in. to inches.

SOLUTION.—

yd.	ft.	in.
5	2	7
3		
15 ft.		
2 ft.		
17 ft.		
12		
34		
17		
204 in.		
7 in.		
211 inches. Ans.		

EXPLANATION.—Since there are 3 feet in 1 yard, in 5 yards there are 5 × 3, or 15 feet, and 15 feet plus 2 feet = 17 feet. There are 12 inches in a foot; therefore, 12 × 17 = 204 inches, and 204 inches plus 7 inches = 211 inches = number of inches in 5 yards 2 feet and 7 inches. Ans.

218. EXAMPLE.—Reduce 6 hours to seconds.

SOLUTION.—

6	hours.
60	
360	minutes.
60	
21600	seconds. Ans.

EXPLANATION.—As there are 60 minutes in one hour, in six hours there are 6×60, or 360 minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

219. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet; as,

rd.	yd.	ft.	in.
3	0	0	6

220. Rule.—*Multiply the number representing the highest denomination by the number of units in the next lower required to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

221. Reduce

(*a*) 4 rd. 2 yd. 2 ft. to ft.
(*b*) 4 bu. 3 pk. 2 qt. to qt.
(*c*) 13 rd. 5 yd. 2 ft. to ft.
(*d*) 5 mi. 100 rd. 10 ft. to ft.
(*e*) 8 lb. 4 oz. 6 pwt. to gr.
(*f*) 52 hhd. 24 gal. 1 pt. to pt.
(*g*) 5 cir. 16° 20′ to minutes.
(*h*) 14 bu. to qt.

Ans. (*a*) 74 ft. (*b*) 154 qt. (*c*) 231.5 ft. (*d*) 28,060 ft. (*e*) 48,144 gr. (*f*) 26,401 pt. (*g*) 108,980′. (*h*) 448 qt.

To reduce lower to higher denominations:

222. EXAMPLE.—Reduce 211 in. to higher denominations.

SOLUTION.—
$$12\overline{)211}\text{ in.}$$
$$3\overline{)17}\text{ ft.} + 7\text{ in.}$$
$$5\text{ yd.} + 2\text{ ft.}\quad \text{Ans.}$$

EXPLANATION.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There

are 3 feet in 1 yard ; therefore, 17 feet divided by 3 = 5 yards and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

223. EXAMPLE.—Reduce 15,735 grains Troy weight to higher denominations.

SOLUTION.—

```
24)15735 gr. (655 pwt.
   144
   ---
    133
    120
    ---
     135
     120
     ---
      15 gr.
20)655 pwt. (32 oz.
   60
   --
    55
    40
    --
    15 pwt.
12)32 oz. (2 lb.
   24
   --
    8 oz.
```

EXPLANATION.—There are 24 grains in 1 pennyweight, and in 15,735 grains there are as many pennyweights as 24 is contained in 15,735, or 655 pennyweights and 15 grains remaining. There are 20 pennyweights in 1 ounce, and in 655 pennyweights there are 32 ounces and 15 pennyweights remaining. There are 12 ounces in 1 pound, and in 32 ounces there are 2 pounds and 8 ounces remaining. The last quotient and the three remainders constitute the answer, 2 pounds 8 ounces 15 pennyweights 15 grains.

The above problem is worked out by long division, because the numbers are too large to solve easily by short division. The student may use either method.

224. Rule.—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by*

the number of units of its denomination required to make one unit of the next higher. Continue until the highest denomination is reached, or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.

EXAMPLES FOR PRACTICE.

225. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 7,896 cu. ft. to cd.; (*e*) 17,651″; (*f*) 1,120 cu. ft. to cd.; (*g*) 8,000 gi.; (*h*) 36,450 lb.

Ans. (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
(*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
(*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
(*d*) 61 cd. 88 cu. ft.
(*e*) 4° 54′ 11″.
(*f*) 8 cd. 96 cu. ft.
(*g*) 3 hhd. 61 gal.
(*h*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

226. EXAMPLE.—Find the sum of 8 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

SOLUTION.—

T.	cwt.	lb.	oz.
0	8	46	12
0	8	12	13
0	12	50	13
0	0	27	4
1	4	37	10

Ans.

EXPLANATION.—Begin to add at the right-hand column: $4 + 13 + 13 + 12 = 42$ ounces; as 16 ounces make 1 pound, 42 ounces $\div 16 = 2$ and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under ounce column, and add 2 pounds to the next or pound column. Then, $2 + 27 + 50 + 12 + 46 = 137$ pounds; as 100 pounds make a hundredweight, $137 \div 100 = 1$ hundredweight and a remainder of 37 pounds. Place the 37 under the pounds column, and add 1 hundredweight to the next or hundredweight column. Next, $1 + 12 + 8 + 3 = 24$ hundredweight.

20 hundredweight make a ton; therefore 24 ÷ 20 = 1 t. and 4 hundredweight remaining. Hence, the sum is 1 t. 4 hundredweight 37 pounds 10 ounces. Ans.

227. EXAMPLE.—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

SOLUTION.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	3½	0	2	
or	26	3	1	8	Ans.

EXPLANATION.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the next column plus 2 yards = 9 yards, or 9 ÷ 5½ = 1 rod and 3½ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the ½ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches. Ans.

228. EXAMPLE.—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

SOLUTION.—When 47 ft. is reduced it equals 2 rd. 4 yd. 2 ft., which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	1½	1	10	
or	6	2	0	4	Ans.

229. Rule.—*Place the numbers so that like denominations are under each other. Begin at the right-hand column, and add. Divide the sum by the number of* ***units*** *of this denomination required to make one unit of the* ***next higher.*** *Place the remainder under the column added,* ***and carry*** *the quotient to the next column. Continue in this* ***manner*** *until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

230. What is the sum of

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 8 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. 9½ lb.; 2½ cwt. 21⅛ lb.?

(*d*) 10 yr. 8 mo. 5 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 5 mo. 18 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 tons 11 cwt. 49 lb. 14 oz.; 16 tons 47 lb. 13 oz.; 20 tons 13 cwt. 14 lb. 6 oz.; 11 tons 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans. (*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 tons 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

231. EXAMPLE.—From 21 rd. 2 yd. 2 ft. 6½ in., take 9 rd 4 yd. 10¼ in.

SOLUTION.—

rd.	yd.	ft.	in.	
21	2	2	6½	
9	4	0	10¼	
11	3¼	1	8¼	Ans.

EXPLANATION.—Since 10¼ inches cannot be taken from 6½ inches, we must borrow 1 foot, or 12 inches, from the 2 feet in the next column and add it to the 6½. 6½ + 12 = 18½. 18½ inches − 10¼ inches = 8¼ inches. Then, 0 foot from the 1 remaining foot = 1 foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or 5½ yards, from 21 rods and add it to 2. 2 + 5½ = 7½; 7½ − 4 = 3½ yards. 9 rods from 20 rods = 11 rods. Hence, the remainder is 11 rods 3½ yards 1 foot 8¼ inches. Ans.

To avoid fractions as much as possible, we reduce the ½ yard to inches, obtaining 18 inches; this added to 8¼ inches, gives 26¼ inches, which equals 2 feet 2¼ inches. Then, 2 feet + 1 foot = 3 feet = 1 yard, and 3 yards + 1 yard = 4 yards. Hence, the above answer becomes 11 rods 4 yards 0 feet 2¼ inches.

232. EXAMPLE.—What is the difference between **3 rd. 2 yd. 2 ft. 10 in.** and **47 ft.** ?

SOLUTION.—47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	$3\frac{1}{2}$	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

233. EXAMPLE.—How many years, months, days, and hours between 4 o'clock P.M. of June 15, 1868, and 10 o'clock A.M., September 28, 1891 ?

SOLUTION.—	yr.	mo.	da.	hr.	
	1891	8	28	10	
	1868	5	15	16	
	23	3	12	18	Ans.

EXPLANATION.—Counting 24 hours in 1 day, 4 o'clock P.M. is the 16th hour from the beginning of the day, or midnight. On September 28, 8 months and 28 days have elapsed, and on June 15, 5 months and 15 days. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

234. Rule.—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above, and place the differences underneath. If the number in the minuend of any denomination is less than the number under it in the subtrahend, one must be borrowed from the minuend of the next higher denomination, reduced and added to it.*

EXAMPLES FOR PRACTICE.

235. From

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.

(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.

(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.

(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.

(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.

(*f*) 14 mi. 34 rd. 16 yd. 13 ft. 11 in. take 3 mi. 27 rd. 11 yd. 4 ft. 10 in

Ans. { (*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

236. EXAMPLE.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

SOLUTION.—

lb.	oz.	pwt.	gr.	
7	5	13	15	
			12	
89	8	3	12	Ans.

EXPLANATION.—15 grains $\times$ 12 = 180 grains. 180 $\div$ 24 = 7 pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next. Now, 13 $\times$ 12 + 7 = 163 pennyweights; 163 $\div$ 20 = 8 ounces and 3 pennyweights remaining. Then, 5 $\times$ 12 + 8 = 68 ounces; 68 $\div$ 12 = 5 pounds and 8 ounces remaining. Then, 7 $\times$ 12 + 5 = 89 pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains. Ans.

237. **Rule.**—*Multiply the number representing each denomination by the multiplier, and reduce each product to the next higher denomination, writing the remainders under each denomination, and carrying the quotient to the next, as in Addition of Denominate Numbers.*

238. NOTE. — In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the above example, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. 43,047 $\times$ 1.2 = 51,656.4 gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, 43,047 $\times$ 12 = 516,564 gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as above. The student may use either method.

EXAMPLES FOR PRACTICE.

239. Multiply

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 10 mo. 4 wk. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 8 gal. 1 qt. 1 pt. by 12.

Ans. (*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 116 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

240. EXAMPLE.—Divide 48 lb. 11 oz. 6 pwt. by 8.

SOLUTION.—

	lb.	oz.	pwt.	gr.
8)	48	11	6	0
	6 lb.	1 oz.	8 pwt.	6 gr. Ans.

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 six times without a remainder. 8 is contained in 11 ounces once with 3 ounces remaining. $3 \times 20 = 60$; $60 + 6 = 66$ pennyweights; 66 pennyweights $\div 8 = 8$ pennyweights and 2 remaining; 2×24 grains $= 48$ grains; 48 grains $\div 8 = 6$ grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains. Ans.

EXAMPLE.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

SOLUTION.—

	lb.	oz.	pwt.
6)	2	8	10
		5 oz.	8 pwt. 8 gr. Ans.

EXPLANATION.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds $= 24$ ounces, and 24 ounces $+ 8$ ounces $= 32$ ounces; 32 ounces $\div 6 = 5$ ounces and 2 ounces over. 2 ounces $= 40$ pennyweights. 40 pennyweights $+ 10$ pennyweights $= 50$ pennyweights, and 50 pennyweights $\div 6 = 8$ pennyweights and 2 pennyweights over. 2 pennyweights $= 48$ grains, and 48 grains $\div 6 = 8$ grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains. Ans.

241. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

SOLUTION.—

```
                    rd.   yd. ft.  rd. yd. ft.   in.
112)820   4   2 ( 7   1   2   5.143  Ans.
    784
    ---
     36 rd. rem.
      5.5
    ---
    180
    180
    ---
    198.0 yd.
      4
    ---
112)202 yd. ( 1 yd.
    112
    ---
     90 yd. rem.
      3
    ---
    270 ft.
      2 ft.
    ---
112)272 ft. ( 2 ft.
    224
    ---
     48 ft. rem.
     12
    ---
     96
     48
    ---
112)576 in. ( 5.1428+ in., or 5.143 in.
    560
    ---
     160
     112
    ----
      480
      448
      ---
       320
       224
       ---
        960
        896
        ---
         64
```

EXPLANATION.—The first quotient is 7 rods with 36 rods remaining. $5.5 \times 36 = 198$ yards; 198 yards + 4 yards = 202 yards; 202 yards $\div$ 112 = 1 yard and 90 yards remaining. $90 \times 3 = 270$ feet; 270 feet + 2 feet = 272 feet; 272 feet $\div$ 112 = 2 feet and 48 feet remaining; $48 \times 12 = 576$ inches; 576 inches $\div$ 112 = 5.143 inches, nearly. Ans.

The preceding example is solved by long division, because the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

242. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination, and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

243. Divide

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 tons 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans. (*a*) 17 mi. $41\frac{7}{11}$ rd.
(*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.
(*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.
(*d*) 4 sq. yd. 4 sq. ft. $2\frac{5}{18}$ sq. in.
(*e*) 12 mi. 112 rd. 2 yd.
(*f*) 6 tons 14 cwt. 41 lb. $3\frac{1}{15}$ oz.
(*g*) 4 lb. 8 oz. 7 pwt. $7\frac{2}{3}$ gr.
(*h*) 1 mi. $38\frac{88}{100}$ rd.

1. A hot water distributing main is composed of the following:

16 ft. 3 in. of 3½-in. pipe.
15 ft. 8 in. of 3-in. pipe.
15 ft. 2 in. of 2½-in. pipe.
14 ft. 6 in. of 2-in. pipe.
12 ft. 10 in. of 1½-in. pipe.

The space taken up by the fittings is included in these measurements If 10 hangers are used, one being at each end of the main, what is the distance between them, supposing them to be spaced equally?

Ans. 8 ft. $3\frac{3}{8}$ in.

2. The water contained in a stand pipe, or water tower, is 47 ft. 3 in. deep. If $\frac{2}{3}$ of the water is run off, what is the depth of the water left?

Ans. 28 ft. $4\frac{1}{2}$ in.

3. A boiler shell is made up of three sheets, each 5 ft. $6\frac{1}{16}$ in. long. If the lap at each of the two middle seams is $2\frac{1}{2}$ in., what is the length of the shell? Ans. 16 ft. $3\frac{1}{16}$ in.

4. In a return tubular boiler the heating surface is divided as follows: Outside of shell, 98 sq. ft. 9.8 sq. in.; heads, 5 sq. ft. $4\frac{1}{2}$ sq. in.; tubes, 683 sq. ft. 10.75 sq. in. What is the total area of the heating surface in square feet and square inches? Ans. 786 sq. ft. 25.05 sq. in.

INVOLUTION.

244. **Involution** is the process of multiplying a number by itself one or more times. The product obtained by multiplying a number by itself is called a **power** of that number.

Thus, the *second* power of 3 is 9, since 3×3 are 9.
The *third* power of 3 is 27, since $3 \times 3 \times 3$ are 27.
The *fifth* power of 2 is 32, since $2 \times 2 \times 2 \times 2 \times 2$ are 32.

245. An **exponent** is a *small figure* placed to the *right* and a little *above* a number to show to what *power* it is to be raised, or how many times the number is to be used as a factor, as the small figures ^{2}, ^{3}, and ^{5} below:

$$3^2 = 3 \times 3 = 9.$$
$$3^3 = 3 \times 3 \times 3 = 27.$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32.$$

246. The **root** of a number is that number which, used the required number of times as a factor, produces the number. In the above cases, 3 is a root of 9, since 3×3 are 9.

It is also a root of 27, since $3 \times 3 \times 3$ are 27. Also, 2 is a root of 32, since $2 \times 2 \times 2 \times 2 \times 2$ are 32.

247. The *second* power of a number is called its **square.**

Thus, 5^2 is called the **square of 5, or 5 *squared*,** and its value is $5 \times 5 = 25$.

248. The *third* power of a number is called its **cube.**

Thus, 5^3 is called the *cube* of 5, or **5 *cubed*,** and its value is $5 \times 5 \times 5 = 125$.

To find any power of a number:

249. EXAMPLE.—What is the third power, or cube, of 35?

SOLUTION.—To find the cube of a number, use the number three times as a factor.

$35 \times 35 \times 35,$

or

$$\begin{array}{r} 35 \\ 35 \\ \hline 175 \\ 105 \\ \hline 1225 \\ 35 \\ \hline 6125 \\ 3675 \\ \hline \end{array}$$

cube = 42875 Ans.

EXAMPLE.—What is the fourth power of 15?

SOLUTION.—

$15 \times 15 \times 15 \times 15,$

or

$$\begin{array}{r} 15 \\ 15 \\ \hline 75 \\ 15 \\ \hline 225 \\ 15 \\ \hline 1125 \\ 225 \\ \hline 3375 \\ 15 \\ \hline 16875 \\ 3375 \\ \hline \end{array}$$

fourth power = 50625 Ans.

250. EXAMPLE.—1.2^3 = what?

SOLUTION.— $1.2 \times 1.2 \times 1.2$,

or

$$\begin{array}{r} 1.2 \\ 1.2 \\ \hline 1.44 \\ 1.2 \\ \hline 1.728 \end{array}$$ Ans.

EXAMPLE.—$.47^5$ = what?

SOLUTION.— $.47 \times .47 \times .47 \times .47 \times .47$,

or

$$\begin{array}{r} .47 \\ .47 \\ \hline 329 \\ 188 \\ \hline .2209 \\ .47 \\ \hline 15463 \\ 8836 \\ \hline .103823 \\ .47 \\ \hline 726761 \\ 415292 \\ \hline .04879681 \\ .47 \\ \hline 34157767 \\ 19518724 \\ \hline .0229345007 \end{array}$$ Ans.

251. EXAMPLE.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $(\frac{3}{8})^3 = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

252. Rule.—I. *To raise a whole number, or a decimal, to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

253. Raise the following to the powers indicated:

(*a*) 85^2.
(*b*) $(\frac{12}{13})^2$.
(*c*) 6.5^2.
(*d*) 14^4.

Ans. (*a*) 7,225. (*b*) $\frac{144}{169}$. (*c*) 42.25. (*d*) 38,416.

(*e*) $(\frac{4}{5})^3$.
(*f*) $(\frac{2}{3})^5$.
(*g*) $(\frac{7}{8})^2$.
(*h*) 1.4^3.

Ans. (*e*) $\frac{64}{125}$. (*f*) $\frac{32}{243}$. (*g*) $\frac{49}{64}$. (*h*) 5.37824.

EVOLUTION.

254. **Evolution** is the reverse of **involution**. It is the process of finding the root of a number which is considered as a power.

255. The **square root** of a number is that number which, when used twice as a factor, produces the number. Thus, 2 is the square root of 4, since 2×2, or $2^2 = 4$.

256. The **cube root** of a number is that number which, when used three times as a factor, produces the number. Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or $3^3 = 27$.

257. The **radical sign** $\sqrt{\ }$, when placed before a number, indicates that some root of that number is to be found.

258. The **index** of the root is a *small figure* placed over and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.
$\sqrt[3]{125}$ denotes the *cube root* of 125.
$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

259. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

SQUARE ROOT.

260. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This

is done by pointing off the number into *periods* of *two* figures each, *beginning at the right.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83'74'08'01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures, since there are (5'06'25) 3 periods.

261. EXAMPLE.—Find the square root of 31,505,769.

SOLUTION.—

				root.	
	(*a*) 5		31'50'57'69	5613	Ans.
	5	(*b*)	25		
(*d*)	100	(*c*)	650		
	6		636		
	106	(*e*)	1457		
	6		1121		
	1120		33669		
	1		33669		
	1121		0		
	1				
	11220				
	3				
	11223				

EXPLANATION.—Pointing off into periods of two figures each, it is seen that there are four figures in the root. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract, and obtain 6 as a remainder.

Bring down the next period 50, and annex it to the remainder 6, as shown at (*c*), which we call the **dividend.** Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, which we call the **trial divisor.** Divide the dividend (*c*) by the trial divisor (*d*), and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it

to 100, the trial divisor, making it 106. This is called the **complete divisor.**

Multiply this by 6, the second figure in the root, and subtract the result from the dividend (*c*). The remainder is 14, to which annex the next period, making it 1,457, as shown at (*e*), which we call the **new dividend.** Add the second figure of the root to the complete divisor 106, and annex a cipher, thus getting 1,120. Dividing 1,457 by 1,120, we get 1 as the next figure of the root. Adding this last figure of the root to 1,120, multiplying the result by it, and subtracting from 1,457, the remainder is 336.

Annexing the next and last period, 69, the result is 33,669. Now, adding the last figure of the root to 1,121, and annexing a cipher as before, the result is 11,220. Dividing 33,669 by 11,220, the result is 3, the fourth figure in the root. Adding it to 11,220, and multiplying the sum by it, the result is 33,669. Subtracting, there is no remainder; hence, $\sqrt{31,505,769} = 5,613$. Ans.

262. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$; $.13^2 = .0169$; $.751^2 = .564001$, etc.

263. It will also be noticed that the number squared is always less than the decimal. Hence, if it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to determine the number of figures in the root of a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

264. EXAMPLE.—What is the square root of .000576?

SOLUTION.—

		root
	2	.00'05'76(.024 Ans.
	2	4
	40	176
	4	176
	44	0

EXPLANATION.—Beginning at the decimal point, and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

265. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

266. EXAMPLE.—What is the square root of 3? Find the result to five decimal places.

SOLUTION.—

		root	
1		3.00'00'00'00'00(	1.73205+ Ans.
1		1	
20		200	
7		189	
27		1100	
7		1029	
340		7100	
3		6924	
343		176000	
3		173205	
3460		2795	
2			
3462			
2			
346400			
5			
346405			

EXPLANATION.—Annexing five periods of two ciphers each to the right of the decimal point, the first figure of the root is 1. To get the second figure, we find that, in dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20, and multiply 29 by 9, the result is 261, a result which is considerably larger than 200; hence,

9 is too large. In the same way it is found that 8 is also too large. Trying 7, 7 times 27 is 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor 34,640 is greater than the new dividend 17,600. In a case of this kind, we annex another cipher to 34,640, thereby making it 346,400, and bring down the next period, making the 17,600, 1,760,000. The next figure of the root is 5, and as we now have five decimal places, we will stop.

The square root of 3 is, then, 1.73205 +. Ans.

267. EXAMPLE.—What is the square root of .3 to five decimal places?

SOLUTION.—

		root
5	.30'00'00'00'00	.54772 + Ans.
5	25	
100	500	
4	416	
104	8400	
4	7609	
1080	79100	
7	76629	
1087	247100	
7	219084	
10940	28016	
7		
10947		
7		
109540		
2		
109542		

EXPLANATION.—In the above example, we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

268. If it is required to find the square root of a mixed number, begin at the decimal point, and point off the

periods both ways. The manner of finding the root will then be exactly the same as in the previous cases.

269. EXAMPLE.—What is the square root of 258.2449?

SOLUTION.—

1	2'58.24'49 (16.07 Ans.
1	1
20	158
6	156
26	22449
6	22449
3200	0
7	
3207	

EXPLANATION.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root, and annex a cipher to 320, making it 3,200. Then, bringing down the next period 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07. Ans.

270. Proof.—*To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.*

271. Rule.—I. *Begin at units place, and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there is any.*

II. *Find the greatest number whose square is contained in the first or left-hand period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period; then annex the second period to the remainder.

III. *Add the first figure of the root to the number in the first column on the left, and annex a cipher to the result; this is the trial divisor. Divide the dividend by the trial divisor*

for the second figure in the root, and add this figure to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure in the root, and subtract this result from the dividend. (If this result is larger than the dividend, a smaller number must be tried for the second figure of the root.) Now bring down the third period, and annex it to the last remainder for a new dividend. Add the second figure of the root to the complete divisor, and annex a cipher for a new trial divisor.

IV. *Continue in this manner to the last period, after which, if any additional places in the root are required, bring down cipher periods, and continue the operation.*

V. *If at any time the trial divisor is not contained in the dividend, place a cipher in the root, annex a cipher to the trial divisor, and bring down another period.*

VI. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say, the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

272. Short Method.—If the number whose root is to be extracted is not an exact square, the root will be an interminable decimal. It is then usual to extract the root to a certain number of decimal places. In such cases, the work may be greatly shortened as follows: Determine to how many decimal places the work is to be carried, say 5, for example; add to this the number of places in the integral part of the root, say 2, for example, thus determining the number of figures in the root, in this case $5 + 2 = 7$. Divide this number by 2 and take the next higher number. In the above case, we have $7 \div 2 = 3\frac{1}{2}$; hence, we take 4, the next higher number. Now extract the root in the usual manner until the same number of figures have been obtained as was expressed by the number obtained above, in this case 4. Then form the trial divisor in the usual manner, but omitting to annex the cipher; divide the last remainder by the trial

divisor, as in long division, obtaining as many figures of the quotient as there are remaining figures of the root, in this case $7 - 4 = 3$. The remainder so obtained is the remaining figures of the root.

Consider the example in Art. **267.** Here there are 5 figures in the root. We therefore extract the root to 3 places in the usual manner, obtaining .547 for the first three root figures. The next trial divisor is 1,094 (with the cipher omitted), and the last remainder is 791. Then, $791 \div 1{,}094 = .723$, and the next two figures of the root are 72, the whole root being .54772 +. Always carry the division one place further than desired, and if the last figure is 5 or greater, increase the preceding figure by 1. This method should not be used unless the root contains five or more figures.

NOTE.—If the last figure of the root found in the regular manner is a cipher, carry the process one place further before dividing as described above.

EXAMPLES FOR PRACTICE.

273. Find the square root of

(*a*) 186,624.		(*a*) 432.
(*b*) 2,050,624.		(*b*) 1,432.
(*c*) 29,855,296.		(*c*) 5,464.
(*d*) .0116964.		(*d*) .1081 +.
(*e*) 198.1369.		(*e*) 14.0761.
(*f*) 994,009.	Ans.	(*f*) 997.
(*g*) 2.375 to four decimal places.		(*g*) 1.5411.
(*h*) 1.625 to three decimal places.		(*h*) 1.275.
(*i*) .3025.		(*i*) .55.
(*j*) .571428.		(*j*) .7559 +.
(*k*) .78125.		(*k*) .8839.

CUBE ROOT.

274. In the same manner as in the case of square root, it can be shown that the periods into which a number is divided, whose cube root is to be extracted, must contain

three figures, except that the first or left-hand period of a whole or mixed number may contain one, two, or three figures.

275. EXAMPLE.—What is the cube root of 875,741,853,696?

SOLUTION.—

(1)	(2)	(3) root
7	49	875'741'853'696 (7216 Ans.
7	98	343
14	14700	32741
7	424	30248
210	15124	2493853
2	428	1557361
212	1555200	936492696
2	2161	936492696
214	1557361	0
2	2162	
2160	155952300	
1	129816	
2161	156082116	
1		
2162		
1		
21630		
6		
21636		

EXPLANATION.—Write the work in three columns as follows: On the right place the number whose cube root is to be extracted, and point it off into periods of three figures each. Call this column (3). Find the largest number whose cube is less than or equal to the first period, in this case 7. Write the 7 on the right, as shown, for the first figure of the root, and also on the extreme left at the head of column (1). Multiply the 7 in column (1) by the first figure of the root 7, and write the product 49 at the head of column (2). Multiply the number in column (2) by the first figure of the root 7, and write the product 343 under the figures in the

first period. Subtract and bring down the next period, obtaining 32,741 for the dividend. Add the first figure of the root to the number in column (1), obtaining 14, which call the *first correction.* Multiply the first correction by the first figure of the root, add the product to the number in column (2), and obtain 147. Add the first figure of the root to the first correction, and obtain 21, which call the *second correction.* Annex *two* ciphers to the number in column (2), and obtain 14,700 for the trial divisor; also, annex *one* cipher to the second correction, and obtain 210. Dividing the dividend by the trial divisor, we obtain $\frac{32741}{14700} = 2+$, and write the 2 as the second figure of the root. Add the 2 to the second correction, and obtain 212, which, multiplied by the second figure of the root, and added to the trial divisor, gives 15,124, the complete divisor. This last result, multiplied by the second figure of the root and subtracted from the dividend, gives a remainder of 2,493. Annexing the third period, we obtain 2,493,853 for the new dividend. Adding the second figure of the root to the number in column (1), we get 214 as the new first correction; this, multiplied by the second figure of the root and added to the trial divisor, gives 15,552. Adding the second figure of the root to the first new correction gives 216 as the second new correction. Annexing two ciphers to the number in column (2) gives 1,555,200, the new trial divisor. Annexing one cipher to the second new correction gives 2,160. Dividing the new dividend by the new trial divisor, we obtain $\frac{2493853}{1555200} = 1+$, and write 1 as the third figure of the root. The remainder of the work should be perfectly clear from what has preceded.

276. In extracting the cube root of a decimal, proceed as above, taking care that each period contains *three* figures. Begin the pointing off at the decimal point, going towards the right. If the last period does not contain three figures, annex ciphers until it does.

277. EXAMPLE.—What is the cube root of .009129329 ?

SOLUTION.—

(1)	(2)	(3)
2	4	.009'129'329 (.209 root
2	8	8
4	120000	1129329
2	5481	1129329
600	125481	0
9		
609		

EXPLANATION.—Beginning at the decimal point, and pointing off as shown, the largest number whose cube is less than 9 is seen to be 2; hence, 2 is the first figure of the root. When finding the second figure, it is seen that the trial divisor 1,200 is greater than the dividend; hence, write a cipher for the second figure of the root; bring down the next period to form the new dividend; annex two ciphers to the trial divisor to form a new trial divisor; also, annex one cipher to the 60 in column (1). Dividing the new dividend by the new trial divisor, we get $\frac{1129329}{120000} = 9+$, and write 9 as the third figure of the root. Complete the work as before.

278. EXAMPLE.—What is the cube root of 78,347.809639 ?

SOLUTION.—

(1)	(2)	(3)
4	16	78'347.809'639 (42.79 root
4	32	64
8	4800	14347
4	244	10088
120	5044	4259809
2	248	3766483
122	529200	493326639
2	8869	493326639
124	538069	0
2	8918	
1260	54698700	
7	115371	
1267	54814071	
7		
1274		
7		
12810		
9		
12819		

EXPLANATION.—Since we have a mixed number, begin at the decimal point and point off periods of three figures each, in both directions. The first period contains but two figures, and the largest number whose cube is less than 78 is 4; consequently, 4 is the first figure of the root. The remainder of the work should be perfectly clear. When dividing the dividend by the trial divisor for the third figure of the root, the quotient was 8 +; but, on trying it, it was found that 8 was too large, the complete divisor being considerably larger than the trial divisor. Therefore, 7 was used instead of 8.

279. EXAMPLE.—What is the cube root of 5 to five decimal places?

SOLUTION.—

			root
1	1	5.000'000'000'000'000(1.70997+	
1	2	1	
2	300	4000	
1	259	3913	
30	559	87000000	
7	308	78443829	
37	8670000	8556171000	
7	45981	7889992299	
44	8715981	666178701000	
7	46062	614014817978	
5100	876204300	52164383027	
9	461511		
5109	876665811		
9	461592		
5118	87712740300		
9	3590839		
51270	87716331139		
9			
51279			
9			
51288			
9			
512970			
7			
512977			

EXPLANATION.—In the preceding example we annex five periods of ciphers, of three ciphers each, to the 5 for the decimal part of the root, placing the decimal point between the 5 and the first cipher. Since it is easy to see that the next figure of the root will be 5, we increase the last figure by 1, obtaining 1.70998 for the correct root to 5 decimal places. Ans.

280. EXAMPLE.—What is the cube root of .5 to four decimal places?

SOLUTION.—

			root
7	49	.500'000'000'000	(.7937+
7	98	343	
14	14700	157000	
7	1971	150039	
210	16671	6961000	
9	2052	5638257	
219	1872300	1322743000	
9	7119	1321748953	
228	1879419	994047	
9	7128		
2370	188654700		
3	166579		
2373	188821279		
3			
2376			
3			
23790			
7			
23797			

EXPLANATION.—In the above example we annex two ciphers to the .5 to complete the first period, and three periods of three ciphers each. The cube root of 500 is 7; this we write as the first figure of the root. The remainder of the work should be perfectly plain from the explanations of the preceding examples.

281. EXAMPLE.—What is the cube root of .05 to four decimal places?

SOLUTION.—

		root
3	9	.050'000'000'000(.3684+
3	18	27
6	2700	23000
3	576	19656
90	3276	3344000
6	612	3180032
96	388800	163968000
6	8704	162685504
102	397504	1282496
6	8768	
1080	40627200	
8	44176	
1088	40671376	
8		
1096		
8		
11040		
4		
11044		

282. Proof.—*To prove cube root, cube the result obtained. If the given number is an exact power, the cube of the root will equal it; if not an exact power, the cube of the root will very nearly equal it.*

283. Rule.—I. *Arrange the work in three columns, placing the number whose cube root is to be extracted in the third or right-hand column. Begin at units place, and separate the number into periods of three figures each, proceeding from the decimal point towards the right with the decimal part, if there is any.*

II. *Find the greatest number whose cube is not greater than the number in the first period. Write this number as the first figure of the root; also, write it at the head of the first column. Multiply the number in the first column by the first figure in the root, and write the result in the second*

column. Multiply the number in the second column by the first figure of the root; subtract the product from the first period, and annex the second period to the remainder for a new dividend; add the first figure of the root to the number in the first column for the first correction. Multiply the first correction by the first figure of the root, and add the product to the number in the second column. Add the first figure of the root to the first correction to form the second correction. Annex one cipher to the second correction, and two ciphers to the last number in the second column; the last number in the second column is the trial divisor.

III. *Divide the dividend by the trial divisor to find the second figure of the root. Add the second figure of the root to the number in the first column, multiply the sum by the second figure of the root, and add the result to the trial divisor to form the complete divisor. Multiply the complete divisor by the second figure of the root, subtract the result from the dividend in the third column, and annex the third period to the remainder for a new dividend. Add the second figure of the root to the number in the first column to form the first correction; multiply the first correction by the second figure of the root, and add the product to the complete divisor. Add the second figure of the root to the first correction to form the second correction. Annex one cipher to the second correction, and two ciphers to the last number in the second column to form the new trial divisor.*

IV. *If there are more periods to be brought down, proceed as before. If there is a remainder after the root of the last period has been found, annex cipher periods, and proceed as before. The figures of the root thus obtained will be decimals.*

V. *If the root contains an interminable decimal, and it is desired to terminate the operation at some point, say the fourth decimal place, carry the operation one place further, and if the fifth figure is 5 or greater, increase the fourth figure by 1 and omit the sign +.*

284. Art. **272** can be applied to cube root (or any other root) as well as to square root. Thus, in the **example,**

Art. **279**, there are to be $5+1=6$ figures in the root. Extracting the root in the usual manner to $6 \div 2 = 3$, say 4 figures, we get for the first four figures 1,709. The last remainder is 8,556,171, and the next trial divisor, with the ciphers omitted, is 8,762,043. Hence, the next two figures of the root are $8{,}556{,}171 \div 8{,}762{,}043 = .976$, say .98. Therefore, the root is 1.70998.

ROOTS OF FRACTIONS.

285. If the given number is in the form of a fraction, and it is required to find some root of it, the simplest and most exact method is to reduce the fraction to a decimal and extract the required root of the decimal. If, however, the numerator and denominator of the fraction are perfect powers, extract the required root of each separately, and write the root of the numerator for a new numerator, and the root of the denominator for a new denominator.

286. EXAMPLE.—What is the square root of $\frac{9}{64}$?

SOLUTION.— $\sqrt{\frac{9}{64}} = \frac{\sqrt{9}}{\sqrt{64}} = \frac{3}{8}$. Ans.

287. EXAMPLE.—What is the square root of $\frac{5}{8}$?

SOLUTION.—Since $\frac{5}{8} = .625$, $\sqrt{\frac{5}{8}} = \sqrt{.625} = .7906$. Ans.

288. EXAMPLE.—What is the cube root of $\frac{27}{64}$?

SOLUTION.— $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$. Ans.

289. EXAMPLE.—What is the cube root of $\frac{1}{4}$?

SOLUTION.—Since $\frac{1}{4} = .25$, $\sqrt[3]{\frac{1}{4}} = \sqrt[3]{.25} = .62996+$. Ans.

290. Rule.—*Extract the required root of the numerator and denominator separately; or, reduce the fraction to a decimal, and extract the root of the decimal.*

EXAMPLES FOR PRACTICE.

291. Find the cube root of

(*a*) $\frac{27}{125}$.
(*b*) 2 to five decimal places.
(*c*) 4,180,769,192.462 to five decimal places.
(*d*) $\frac{27}{125}$.
(*e*) $\frac{7}{8}$.
(*f*) 518,229.783302144 to three decimal places.

Ans. (*a*) $\frac{3}{5}$. (*b*) 1.25992+. (*c*) 1,610.96238. (*d*) .8862+. (*e*) .7211+. (*f*) 80.064.

TO EXTRACT OTHER ROOTS THAN THE SQUARE AND CUBE ROOTS.

292. EXAMPLE.—What is the fourth root of 256?

SOLUTION.— $\sqrt{256} = 16.$

$\sqrt{16} = 4.$

Therefore, $\sqrt[4]{256} = 4.$ Ans.

In this example, $\sqrt[4]{256}$, the index is 4, which equals 2×2. The root indicated by 2 is the square root; therefore, the square root is extracted twice.

293. EXAMPLE.—What is the sixth root of 64?

SOLUTION.— $\sqrt{64} = 8.$

$\sqrt{8} = 2.$

Therefore, $\sqrt[6]{64} = 2.$ Ans.

In this example, $\sqrt[6]{64}$, the index is 6, which equals 2×3. The root indicated by 3 is the cube root; therefore, the square and cube roots are extracted in succession.

294. Rule.—*Separate the index of the required root into its factors (2's and 3's), and extract successively the roots indicated by the several factors obtained. The final result will be the required root.*

295. EXAMPLE.—What is the sixth root of 92,873,580 to two decimal places?

SOLUTION.— $6 = 3 \times 2$. Hence, extract the cube root, and then extract the square root of the result. $\sqrt[3]{92,873,580} = 452.8601$, and $\sqrt{452.8601} = 21.28+$. Ans.

296. It matters not which root is extracted first, but it is probably easier and more exact to extract the cube root first.

EXAMPLES FOR PRACTICE.

297. Extract the

(*a*) Fourth root of 100.
(*b*) Fourth root of 3,049,800,625.
(*c*) Sixth root of 9,474,296,896.

Ans. (*a*) 3.16227+. (*b*) 235. (*c*) 46.

RATIO.

298. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$ or .2 of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

299. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4, and express this comparison as a ratio, it may be done as follows: $20 \div 4$; $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4.* The ratio of 4 to 20 would be expressed thus: $4 \div 20$; $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third is rapidly growing in favor, and is likely to supersede the second. The third form, called the fractional form, is preferred by modern mathematicians, and possesses great advantages to students of algebra and of higher mathematical subjects. The second form seems to be better adapted to arithmetical subjects, and is one we shall ordinarily adopt. There is still another way of expressing a ratio, though seldom or never used in the case of a simple ratio like that given above. Instead of the colon, a straight vertical line is used; thus, 20 | 4.

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300. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet;** when considered separately, the first term is called the **antecedent,** and the second term the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent, and 4 the consequent.

301. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal** ratio. The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{5}$ is $1 \div \frac{3}{5} = \frac{5}{3}$; i.e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

302. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

303. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20:4 is 5, it is the quotient obtained by dividing the antecedent by the consequent.

304. By expressing the ratio in the fractional form, for example, the ratio of 20 to 4 as $\frac{20}{4}$, it is easy to see, from the laws of fractions, that if both terms be multiplied, or both divided by the same number, it will not alter the value of the ratio. Thus,

$$\frac{20}{4} = \frac{20 \times 5}{4 \times 5} = \frac{100}{20}; \text{ and } \frac{20}{4} = \frac{20 \div 4}{4 \div 4} = \frac{5}{1}.$$

305. It is also evident, from the laws of fractions, that multiplying the antecedent or dividing the consequent multiplies the ratio; and dividing the antecedent or multiplying the consequent divides the ratio.

306. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

EXAMPLES FOR PRACTICE.

307. What is the value of the ratio of

(*a*)	98 to 49?	(*a*)	2.
(*b*)	\$45 to \$9?	(*b*)	5.
(*c*)	$6\frac{1}{2}$ to $\frac{1}{4}$?	(*c*)	$12\frac{1}{2}$.
(*d*)	3.5 to 4.5?	(*d*)	$.77\frac{7}{9}$.
(*e*)	The inverse ratio of 76 to 19?	(*e*)	$\frac{1}{4}$.
(*f*)	The inverse ratio of 49 to 98?	(*f*)	2.
(*g*)	The inverse ratio of 18 to 24?	(*g*)	$1\frac{1}{3}$.
(*h*)	The inverse ratio of 9 to 15?	(*h*)	$1\frac{2}{3}$.
(*i*)	The ratio of 10 to 3, multiplied by 3?	(*i*)	10.
(*j*)	The ratio of 35 to 49, multiplied by 7?	(*j*)	5.
(*k*)	The ratio of 18 to 64, divided by 9?	(*k*)	$\frac{1}{32}$.
(*l*)	The ratio of 14 to 28, divided by 5?	(*l*)	$\frac{1}{10}$.

Ans.

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308. Instead of expressing the value of a ratio by a single number, as above, it is customary to express it by

means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30, the consequent, we obtain $\frac{1\frac{1}{2}}{1}$ or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

309. A ratio may be squared, cubed, or raised to any power, or any root of it may be taken. Thus, if the ratio of two numbers is 105 : 63, and it is desired to cube this ratio, the cube may be expressed as $105^3 : 63^3$. That this is correct is readily seen; for, expressing the ratio in the fractional form, it becomes $\frac{105}{63}$, and the cube is $\left(\frac{105}{63}\right)^3 = \frac{105^3}{63^3} = 105^3 : 63^3$. Also, if it is desired to extract the cube root of the ratio $105^3 : 63^3$, it may be done by simply dividing the exponents by 3, obtaining 105 : 63. This may be proved in the same way as in the case of cubing the ratio. Thus, $105^3 : 63^3 = \left(\frac{105}{63}\right)^3$, and $\sqrt[3]{\left(\frac{105}{63}\right)^3} = \frac{105}{63} = 105 : 63$.

310. Since $\left(\frac{105}{63}\right)^3 = \left(\frac{5}{3}\right)^3$, it follows that $105^3 : 63^3 = 5^3 : 3^3$ (this expression is read: the ratio of 105 cubed to 63 cubed equals the ratio of 5 cubed to 3 cubed), and, hence, that the antecedent and consequent may both be multiplied or both divided by the same number, irrespective of any indicated powers or roots, without altering the value of the ratio. Thus, $24^2 : 18^2 = 4^2 : 3^2$. For, performing the operations indicated by the exponents, $24^2 = 576$ and $18^2 = 324$. Hence, $576 : 324 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$. Also, $4^2 = 16$ and $3^2 = 9$; hence, $16 : 9 = 1\frac{7}{9}$ or $1\frac{7}{9} : 1$, the same result as before. Also, $24^2 : 18^2 = \frac{24^2}{18^2} = \left(\frac{24}{18}\right)^2 = \left(\frac{4}{3}\right)^2 = \frac{4^2}{3^2} = 4^2 : 3^2$.

The statement may be proved for roots in the same manner. Thus, $\sqrt[3]{24^3} : \sqrt[3]{18^3} = \sqrt[3]{4^3} : \sqrt[3]{3^3}$. For the $\sqrt[3]{24^3} = 24$ and $\sqrt[3]{18^3} = 18$; and, $24 : 18 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$. Also, $\sqrt[3]{4^3} = 4$ and $\sqrt[3]{3^3} = 3$; $4 : 3 = 1\frac{1}{3}$ or $1\frac{1}{3} : 1$.

NOTE.—If the numbers composing the antecedent and consequent have different exponents, or if different roots of those numbers are indicated, the operations described in Art. **310** cannot be performed. This is evident; for, consider the ratio $4^2 : 8^3$. When expressed in the fractional form, it becomes $\frac{4^2}{8^3}$, which cannot be expressed either as $\left(\frac{4}{8}\right)^2$ or as $\left(\frac{4}{8}\right)^3$, and, hence, cannot be reduced as described above.

PROPORTION.

311. **Proportion** is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8:4 and 6:3, which both have the same value 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$

$$8 : 4 = 6 : 3 \qquad (2)$$

$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

312. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and, in time, will probably entirely supersede the first form. In this paper we shall adopt the second form, unless some statement can be made clearer by using the third form.

313. A proportion may be *read* in two ways. The old way to read the above proportion was—*8 is to 4 as 6 is to 3;* the new way is—*the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

314. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8:4 is a couplet, and so is 6:3.

315. The numbers forming the proportion are called **terms**; and they are numbered consecutively from left to right, thus:

first second third fourth

$$8 : 4 = 6 : 3$$

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

316. The first and fourth terms of a proportion are called the **extremes**, and the second and third terms, the **means**. Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

317. A **direct proportion** is one in which both couplets are direct ratios.

318. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written $8 : 4 = 6 : 3$; i. e., the second ratio (couplet) must be inverted.

319. Proportion forms one of the most useful sections of Arithmetic. In our grandfathers' Arithmetics, it was called "The rule of three."

320. Rule I.—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

$$17 : 51 = 14 : 42.$$

$17 \times 42 = 51 \times 14$, since both products equal 714.

321. Rule II.—*The product of the extremes divided by either mean gives the other mean.*

EXAMPLE.—What is the third term of the proportion $17 : 51 = \quad : 42$?

SOLUTION.—Applying rule II, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

322. Rule III.—*The product of the means divided by either extreme gives the other extreme.*

EXAMPLE.—What is the first term of the proportion $\quad : 51 = 14 : 42$?

SOLUTION.—Applying rule III, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans.

323. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written,

$$x : 51 = 14 : 42$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

324. If the same (addition and subtraction excepted) operations be performed upon *all* of the terms of a proportion, the proportion is not thereby destroyed. In other words, if all of the terms of a proportion be (1) multiplied or (2) divided by the same number; (3) if all the terms be raised to the same power; if (4) the same root of all the terms be taken, or (5) if both couplets be inverted, the proportion still holds. We will prove these statements by a numerical example, and the student can satisfy himself by other similar ones. The fractional form will be used, as it is better suited to the purpose. Consider the proportion $8 : 4 = 6 : 3$. Expressing it in the third form, it becomes $\frac{8}{4} = \frac{6}{3}$. What we are to prove is that, if any of the five operations enumerated above be performed upon all of the terms of this proportion, the first fraction will still equal the second fraction.

1. Multiplying all the terms by any number, say 7, $\frac{8 \times 7}{4 \times 7} = \frac{6 \times 7}{3 \times 7}$; or $\frac{56}{28} = \frac{42}{21}$. Now $\frac{56}{28}$ evidently equals $\frac{42}{21}$, since the value of either ratio is 2, and the same is true of the original proportion.

2. Dividing all the terms by any number, say 7, $\frac{8 \div 7}{4 \div 7} = \frac{6 \div 7}{3 \div 7}$; or $\frac{\frac{8}{7}}{\frac{4}{7}} = \frac{\frac{6}{7}}{\frac{3}{7}}$. But $\frac{8}{7} \div \frac{4}{7} = 2$, and $\frac{6}{7} \div \frac{3}{7} = 2$ also, the same as in the original proportion.

3. Raising all the terms to the same power, say the cube, $\frac{8^3}{4^3} = \frac{6^3}{3^3}$. This is evidently true, since $\frac{8^3}{4^3} = \left(\frac{8}{4}\right)^3 = 2^3 = 8$, and $\frac{6^3}{3^3} = \left(\frac{6}{3}\right)^3 = 2^3 = 8$ also.

4. Extracting the same root of all the terms, say the cube root, $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \frac{\sqrt[3]{6}}{\sqrt[3]{3}}$. It is evident that this is likewise true, since $\frac{\sqrt[3]{8}}{\sqrt[3]{4}} = \sqrt[3]{\frac{8}{4}} = \sqrt[3]{2}$, and $\frac{\sqrt[3]{6}}{\sqrt[3]{3}} = \sqrt[3]{\frac{6}{3}} = \sqrt[3]{2}$ also.

5. Inverting both couplets, $\frac{4}{8} = \frac{3}{6}$, which is true, since both equal $\frac{1}{2}$.

325. If both terms of either couplet be multiplied or both divided by the same number, the proportion is not destroyed. This should be evident from the preceding article, and also from Art. **304.** Hence, in any proportion, equal factors may be canceled from the terms of a couplet, before applying rules II or III. Thus, the proportion $45 : 9 = x : 7.1$, we may divide both terms of the first couplet by 9 (that is, cancel 9 from both terms), obtaining $5 : 1 = x : 7.1$, whence $x = 7.1 \times 5 \div 1 = 35.5$. (See note in Art. **310.**)

326. The principle of all calculations in proportion is this: *Three of the terms are always given, and the remaining one is to be found.*

327. EXAMPLE.—If 4 men can earn $25 in one week, how much can 12 men earn in the same time?

SOLUTION.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is:

"What is it I want to find?"

In this case it is dollars. We have two sets of men, one set earning $25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to $25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

The amount 12 men earn : $25 = 12 men : 4 men,

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x or the required term occupies, the problem could be stated as any of the following forms, the value of x being the same in each:

(a) \$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

(b) 4 men : 12 men = \$25 : the amount 12 men earn; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.

(c) 12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn $= \frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.

328. If the proportion is an inverse one, first form it as though it were a direct proportion, and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

329. Find the value of x in each of the following:

(a) \$16 : \$64 :: x : \$4.
(b) x : 85 :: 10 : 17.
(c) 24 : x :: 15 : 40.
(d) 18 : 94 :: 2 : x.
(e) \$75 : \$100 = x : 100.
(f) 15 pwt. : x = 21 : 10.
(g) x : 75 yd. = \$15 : \$5.

Ans. (a) x = \$1. (b) x = 50. (c) x = 64. (d) $x = 10\frac{1}{2}$. (e) x = 75. (f) $x = 7\frac{1}{2}$ pwt. (g) x = 225 yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. $97\frac{3}{7}$ inches.

INVERSE PROPORTION.

330. In Art. **318**, an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the

statement of the example; in such cases the proportion can be written directly, merely inverting one of the couplets. But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term. Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example, and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

331. EXAMPLE.—If A's *rate* of doing work is to B's as 5 : 7, and A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a less number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5}$ $= 58.8$ days.

EXAMPLES FOR PRACTICE.

332. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. 6⅔ hr.

2. If a pump discharges 90 gal. of water in 20 hr., in what time will it discharge 144 gal.? Ans. 32 hr.

3. If 50 cu. ft. of air weigh 4.2 pounds when the absolute temperature is 562°, what will be the absolute temperature when the same volume weighs 5.8 pounds, the pressure being the same in both cases?

Ans. 407°, very nearly.

CAUSES AND EFFECTS.

333. Many examples in proportion may be more easily solved by using the principle of *cause and effect*. That which may be regarded as producing a change or alteration in something, or as accomplishing something, may be called a **cause**, and the change or alteration, or thing accomplished, is the **effect**.

334. *Like causes produce like effects.* Hence, when two causes of the same kind produce two effects of the same kind, the ratio of the causes equals the ratio of the effects; in other words, the first cause is to the second cause as the first effect is to the second effect. Thus, in the question, if 3 men can lift 1,400 pounds, how many pounds can 7 men lift? we call 3 men and 7 men the *causes* (since they accomplish something, viz., the lifting of the weight); the number of pounds lifted, viz., 1,400 pounds and x pounds, are the effects. If we call 3 men the first cause, 1,400 pounds is the first effect; 7 men is the second cause, and x pounds is the second effect. Hence, we may write

1st cause	*2d cause*		*1st effect*		*2d effect*
3	: 7	=	1,400	:	x,

whence $x = \dfrac{7 \times 1{,}400}{3} = 3{,}266\frac{2}{3}$ pounds.

The principle of cause and effect is extremely useful in the solution of examples in compound proportion, as we shall now show.

COMPOUND PROPORTION.

335. All the cases of proportion so far considered have been cases of **simple proportion**; i. e., each term has been composed of but one number. There are many cases, however, in which two or all of the terms have more than one number in them; all such cases belong to **compound proportion.** In all examples in compound proportion, both causes or both effects or all four consist of more than two numbers. We will illustrate this by an

EXAMPLE.—If 40 men earn $1,280 in 16 days, how much will 36 men earn in 31 days?

SOLUTION.—Since 40 men earn something, 40 men is a cause, and since they take 16 days in which to earn something, 16 days is also a cause. For the same reason, 36 men and 31 days are also causes. The effects, that which is earned, are 1,280 dollars and x dollars. Then, 40 men and 16 days make up the first cause, and 36 men and 31 days make up the second cause. $1,280 is the first effect and $\$x$ is the second effect. Hence, we write

1st cause		*2d cause*		*1st effect*		*2d effect*
40 16	:	36 31	=	1,280	:	x

Now, instead of using the colon to express the ratio, we shall use the vertical line (see Art. **299**), and the above becomes

$$\begin{matrix}40\\16\end{matrix} \;\Big|\; \begin{matrix}36\\31\end{matrix} = 1{,}280 \;\Big|\; x.$$

In the last expression, the product of all of the numbers included between the vertical lines must equal the product of all the numbers without them; i. e., $36 \times 31 \times 1{,}280 = 40 \times 16 \times x$.

$$\text{Or } x = \frac{36 \times 31 \times \overset{2}{\cancel{1280}}}{\underset{\cancel{16}}{\cancel{40}} \times \cancel{16}} = \$2{,}232. \quad \text{Ans.}$$

336. The above might have been solved by canceling factors of the numbers in the original proportion. For if any number within the lines has a factor common to any number without the lines, that factor may be canceled from both numbers. Thus, 16 is contained in

$$\begin{matrix}\cancel{40}\\\cancel{16}\end{matrix} \;\Big|\; \begin{matrix}36\\31\end{matrix} = \begin{matrix}2\\\cancel{80}\\\cancel{1280}\end{matrix} \;\Big|\; x,$$

1,280, 80 times. Cancel 16 and 1,280, and write 80 above 1,280. 40 is contained in 80, 2 times. Cancel 40 and 80,

and write 2 above 80. Now, since there are no more numbers that can be canceled, $x = 36 \times 31 \times 2 = \$2{,}232$, the same result as was obtained in the last article.

337. Rule.—*Write all the numbers forming the first cause in a vertical column, and draw a vertical line; on the other side of this line write in a vertical column all of the numbers forming the second cause. Write the sign of equality to the right of the second column, and on the right of this form a third column of the numbers composing the first effect, drawing a vertical line to the right; on the other side of this line, write, for a fourth column, the numbers composing the second effect. There must be as many numbers in the second cause as in the first cause, and in the second effect as in the first effect; hence, if any term is wanting, write x in its place. Multiply together all of the numbers within the vertical lines, and also all those without the lines (canceling previously, if possible), and divide the product of those numbers which do not contain x by the product of the others in which x occurs, and the result will be the value of x.*

EXAMPLE.—If 40 men can dig a ditch 720 feet long, 5 feet wide, and 4 feet deep in a certain time, how long a ditch 6 feet deep and 3 feet wide could 24 men dig in the same time?

SOLUTION.—Here 40 men and 24 men are the causes and the two ditches are the effects. Hence,

$$\begin{array}{|c|c|c|c|} 40 & 24 & = & x \\ & & 5 & 3 \\ & & 4 & 6 \end{array}$$

whence, $x = 24 \times 5 \times 4 = 480$ feet. Ans.

EXAMPLE.—The volume of a cylinder varies directly as its length and directly as the square of its diameter. If the volume of a cylinder 10 inches in diameter and 20 inches long is 1,570.8 cubic inches, what is the volume of another cylinder 16 inches in diameter and 24 inches long?

SOLUTION.—In this example, either the dimensions or the volumes may be considered the causes; say we take the dimensions for the causes. Then, squaring the diameters,

$$\begin{array}{|c|c|c|c|c|c|c|c|} 10^2 & 16^2 & = 1{,}570.8 & x, & \text{or} & 100 & 256 & = 1{,}570.8 & x; \\ 20 & 24 & & & & 20 & 24 & & \\ & & & & & 5 & 6 & & \end{array}$$

whence, $x = \dfrac{256 \times 6 \times 1{,}570.8}{5 \times 100} = 4{,}825.4976$ cubic inches. Ans.

EXAMPLE.—If a block of granite 8 ft. long, 5 ft. wide, and 3 ft. thick weighs 7,200 lb., what will be the weight of a block of granite 12 ft. long, 8 ft. wide, and 5 ft. thick?

SOLUTION.—Taking the weights as the effects, we have

$$\begin{array}{c|c|c} 8 & 4 & \\ 5 & 12 & \\ 3 & 8 = 7{,}200 & x, \text{ or } x = 4 \times 7{,}200 = 28{,}800 \text{ pounds. Ans.} \\ & 5 & \end{array}$$

EXAMPLE.—If 12 compositors in 30 days of 10 hours each set up 25 sheets of 16 pages each, 32 lines to the page, in how many days 8 hours long can 18 compositors set up, in the same type, 64 sheets of 12 pages each, 40 lines to the page?

SOLUTION.—Here compositors, days, and hours compose the causes, and sheets, pages, and lines the effects. Hence,

$$\begin{array}{c|c|c|c} 3 & 3 & 5 & 2 \\ 12 & 18 & 25 & 64 \\ & & 4 & \\ 30 & x = & 16 & 12, \text{ or } x = 8 \times 10 \times 2 = 60 \text{ days. Ans.} \\ 8 & & & 4 \\ 10 & 8 & 32 & 40 \\ & & & 5 \end{array}$$

338. In examples stated like the second in Art. **337**, should an inverse proportion occur, write the various numbers as in the preceding examples, and then transpose those numbers which are said to vary inversely from one side of the vertical line to the other side.

EXAMPLE.—The centrifugal force of a revolving body varies directly as its weight, as the square of its velocity and inversely as the radius of the circle described by the center of the body. If the centrifugal force of a body weighing 15 pounds is 187 pounds when the body revolves in a circle having a radius of 12 inches, with a velocity of 20 feet per second, what will be the centrifugal force of the same body when the radius is increased to 18 inches and the speed is increased to 24 feet per second?

SOLUTION.—Calling the centrifugal force the effect, we have

$$\begin{array}{c|c|c|c} 15 & 15 & & \\ 20^2 & 24^2 & = 187 & x. \\ 12 & 18 & & \end{array}$$

Transposing 12 and 18 (since the radii are to vary inversely) and squaring 20 and 24,

$$\begin{array}{c|c|c} 15 & 15 & \\ & 2 & \\ 25 & 36 = 187 & x, \text{ or } x = \dfrac{12 \times 2 \times 187}{25} = 179.52 \text{ pounds. Ans.} \\ 400 & 576 & \\ 18 & 12 & \end{array}$$

EXAMPLES FOR PRACTICE.

339. Solve the following by compound proportion:

1. If 12 men dig a trench 40 rods long in 24 days of 10 hours each, how many rods can 16 men dig in 18 days of 9 hours each?

Ans. 36 rods.

2. If a piece of iron 7 ft. long, 4 in. wide, and 6 in. thick weighs 600 lb., how much will a piece of iron weigh that is 16 ft. long, 8 in. wide and 4 in. thick? Ans. 1,828$\frac{4}{7}$ lb.

3. If 24 men can build a wall 72 rods long, 6 feet wide, and 5 feet high in 60 days of 10 hours each, how many days will it take 32 men to build a wall 96 rods long, 4 feet wide and 8 feet high, working 8 hours a day? Ans. 80 days.

4. The horsepower of an engine varies as the mean effective pressure, as the piston speed and as the square of the diameter of the cylinder. If an engine having a cylinder 14 inches in diameter develops 112 horsepower when the mean effective pressure is 48 pounds per square inch and the piston speed is 500 feet per minute, what horsepower will another engine develop if the cylinder is 16 inches in diameter, piston speed is 600 feet per minute, and mean effective pressure is 56 pounds per square inch? Ans. 204.8 horsepower.

5. Knowing that the product of $3 \times 5 \times 7 \times 9$ is 945, what is the product of $6 \times 15 \times 14 \times 36$? Ans. 45,360.

MENSURATION AND USE OF LETTERS IN FORMULAS.

FORMULAS.

340. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note-book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities;" the word quantity is more general in its meaning than the word number.

341. The signs used in formulas are the ordinary signs indicative of operations, and the signs of aggregation. All these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

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The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, $\surd$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 \,/\, 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\surd$) indicates that some root of the quantity to the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\surd 25$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

The signs of aggregation are four in number; viz., ——, ( ), [ ], and { }, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8} \times 7$, $(5+8) \times 7$, $[5+8] \times 7$, $\{5+8\} \times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operations.

While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{(5+8)}$, $\sqrt[3]{[5+8]}$, or $\sqrt[3]{\{5+8\}}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis

is always used. Hence, $(5 + 8) \times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20 - 5) \div 3] \times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20 - 5) \div 3] \times 9 - 21\} \div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20 - 5} \div 3) \times 9 - 21] \div 8\} \times 12$.

As stated in arithmetic, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2 + 3 \times 4$ equals 14, 3 being multiplied by 4, before adding to 2. Similarly, $10 \div 2 \times 5$ equals 1, since 2×5 equals 10, and $10 \div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5 - 9 \div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

There is one other sign used, which is neither a sign of aggregation nor a sign indicative of an operation to be

performed; it is (=), and is called the sign of *equality;* it means that all on one side of it is exactly equal to all on the other side. For example, $2 = 2$, $5-3 = 2$, $5\times(14-9) = 25$.

342. Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam-engine, which may be stated as follow:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (viz., the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\text{Horsepower} = \frac{\text{mean effective pressure (in pounds per square inch)} \times \text{stroke (in feet)} \times \text{area of piston (in square inches)} \times \text{number of strokes (per minute)}}{33{,}000}.$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33{,}000},$$

a much simpler and shorter expression. This last expression is called a *formula.*

The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33{,}000}.$$

The sign of multiplication, evidently, can not be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1{,}006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum, and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{\overline{b+c}} \times a$, etc., are always written as here printed.

343. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial

letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further, and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned, when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam, at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter, and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure, and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure, or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented; in many instances, however, it is not convenient to use them, in which case

primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^{1}, p^{2}, p^{3}, p^{a}, etc., as they are liable to be mistaken for exponents.

The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body; etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters, and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, $3\frac{1}{4}$ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\frac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\frac{1}{4} \times .1138} =$$

$$\frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

344. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\frac{160}{\frac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\frac{\frac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\frac{160}{7+\frac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7+\frac{660}{25}=\frac{175+660}{25}$, if we substitute $\frac{175+660}{25}$ for $7+\frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7+\frac{660}{25}}=\frac{160}{\frac{175+660}{25}}=\frac{160}{\frac{835}{25}}$$

Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc., of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the

denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1+a^2\,b)^3}$ may also be written $(1+a^2\,b)^{\frac{3}{5}}$, a much simpler expression.

345. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r-h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
and h = height of segment.

EXAMPLE.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

SOLUTION.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r-h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5-1.91) =$$

40.968 − 27.95 = 13.018 square inches, nearly. Ans.

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2+b^2-c^2}{2\,b}\right)^2}.$$

EXAMPLE.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

SOLUTION.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then substituting in the formula,

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50}\right)^2}$$

$$= \frac{50}{2}\sqrt{441 - \left(\frac{441 + 2{,}500 - 2{,}116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$

$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$

$$= 25 \times 19.312 = 482.8 \text{ square feet, nearly. Ans.}$$

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}},$$

in which P = load (pressure) in pounds;

S = ultimate strength (in pounds per square inch) of the material composing the column;

A = area of cross-section of column in square inches;

q = a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;

l = length of column in inches;

and G = least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow wrought-iron column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and $q = \frac{1}{25{,}000}$ for flat-ended steel columns; A, the area of the cross-section, $= .7854\,(d_1^2 - d_2^2) = .7854\,(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters,

respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula,

$$P = \frac{SA}{1 + q\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}} =$$

$$\frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ pounds.} \quad \text{Ans.}$$

4. EXAMPLE.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following:

(*a*) $E = \sqrt[3]{\frac{BCD}{A\left(2 + \frac{D^2}{C^2}\right)}}$? (*b*) $E = \frac{A - \frac{1}{2}D + \frac{4B^2}{A + C}}{A - \sqrt{\frac{2B^2}{A + 22}}}$?

SOLUTION.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \quad \text{Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{1}{2} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}} =$$

$$\frac{7 + 17.066+}{10 - \sqrt{4}} = \frac{24.066+}{8} = 3.008+. \quad \text{Ans.}$$

345.—In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example,

let it be required to find how many strokes per minute an engine having a piston area of 78.54 sq. in. must make in order to develop 60 horsepower, if the mean effective pressure is 40 lb. per sq. in., and the length of stroke $1\frac{1}{4}$ ft. By substituting the given values in the formula $H = \frac{PLAN}{33{,}000}$, we have

$$60 = \frac{40 \times 1\frac{1}{4} \times 78.54 \times N}{33{,}000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33{,}000} \times N$, a fraction whose numerator is composed of 3 factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1{,}000} \times N = .119\,N.$$

If 60 equals .119 N, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{m\,v^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4{,}500}{r}.$$

But if 375 equals 4,500 divided by r, then $375 \times r = 4{,}500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4{,}500}{375} = 12$.

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{50}$.

2. $x = \frac{\frac{1}{2}(A + e)}{ce}$. Ans. $x = 1\frac{1}{8}$.

3. $x = \sqrt{\frac{d^3}{2c} + \sqrt{AB^3}}$. Ans. $x = 29$.

4. $x = \frac{Ae}{\sqrt{16Bc}} + \frac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2e)\left(\sqrt[3]{B} - \frac{1}{c}\right) + \frac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{17}$.

6. $x = \sqrt{\frac{Bcd}{A\left(2 + \frac{d^2}{e^2}\right)}}$. Ans. $x = .396+$.

MENSURATION.

346. **Mensuration** treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

347. A straight line is one that does not change its direction throughout its whole length. To distinguish one straight line from another, its two extreme points are designated by letters. The line shown in Fig. 1 would be called the line $A\ B$.

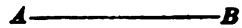

FIG. 1.

348. A **curved line** changes its direction at every point. Curved lines are designated by three or more letters, as the curved line $A\ B\ C$, Fig. 2.

FIG. 2.

349. **Parallel lines** (Fig. 3) are those which are equally distant from each other at all points.

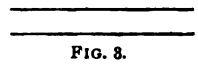
FIG. 3.

350. A line is **perpendicular** to another (see Fig. 4) when it meets that line so as not to incline towards it on either side.

FIG. 4.

351. A **vertical line** is one that points towards the center of the earth, and is also known as a *plumb* line.

352. A **horizontal line** (see Fig. 5) is one that makes a right angle with any vertical line.

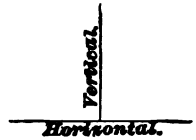

FIG. 5.

353. An **angle** is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines $A\ B$ and $C\ B$ is called the angle $A\ B\ C$, or the angle $C\ B\ A$; the letter at the vertex is always placed in the middle. When an angle stands alone so that it can not be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle B.

FIG. 6.

354. If one straight line meets another straight line at a point between its ends, as in Fig. 7, two angles, $A\ B\ C$ and $A\ B\ D$, are formed, which are called **adjacent angles.**

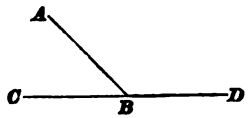

FIG. 7.

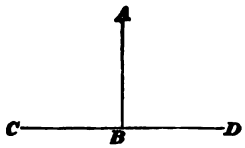

FIG. 8.

355. When these adjacent angles, $A\ B\ C$ and $A\ B\ D$, are equal, as in Fig. 8, they are called **right angles.**

356. An **acute angle** is less than a right angle. $A\ B\ C$, Fig. 9, is an acute angle.

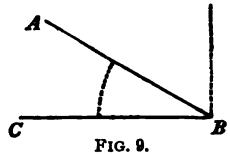

FIG. 9.

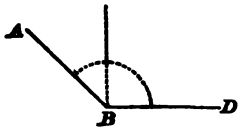

FIG. 10.

357. An **obtuse angle** is greater than a right angle. $A\ B\ D$ (Fig. 10) is an obtuse angle.

358. A **circle** (see Fig. 11) is a figure bounded by a curved line, called the **circumference**, every point of which is equally distant from a point within, called the **center.**

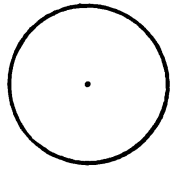
FIG. 11.

359. An **arc** of a circle is any part of its circumference; thus *a e b*, Fig. 12, is an arc of the circle.

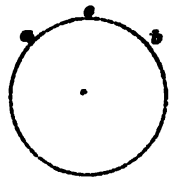

FIG. 12.

The circumference of every circle is considered to be divided into 360 equal parts, or arcs, called **degrees**; every degree is subdivided into 60 equal parts, called **minutes**, and every minute is again divided into 60 equal parts, called **seconds**.

Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of a degree will be different in circles of different sizes, but the proportion of the length of an arc of one degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Degrees, minutes, and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″ is read 37 degrees, 14 minutes, and 44 seconds.

360. The arcs of circles are used to measure angles. An angle having its vertex at the center of a circle is measured by the arc included between its sides; thus, in Fig. 13, the arc *F B* measures the angle *F O B*. If the arc *F B* contains 20°, or $\frac{20}{360}$ of the circumference, the angle *F O B* would be an angle of 20°; if it contained 20° 14′ 18″, it would be an angle of 20° 14′ 18″, etc.

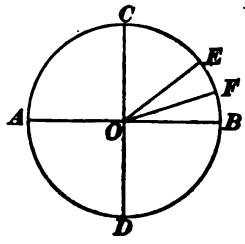

FIG. 13.

In the figure, if the line *C D* be drawn perpendicular to *A B*, the adjacent angles will be equal, and the circle will be divided into four equal angles, each of which will be a right angle. A right angle, therefore, is an angle of $\frac{360°}{4}$, or 90°; two right angles are measured by 180°, or half the circumference, and four right angles by the whole circumference, or 360°. One-half of a right angle, as *E O B*, is an

angle of 45°. An *acute* angle may now be defined as an angle of *less* than 90°, and an *obtuse* angle as one of *more* than 90°. These values are important, and should be remembered.

361. From the foregoing it will be evident that if a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles, or 180°. Thus, in Fig. 14, angles $C\,O\,B + D\,O\,C + E\,O\,D + F\,O\,E + A\,O\,F$ = 2 right angles, or 180°.

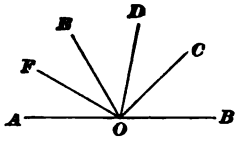

FIG. 14.

362. Also, if through a given point any number of straight lines be drawn, the sum of all the angles formed about the points of intersection equals four right angles, or 360°. Thus, in Fig. 15, angles $H\,O\,F + F\,O\,C + C\,O\,A + A\,O\,G + G\,O\,E + E\,O\,D + D\,O\,B + B\,O\,H$ = four right angles, or 360°.

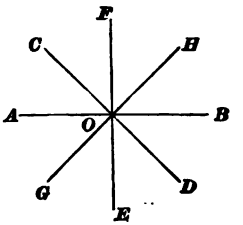

FIG. 15.

EXAMPLE.—In a fly-wheel, with 12 arms, how many degrees are there in the angle included between the center lines of any two arms, the arms being spaced equally?

SOLUTION.—Since there are 12 arms, there are 12 angles, which together equal 360°. Hence, one angle equals $\frac{1}{12}$ of 360°, or $\frac{360°}{12} = 30°$

EXAMPLES FOR PRACTICE.

1. How many seconds in 32° 14′ 6″? Ans. 116,046 sec.
2. How many degrees, minutes, and seconds do 38,582 seconds amount to? Ans. 10° 43′ 2″.
3. How many right angles are there in an angle of 170°? Ans. $1\frac{7}{9}$ right angles.
4. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms? Ans. $\frac{1}{5}$ of a right angle.
5. If one straight line meets another so as to form an angle of 20° 10′, what does its adjacent angle equal? Ans. 159° 50′.
6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle? Ans. 30°.

QUADRILATERALS.

363. A **plane figure** is any part of a plane or flat surface bounded by straight or curved lines.

364. A **quadrilateral** is a plane figure bounded by four straight lines.

365. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

366. A **rectangle** (Fig. 16) is a parallelogram whose angles are all right angles.

FIG. 16.

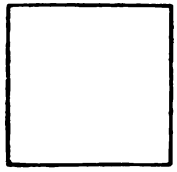

FIG. 17.

367. A **square** (Fig. 17) is a rectangle whose sides are all of the same length.

368. A **rhomboid** (Fig. 18) is a parallelogram whose opposite sides are equal and parallel, and whose angles are not right angles.

FIG. 18.

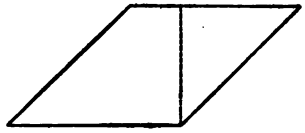

FIG. 19.

369. A **rhombus** (Fig. 19) is a parallelogram having equal sides, and whose angles are not right angles.

370. A **trapezoid** (Fig. 20) is a quadrilateral which has only two of its sides parallel.

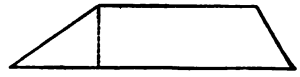

FIG. 20.

371. The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between the parallel lines, as shown by the dotted lines in Figs. 18, 19, and 20.

372. The **base** of *any* plane figure is the side on which it is supposed to stand.

373. The **area** of a surface is expressed by the number of unit squares it will contain.

374. A **unit square** is the square having a unit for its side. For example, if the unit is 1 inch, the unit square is the square each of whose sides measures 1 inch in length, and the area of a surface would be expressed by the number of square inches it would contain. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area of the given surface would be the number of square feet it would contain, etc.

The square that measures one inch on a side is called a **square inch,** and the one that measures one foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft.

375. To find the area of any parallelogram:

Rule.—*Multiply the base by the altitude.*

NOTE.—Before multiplying, the base and altitude must be reduced to the same kind of units; that is, if the base should be given in feet and the altitude in inches, they could not be multiplied together until either the altitude had been reduced to feet, or the base to inches. This principle holds throughout the subject of mensuration.

EXAMPLE.—The sides of a square piece of sheet iron are each $10\frac{1}{4}$ inches long. How many square inches does it contain?

SOLUTION.—$10\frac{1}{4}$ inches $= 10.25$ inches when reduced to a decimal. The base and altitude are each 10.25 inches. Multiplying them together, $10.25 \times 10.25 = 105.06+$ sq. in. Ans.

EXAMPLE.—What is the area in square rods of a piece of land in the shape of a rhomboid, one side of which is 8 rods long, and whose length, measured on a line perpendicular to this side, is 200 feet?

SOLUTION.—The base is 8 rods and the altitude 200 feet. As the answer is to be in rods, the 200 feet should be reduced to rods. Reducing $200 \div 16\frac{1}{2} = 200 \div \frac{33}{2} = 12.12$ rods. Hence, area $= 8 \times 12.12 = 96.96$ sq. rd. Ans.

376. To find the area of a trapezoid:

Rule.—*Multiply one-half the sum of the parallel sides by the altitude.*

EXAMPLE.—A board 14 feet long is 20 inches wide at one end and 16 inches wide at the other. If the ends are parallel, how many square feet does the board contain?

SOLUTION.—One-half the sum of the parallel sides $= \frac{20+16}{2} = 18$ inches $= 1\frac{1}{2}$ feet. The length of the board corresponds to the altitude of a trapezoid. Hence, $14 \times 1\frac{1}{2} = 21$ sq. ft. Ans.

377. Having given the area of a parallelogram and one dimension, to find the other dimension:

Rule.—*Divide the area by the given dimension.*

EXAMPLE.—What is the width of a parallelogram whose area is 212 square feet and whose length is $26\frac{1}{2}$ feet?

SOLUTION.—$212 \div 26\frac{1}{2} = 212 \div \frac{53}{2} = 8$ feet. Ans.

The following examples illustrate a few special cases:

EXAMPLE.—An engine room is 22 feet by 32 feet. The engine-bed occupies a space of 3 feet by 12 feet; the fly-wheel pit, a space of 2 feet by 6 feet, and the outer bearing, a space of 2 feet by 4 feet. How many square feet of flooring will be required for the room?

SOLUTION.—Area of engine-bed $= 3 \times 12 = 36$ sq. ft.
Area of fly-wheel pit $= 2 \times 6 = 12$ sq. ft.
Area of outer bearing $= 2 \times 4 = 8$ sq. ft.
Total, 56 sq. ft.
Area of engine room $= 22 \times 32 = 704$ sq. ft.
$704 - 56 = 648$ square feet of flooring required. Ans.

EXAMPLE.—How many square yards of plaster will it take to cover the sides and ceiling of a room 16×20 feet and 11 feet high, having four windows, each 7×4 feet, and three doors, each 9×4 feet over all, the baseboard coming 6 inches above the floor?

SOLUTION.—
Area of ceiling $= 16 \times 20 = 320$ sq. ft.
Area of end walls $= 2(16 \times 11) = 352$ sq. ft.
Area of side walls $= 2(20 \times 11) = 440$ sq. ft.
Total area $= 1{,}112$ sq. ft.

From the above must be deducted:
Windows $= 4(7 \times 4) = 112$ sq. ft.
Doors $= 3(9 \times 4) = 108$ sq. ft.

Baseboard less the width of three doors $= (72 - 12) \times \frac{6}{12} = 30$ sq. ft.

Total number of feet to be deducted $= 112 + 108 + 30 = 250$ sq. ft.

Hence, number of square feet to be plastered $= 1{,}112 - 250 = 862$ sq. ft., or $95\frac{7}{9}$ sq. yd. Ans.

EXAMPLE.—How many acres are contained in a rectangular tract of land 800 rods long and 520 rods wide?

SOLUTION.—$800 \times 520 = 416{,}000$ sq. rd. Since there are 160 square rods in one acre, the number of acres $= 416{,}000 \div 160 = 2{,}600$ acres. Ans

EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches, and whose altitude is 3 feet? Ans. 21 sq. ft.

2. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roof? Ans. 1,840 lb.

3. One side of a room measures 16 ft. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. How many square feet in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

5. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case.

6. How many square yards of plastering will be required for the ceiling and walls of a room 10×15 feet and 9 feet high; the room contains one door $3\frac{1}{2} \times 7$ feet, three windows $3\frac{1}{2} \times 6$ feet, and a baseboard 8 inches high? Ans. 53.5 sq. yd.

THE TRIANGLE.

378. A **triangle** is a plane figure having three sides.

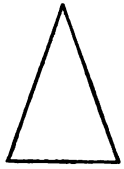

FIG. 21.

379. An **isosceles** triangle is one having two of its sides equal; see Fig. 21.

380. An **equilateral** triangle (Fig. 22) is one having all of its sides of the same length.

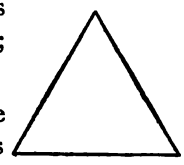

FIG. 22.

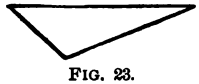

FIG. 23.

381. A **scalene** triangle (Fig. 23) is one having no two of its sides equal.

382. A **right-angled** triangle (Fig. 24) is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse.**

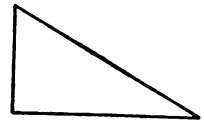

FIG. 24.

In any triangle the sum of the three angles equals two right angles, or 180°. Thus, in Fig. 25, the sum of the angles A, B, and C equals two right angles, or 180°. Hence, if any two angles of a triangle are given and it is required to find the third angle:

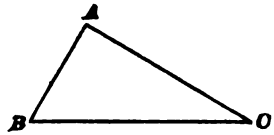

FIG. 25.

Rule.—*Add together the two given angles, and subtract their sum from 180°; the result will be the third angle.*

EXAMPLE.—If two angles of a triangle = 48° 16′ and 47° 50′ respectively, what does the third angle equal?

SOLUTION.—First reduce 48° 16′ and 47° 50′ to minutes, for convenience in adding and subtracting the angles. $48° = 48 \times 60 = 2{,}880'$; $2{,}880' + 16' = 2{,}896'$; hence, $48° 16' = 2{,}896'$. In like manner, $47° 50' = 2{,}820' + 50' = 2{,}870$. Adding the two angles together, and subtracting from 180° reduced to minutes, $2{,}896' + 2{,}870' = 5{,}766'$; $180° = 180 \times 60 = 10{,}800'$; $10{,}800 - 5{,}766 = 5{,}034'$. Reducing this last number to degrees and minutes, $\frac{5{,}034}{60} = 83\frac{54}{60}° = 83° 54'$. Hence, the third angle in the triangle = 83° 54′. Ans.

383. In any right-angled triangle there can be but one right angle, and since the sum of all the angles is two right angles, it is evident that the sum of the two acute angles must equal one right angle, or 90°. Therefore, if in any right-angled triangle one acute angle is known, to find the other acute angle:

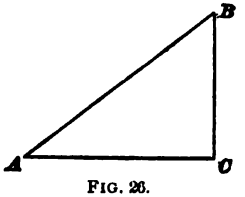

FIG. 26.

Rule.—*Subtract the known acute angle from 90°; the result will be the other acute angle.*

EXAMPLE.—If one acute angle, as A, of the right-angled triangle $A\,B\,C$, Fig. 26, equals 30°, what does the angle B equal?

SOLUTION.—90° − 30° = 60°. Ans.

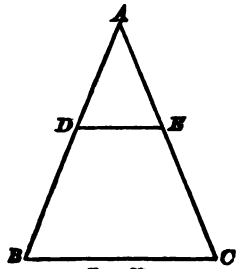

FIG. 27.

384. If a straight line be drawn through two sides of a triangle, parallel to the third side, a second triangle will be formed whose sides will be proportional to the corresponding sides of the first triangle. Thus, in the triangle $A\,B\,C$, Fig. 27, if the line $D\,E$ be drawn parallel to the side $B\,C$, the triangle $A\,D\,E$ will be formed and we shall have

(1) Side $A\,D$: side $D\,E$ = side $A\,B$: side $B\,C$; and,

(2) Side $A\,E$: side $D\,E$ = side $A\,C$: side $B\,C$; also,

(3) Side $A\,D$: side $A\,E$ = side $A\,B$: side $A\,C$.

EXAMPLE.—In Fig. 27, if $A\ B = 24$, $B\ C = 18$, and $D\ E = 8$, what does $A\ D$ equal?

SOLUTION.—Writing these values for the sides in (1),

$$A\ D : 8 = 24 : 18; \text{ whence } A\ D = \frac{24 \times 8}{18} = 10\tfrac{2}{3}. \quad \text{Ans.}$$

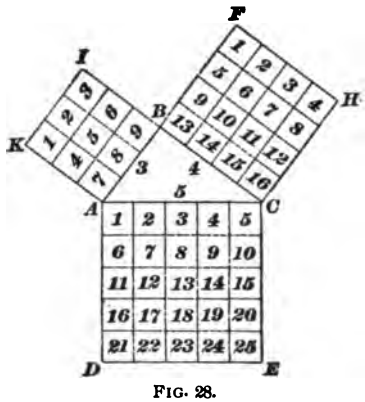

FIG. 28.

385. In any right-angled triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If $A\ B\ C$, Fig. 28, is a right-angled triangle, right-angled at B, then the square described upon the hypotenuse $A\ C$ is equal to the sum of the squares described upon the sides $A\ B$ and $B\ C$. Hence, having given the two sides forming the right angle in a right-angled triangle, to find the hypotenuse:

Rule.—*Square each of the sides forming the right angle; add the squares together, and take the square root of the sum.*

EXAMPLE.—If $A\ B = 3$ inches and $B\ C = 4$ inches, what is the length of the hypotenuse $A\ C$?

SOLUTION.—Squaring each of the given sides, $3^2 = 9$ and $4^2 = 16$. Taking the square root of the sum of 9 and 16, the hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$ inches. Ans.

If the hypotenuse and one side are given, the other side can be found as follows:

Rule.—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

EXAMPLE.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

SOLUTION.—$3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and the $\sqrt{16} = 4$ inches. Ans.

EXAMPLE.—If from a church steeple which is 150 feet high, a rope is to be attached to the top, and to a stake in the ground, which is 85 feet from the center of the base (the ground being supposed to be level), what must be the length of the rope?

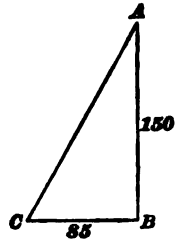

FIG. 29.

SOLUTION.—In Fig. 29, $A\ B$ represents the steeple, 150 feet high; C a stake 85 feet from the foot of the steeple, and $A\ C$ the rope. Here we have a right-angled triangle, right-angled at B, and $A\ C$ is the hypotenuse. The square of $A\ B = 150^2 = 22{,}500$; of $C\ B$, $85^2 = 7{,}225$. $22{,}500 + 7{,}225 = 29{,}725$; $\sqrt{29{,}725} = 172.4$ feet, nearly. Ans.

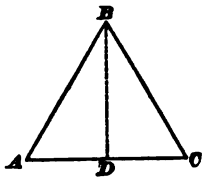

FIG. 30.

386. The **altitude** of any triangle is a line, as $B\ D$, drawn from the vertex B of the angle opposite the base $A\ C$, perpendicular to the base, as in Fig. 30, or to the base extended, as in Fig. 31.

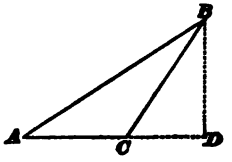

FIG. 31.

If in any parallelogram a straight line, called the **diagonal,** be drawn, connecting two opposite corners it will divide the parallelogram into two equal triangles, as $A\ D\ B$ and $D\ B\ C$ in Fig. 32. The area of each triangle will equal one-half the area of the parallelogram, or one-half the product of the base and the altitude. Hence, to find the area of any triangle:

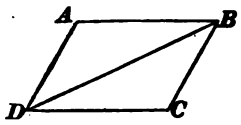

FIG. 32.

Rule.—*Multiply the base by the altitude, and divide the product by 2.*

EXAMPLE.—What is the area in square feet of a triangle whose base is 18 feet, and whose altitude is 7 feet 9 inches?

SOLUTION.—7 feet 9 inches $= 7\frac{3}{4}$ feet $= \frac{31}{4}$ feet. $18 \times \frac{31}{4} = 139\frac{1}{2}$, and one-half of $139\frac{1}{2} = 69\frac{3}{4}$ square feet. Ans.

To find the altitude or base of a triangle, having given the area and the base or altitude:

Rule.—*Multiply the area by 2, and divide by the given dimension.*

EXAMPLE.—What must be the height of a triangular piece of sheet metal to contain 100 square inches, if the base is 10 inches long?

SOLUTION.—$100 \times 2 = 200$; $200 \div 10 = 20$ inches. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose base is 18 feet long, and whose altitude is 10 feet 6 inches? Ans. 94.5 sq. ft.

2. Two angles of a scalene triangle together equal 100° 4′. What is the size of the third angle? Ans. 79° 56′.

3. One angle of a right-angled triangle equals 20° 10′ 5″. What is the size of the other acute angle? Ans. 69° 49′ 55″.

4. A ladder 65 feet long reaches to the top of a wall when its foot is 25 feet from the wall. How high is the wall? Ans. 60 ft.

5. Draw a triangle, and through two of its sides draw a line parallel to the base. Letter the different lines, and then, without referring to the text, write out the proportions existing between the sides of the two triangles.

6. A triangular piece of sheet metal weighs 24 pounds. If the base of the triangle is 4 feet and its height 6 feet, how much does the metal weigh per square foot? Ans. 2 lb.

7. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

8. Two sides of a right-angled triangle are 92 feet and 69 feet long. How long is the hypotenuse? Ans. 115 ft.

POLYGONS.

387. A **polygon** is a plane figure bounded by straight lines. The term is usually applied to a figure having more than four sides. The bounding lines are called the **sides,** and the sum of the lengths of all the sides is called the **perimeter** of the polygon.

388. A **regular polygon** is one in which all of the sides and all of the angles are equal.

389. A polygon of five sides is called a **pentagon;** one with six sides a **hexagon;** one of seven sides a **hep-**

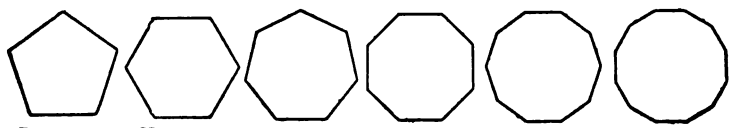

FIG. 33.

tagon, etc. Regular polygons having from five to twelve sides are shown in Fig. 33. In any polygon, the sum of all

the interior angles, as $A + B + C + D + E$, Fig. 34, equals 180° multiplied by a number which is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a *regular* polygon:

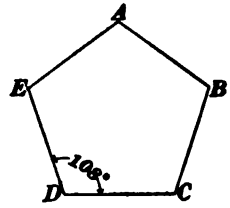

FIG. 34.

Rule.—*Multiply 180° by the number of sides less two, and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle.*

EXAMPLE.—If Fig. 84 is a regular pentagon, how many degrees are there in each interior angle?

SOLUTION.—In a pentagon there are five sides; hence, $5 - 2 = 3$ and $180 \times 3 = 540$; $540 \div 5 = 108°$ in each angle. Ans.

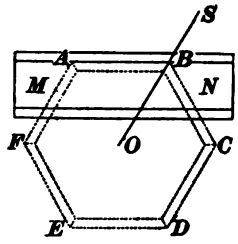

FIG. 35.

EXAMPLE.—It is desired to make a miter-box in which to cut a strip of molding to fit around a column having the shape of a regular hexagon. At what angle should the saw run across the miter-box?

SOLUTION.—In Fig. 35, let AB, BC, CD, etc., represent the pieces of molding as they will fit around the column. First find the size of one of the equal angles of the polygon by the above rule. Number of sides = 6; $6 - 2 = 4$; hence, $180 \times 4 = 720$, and $720 \div 6 = 120°$ in each angle. Now, let MN represent the miter-box, and OS the direction in which the saw should run; then, ABO is the angle made by the saw with the side of the miter-box; but, as the polygon is a regular one, this angle is one-half the interior angle ABC, which we have found to be 120°. Hence, the saw should run at an angle of $\frac{120}{2} = 60°$ with the side of the miter-box.

390. The area of any regular polygon may be found by drawing lines from the center to each angle, and computing the area of each triangle thus formed. Hence, to find the area of any regular polygon:

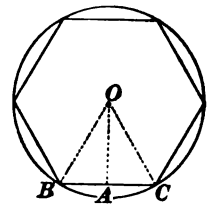

FIG. 36.

Rule.—*Multiply the length of a side by half the distance from the side to the center, and that product by the number of sides. The last product will be the area of the figure.*

EXAMPLE.—In Fig. 36 the side BC of the regular hexagon is 12 inches and the distance AO is 10.4 inches; required, the area of the polygon.

SOLUTION.—$10.4 + 2 = 5.2$; $12 \times 5.2 \times 6 = 374.4$ sq. in. Ans.

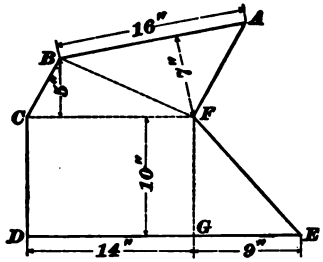

FIG. 37.

391. To obtain the area of any irregular polygon, draw diagonals dividing the polygon into triangles and quadrilaterals, and compute the areas of these separately; their sum will be the area of the figure.

EXAMPLE.—It is required to find the area of the polygon $ABCDEF$, Fig. 37.

SOLUTION.—Draw the diagonals BF, and CF and the line FG perpendicular to DE, dividing the figure into the triangles ABF, BCF, and FGE, and the rectangle $FCDG$. Let it be supposed that the altitudes of the figures and the lengths of the sides AB, DG, and GE are as indicated in the polygon above. Then,

$$\text{Area } ABF = \frac{16 \times 7}{2} = 56 \text{ sq. in.}$$

$$\text{Area } BCF = \frac{14 \times 5}{2} = 35 \text{ sq. in.}$$

$$\text{Area } FCDG = 14 \times 10 = 140 \text{ sq. in.}$$

$$\text{Area } FGE = \frac{9 \times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56 + 35 + 140 + 45 = 276$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. How many degrees are there in one of the angles of a regular octagon? Ans. 135°.

2. Find the area of the polygon $ABCDEF$ (see Fig. 37), supposing each of the given dimensions to be increased to $1\frac{1}{2}$ times the length given in the figure. Ans. 621 sq. in.

3. What is the area of a regular heptagon, whose sides are 4 inches long, the distance from one side to the center being 4.15 inches? Ans. 58.1 sq. in.

4. At what angle should the saw run in a miter-box to cut strips to fit around the edge of a table top, made in the shape of a regular pentagon? Ans. 54°.

THE CIRCLE.

392. A **circle** (Fig. 38) is a figure bounded by a curved line, called the **circumference,** every point of which is equally distant from a point within, called the **center.**

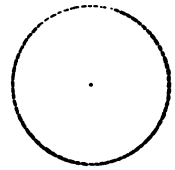

FIG. 38.

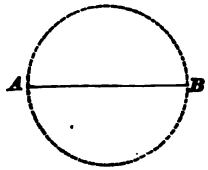

FIG. 39.

393. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference; thus, $A\ B$ (Fig. 39) is a diameter of the circle.

394. The **radius** of a circle, $A\ O$ (Fig. 40), is a straight line drawn from the center O to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii,** and all radii of a circle are equal.

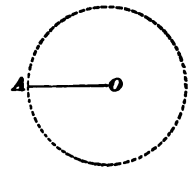

FIG. 40.

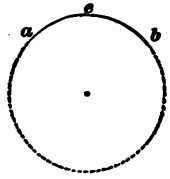

FIG. 41.

395. An **arc** of a circle (see $a\ e\ b$, Fig. 41) is any part of its circumference.

396. A **chord** is a straight line joining any two points in a circumference; or it is a straight line joining the extremities of an arc; thus, the straight line $a\ b$, Fig. 42, is a chord of the circle whose corresponding arc is $a\ e\ b$.

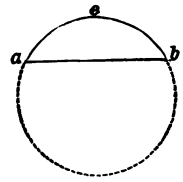

FIG. 42.

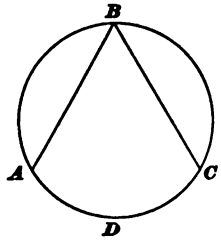

FIG. 43.

397. An **inscribed angle** is one whose vertex lies on the circumference of a circle, and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 43, $A\ B\ C$ is an inscribed angle, and it is measured by one-half the arc $A\ D\ C$.

EXAMPLE.—If in figure 43, the arc $A\,D\,C = \frac{1}{6}$ of the circumference, what is the measurement of the inscribed angle $A\,B\,C$?

SOLUTION.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$ of the circumference. The whole circumference $= 360°$; hence, $360° \times \frac{1}{12} = 72°$; therefore, angle $A\,B\,C$ is an angle of 72°.

398. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semicircumference.**

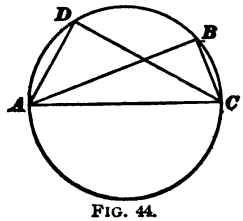

FIG. 44.

Any angle inscribed in a semicircle is a right angle, since it is measured by one-half a semicircumference, or $180° \div 2 = 90°$. Thus, the angles $A\,D\,C$ and $A\,B\,C$, Fig. 44, are right angles, since they are inscribed in a semicircle.

399. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle, and whose sides are chords, as $A\,B\,C\,D\,E$, Fig. 45.

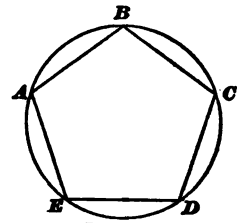

FIG. 45.

The sides of an inscribed regular hexagon have the same length as the radius of the circle.

If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius $O\,C$, Fig. 46, is perpendicular to the chord $A\,B$, $A\,D = D\,B$.

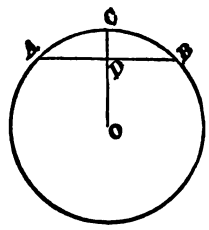

FIG. 46.

EXAMPLE.—If a regular pentagon be inscribed in a circle, and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

SOLUTION.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths; the perimeter of a pentagon, which equals the distance around it, or equals the sum of all the sides, is 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, which equals the length of each of the parts of the side, cut by a radius perpendicular to it.

400. If, from any point on the circumference of a circle, a perpendicular be let fall upon a given diameter, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other. That is, the perpendicular will be a *mean proportional* between the two parts.

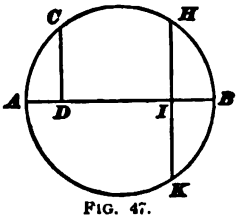

FIG. 47.

If $A\,B$, Fig. 47, is the given diameter, and C any point on the circumference, then $A\,D : C\,D :: C\,D : D\,B$, $C\,D$ being a mean proportional between $A\,D$ and $D\,B$.

EXAMPLE.—If $H\,K = 30$ feet, and $I\,B = 8$ feet, what is the diameter of the circle, $H\,K$ being perpendicular to $A\,B$?

SOLUTION.—30 feet + 2 feet = 15 feet = $I\,H$. And $B\,I : I\,H :: I\,H : I\,A$, or $8 : 15 :: 15 : I\,A$.

Therefore, $I\,A = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet, and $I\,A + I\,B = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= A\,B$ the diameter of the circle. Ans.

401. When the diameter of a circle and the lengths of the two parts into which it is divided are given, the length of the perpendicular may be found by multiplying the lengths of the two parts together and extracting the square root of the product.

EXAMPLE.—In Fig. 47, the diameter of the circle $A\,B$ is $36\frac{1}{8}$ feet, and the distance $B\,I$ is 8 feet; what is the length of the line $H\,K$?

SOLUTION.—As the diameter of the circle is $36\frac{1}{8}$ feet, and as $B\,I$ is 8 feet, $I\,A$ is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. The two parts, therefore, are 8 and $28\frac{1}{8}$ feet, and their product $= 8 \times 28\frac{1}{8} = 8 \times \frac{225}{8} = 225$; the square root of their product $= \sqrt{225} = 15$ feet, and as $H\,K = I\,H + I\,K$, or $2\,I\,H$, $H\,K = 15 \times 2 = 30$ feet. Ans.

402. To find the circumference of a circle, the diameter being given:

Rule.—*Multiply the diameter by 3.1416.*

EXAMPLE.—What is the circumference of a circle whose diameter is 15 inches?

SOLUTION.—$15 \times 3.1416 = 47.124$ inches. Ans.

403. To find the diameter of a circle, the circumference being given:

Rule.—*Divide the circumference by 3.1416.*

EXAMPLE.—What is the diameter of a circle whose circumference is 65.973 inches?

SOLUTION.—65.973 ÷ 3.1416 = 21 inches. Ans.

404. To find the length of an arc of a circle:

Rule.—*Multiply the length of the circumference of the circle of which the arc is a part by the number of degrees in the arc, and divide by 360.*

EXAMPLE.—What is the length of an arc of 24°, the radius of the arc being 18 in.?

SOLUTION.—18 × 2 = 36 in. = the diameter of the circle. 36 × 3.1416 = 113.1 in., the circumference of the circle of which the arc is a part.

$113.1 \times \frac{24}{360} = 7.54$ in., or the length of the arc. Ans.

405. To find the area of a circle:

Rule.—*Square the diameter, and multiply by .7854.*

EXAMPLE.—What is the area of a circle whose diameter is 15 inches?

SOLUTION.—$15^2 = 225$; and $225 \times .7854 = 176.72$ sq. in. Ans.

406. Given the area of a circle, to find its diameter:

Rule.—*Divide the area by .7854, and extract the square root of the quotient.*

EXAMPLE.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

SOLUTION.—$\sqrt{\frac{17,671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, or the diameter. Ans.

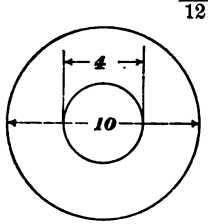

FIG. 48.

EXAMPLE.—What is the area of a flat circular ring, Fig. 48, whose outside diameter is 10 inches, and whose inside diameter is four inches?

SOLUTION.—The area of the large circle = $10^2 \times .7854 = 78.54$ square inches; the area of the small circle = $4^2 \times .7854 = 12.57$ square inches. The area of the ring is the difference between these areas, or 78.54 − 12.57 = 65.97 square inches. Ans.

407. To find the area of a sector (a **sector** of a circle is the area included between two radii and the circumference, as, for example, the area *C O E*, Fig. 13):

Rule.—*Divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

EXAMPLE.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

SOLUTION.—$\frac{75}{360} = \frac{5}{24}$; and $12^2 \times .7854 = 113.1$ square inches.

$113.1 \times \frac{5}{24} = 23.56$ square inches, the area. Ans.

408. To find the area of a segment of a circle (a **segment** of a circle is the area included between a chord and its arc; for example, the area $A\ B\ C$, Fig. 49):

Rule.—*Divide the diameter by the height of the segment; subtract .608 from the quotient, and extract the square root of the remainder. This result multiplied by 4 times the square of the height of the segment, and then divided by 3, will give the area, very nearly.*

The rule, expressed as a formula, is as follows, where D = the diameter of the circle, and h = the height of the segment, see Fig. 49:

$$\text{Area of } A\ B\ C\ A = \frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}.$$

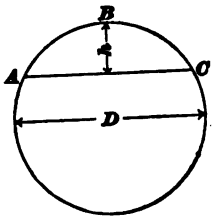

FIG. 49.

EXAMPLE.—What is the area of the segment of a circle whose diameter is 54 inches, the height of the segment being 20 inches?

SOLUTION.—Substituting in the formula,

$$\text{Area} = \frac{4 \times 20^2}{3}\sqrt{\frac{54}{20} - .608}. \quad \frac{4 \times 20^2}{3} = \frac{4 \times 400}{3} = \frac{1{,}600}{3};$$

$\sqrt{\frac{54}{20} - .608} = \sqrt{2.092} = 1.447$; $\frac{1{,}600}{3} \times 1.447 = 771.7$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. An angle inscribed in a circle intercepts one-third of the circumference. How many degrees are there in the angle? Ans. 60°.

2. Suppose that in figure 47, the diameter $A\ B = 15$ feet, and the distance $B\ I = 3$ feet. What is the length of the line $H\ K$?

Ans. 12 ft.

3. The diameter of a fly-wheel is 18 feet. What is the distance around it to the nearest 16th of an inch? Ans. 56 ft. $6\frac{9}{16}$ in.

4. A carriage wheel was observed to make $71\frac{3}{5}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

5. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

6. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

7. What must be the diameter of a circle to contain 100 square inches? Ans. 11.28 in.

8. Compute the area of a segment, whose height is 11 inches, and the radius of whose arc is 21 inches. Ans. 289.04 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

THE PRISM AND CYLINDER.

409. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their intersections are called **edges.**

410. A **prism** is a solid whose ends are equal and parallel polygons, and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base; a hexagonal prism is one having a hexagon for its base, etc.

411. A **cylinder** is a body of uniform diameter whose ends are equal parallel circles.

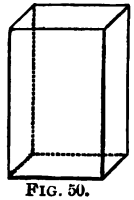

FIG. 50.

412. A **parallelopipedon** (Fig. 50) is a prism whose bases (ends) are parallelograms.

413. A **cube** (Fig. 51) is a prism whose faces and ends are squares. All the faces of a cube are equal.

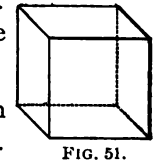

FIG. 51.

In the case of plane figures, we have had to do with perimeters and areas. In the case of solids, we have to do with the areas of their outside surfaces, and with their contents or volumes.

414. The **entire surface** of any solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

415. A **unit of volume** is a cube each of whose edges is equal in length to the unit. The **volume** is expressed by the number of times it will contain a *unit of volume.*

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges each measure 1 inch, this cube being 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

Instead of the word *volume*, the expression **cubical contents** is sometimes used.

416. To find the area of the convex surface of a prism or cylinder:

Rule.—*Multiply the perimeter of the base by the altitude.*

EXAMPLE.—A block of marble is 24 inches long, and its ends are 9 inches square. What is the area of its convex surface?

SOLUTION.—$9 \times 4 = 36$ = the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1{,}026$ sq. in. Ans.

EXAMPLE.—How many square feet of sheet iron will be required for a pipe $1\frac{1}{2}$ feet in diameter and ten feet long, neglecting the amount necessary for lapping?

SOLUTION.—The problem is to find the convex surface of a cylinder $1\frac{1}{2}$ feet in diameter and 10 feet long. The perimeter, or circumference, of the base $= 1\frac{1}{2} \times 3.1416 = 1.5 \times 3.1416 = 4.712$ feet. The convex surface $= 4.712 \times 10 = 47.12$ sq. ft. of metal. Ans.

417. To find the volume of a prism or a cylinder:

Rule.—*Multiply the area of the base by the altitude.*

EXAMPLE.—What is the weight of a length of wrought-iron shafting 16 feet long and 2 inches in diameter? Wrought iron weighs .28 pound per cubic inch.

SOLUTION.—The shaft is a cylinder 16 feet long. The area of one end, or the base, $= 2^2 \times .7854 = 3.1416$ sq. in. Since the weight of the iron is given per cubic inch, the contents of the shaft must be found in cubic inches. The length, 16 feet, reduced to inches $= 16 \times 12 =$ 192 in.; $3.1416 \times 192 = 603.19$ cu. in. = the volume. The weight $=$ $603.19 \times .28 = 168.89$ lb. Ans.

EXAMPLE.—Find the cubical contents of a hexagonal prism, Fig. 52, 12 inches long, each edge of the base being one inch long.

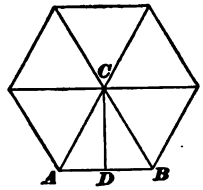

FIG. 52.

SOLUTION.—In order to obtain the area of one end, the distance $C\,D$ from the center C to one side must be found.

In the right-angled triangle $C\,D\,A$, side $A\,D = \frac{1}{2}\,A\,B$, or one-half inch, and since the polygon is a hexagon, side $C\,A =$ distance $A\,B$, or one inch. Hence, $C\,A$ being the hypotenuse, the length of side $C\,D = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{1^2 - .5^2} = \sqrt{.75}$, or .866 inch. Area of triangle $A\,C\,B = \frac{1 \times .866}{2} = .433$ sq. in.; area of the whole polygon $= .433 \times 6 = 2.598$ sq. in. Hence, the contents of the prism $= 2.598 \times 12 = 31.176$ cu. in. Ans.

EXAMPLE.—It is required to find the number of cubic feet of steam space in the boiler shown in Fig. 53. The boiler is 16 feet long between

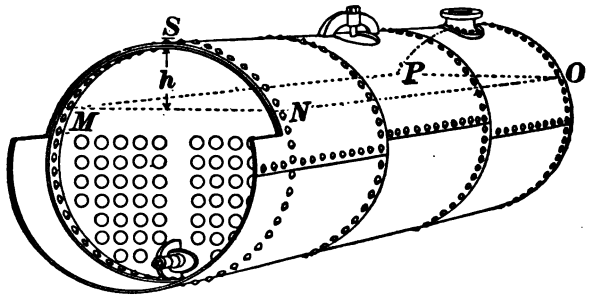

FIG. 53.

heads, 54 inches in diameter, and the mean water line $M\,N$ is at a distance of 16 inches from the top of the boiler. The volume of the steam outlet casting may be neglected.

SOLUTION.—The volume of the steam space, which is that space within the boiler above the surface $M\,N\,O\,P$ of the water, is found by the rule for finding the volume of a prism or cylinder, the area $M\,N\,S$ being the base, and the length $N\,O$ the altitude. First obtain the area of the segment $M\,N\,S$, whose height h is 16 inches, in square feet; then multiply the result by 16, the length of the boiler.

By the formula previously given, the area of the segment =

$$\frac{4h^2}{3}\sqrt{\frac{D}{h}-.608}=\frac{4\times16^2}{3}\sqrt{\frac{54}{16}-.608};$$

$$\frac{4\times16^2}{3}=341.33;\ \sqrt{\frac{54}{16}-.608}=\sqrt{2.767}=1.663.$$

Hence, the area $= 341.33 \times 1.663 = 567.63$ sq. in. This, reduced to square feet, $= 567.63 \div 144 = 3.942$ sq. ft., and the volume, therefore, $= 3.942 \times 16 = 63.07$ cu. ft. Ans.

In the above solution, the space occupied by the stays is not considered, for sake of simplicity. They are not shown in the figure.

EXAMPLE.—In the above boiler there are 60 tubes, 3¼ inches outside diameter. How many gallons of water will it take to fill the boiler up to the mean water level, there being 231 cubic inches in a gallon?

SOLUTION.—Find the volume in cubic inches of that part of the boiler below the surface of the water $MNOP$, since the contents of a gallon is given in cubic inches, and from it subtract the volume of the tubes in cubic inches.

This may be done by first finding the *total* area of one end of the boiler in square inches, from it subtracting the area of the segment MNS, and the areas of the ends of the tubes in square inches, and then by multiplying the result by the length of the boiler *in inches*.

Total area of one end $= 54^2 \times .7854 = 2{,}290.23$ sq. in.

Area of segment MNS, as found in last example, $= 567.63$ sq. in.

Area of the end of one tube $= 3.25^2 \times .7854 = 8.2958$ sq. in.

Area of the ends of the 60 tubes $= 8.2958 \times 60 = 497.75$ sq. in.

Hence, the area to be subtracted $= 567.63 + 497.75 = 1{,}065.38$ sq. in. Subtracting, $2{,}290.23 - 1{,}065.38 = 1{,}224.85$ sq. in. $=$ net area.

The cubical contents $= 1{,}224.85 \times 16 \times 12 = 235{,}171.2$ cu. in. This, divided by 231, will give the number of gallons; whence, $235{,}171.2 \div 231 = 1{,}018.06$ gallons of water. Ans.

EXAMPLES FOR PRACTICE.

1. Find the area in square inches of the convex surface of a bar of iron 4¼ inches in diameter, and 8 feet 5 inches long. Ans. 1,348.53 sq. in.

2. Find the area of the entire surface of the above bar.

Ans. 1,376.9 sq. in.

3. What is the area of the entire surface of the hexagonal prism whose base is shown in Fig. 52? Ans. 77.196 sq. in.

4. A multitubular boiler has the following dimensions: diameter, 50 inches; length between heads, 15 feet; number of tubes, 56; outside diameter of tubes, 3 inches; distance of mean water line from top of boiler, 16 inches. (*a*) Compute the steam space in cubic feet. (*b*) Find the number of gallons of water required to fill the boiler up to the mean water line.

Ans. { (*a*) 56.4 cu. ft.
(*b*) 800 gallons.

THE PYRAMID AND CONE.

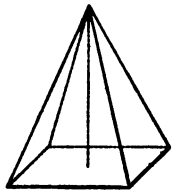

FIG. 54.

418. A **pyramid** (Fig. 54) is a solid whose base is a polygon, and whose sides are triangles uniting at a common point, called the **vertex.**

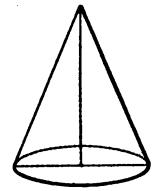

FIG. 55.

419. A **cone** (Fig. 55) is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

420. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

421. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

422. To find the convex area of a pyramid or cone:

Rule.—*Multiply the perimeter of the base by one-half the slant height.*

EXAMPLE.—What is the convex area of a pentagonal pyramid, if one side of the base measures 6 inches, and the slant height = 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, has five sides.

$6 \times 5 = 30$ inches, or the perimeter of the base. $30 \times \frac{14}{2} = 210$ sq. in., or the convex area. Ans.

EXAMPLE.—What is the entire area of a right cone whose slant height is 17 inches, and whose base is 8 inches in diameter?

SOLUTION.—The perimeter of the base $= 8 \times 3.1416 = 25.1328$ in.

Convex area $= 25.1328 \times \frac{17}{2} = 213.63$ sq. in.

Area of base $= 8^2 \times .7854 = 50.27$ sq. in.

Entire area $= 263.90$ sq. in. Ans.

423. To find the volume of a pyramid or cone:

Rule.—*Multiply the area of the base by one-third of the altitude.*

EXAMPLE.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches, and whose altitude is 8 inches?

SOLUTION.—Draw the base as shown in Fig. 56; it will be an equilateral triangle, all of whose sides are 6 inches long.

Draw a perpendicular BD from the vertex to the base; it will divide the base into two equal parts, since an equilateral triangle is also isosceles, and will be the altitude of the triangle. In order to obtain the area of the base, this altitude must be determined.

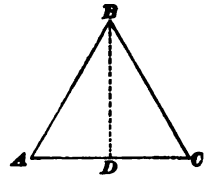

FIG. 56.

In the right-angled triangle BDA, the hypotenuse $BA = 6$ inches, and side $AD = 3$ inches, to find the other side,

$$BD = \sqrt{6^2 - 3^2} = 5.2 \text{ inches, nearly.}$$

Area of the base, or $BAC = \frac{6 \times 5.2}{2} = 15.6$ sq. in. Hence, the volume $= 15.6 \times \frac{8}{3} = 41.6$ cu. in. Ans.

EXAMPLE.—What is the volume of a cone whose altitude is 18 inches, and whose base is 14 inches in diameter?

SOLUTION.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of a square pyramid whose slant height is 28 inches, and one edge of whose base is 7½ inches long.

Ans. 420 sq. in.

2. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches, and whose altitude is 4 inches?

Ans. 5.2 cu. in.

3. Find the volume of a cone whose altitude is 12 inches, and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base and then its area.

THE FRUSTUM OF A PYRAMID OR CONE.

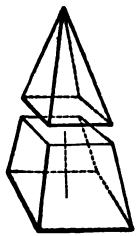
FIG. 57.

424. If a pyramid be cut by a plane, parallel to the base, so as to form two parts, as in Fig. 57, the lower part is called the **frustum** of the pyramid.

If a cone be cut in a similar manner, as in Fig. 58, the lower part is called the **frustum** of the cone.

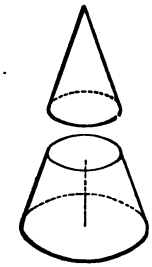
FIG. 58.

425. The upper end of the frustum of a pyramid or cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

426. To find the convex surface of a frustum of a pyramid or cone:

Rule.—*Multiply one-half the sum of the perimeters of the two bases by the slant height of the frustum.*

EXAMPLE.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches; find the area of the convex surface.

SOLUTION.—10 in. $\times$ 3 = 30 in., the perimeter of the lower base.

6 in. $\times$ 3 = 18 in., the perimeter of the upper base.

$\frac{30+18}{2} = 24$ in., or one-half the sum of the perimeters of the two bases. $24 \times 9 = 216$ sq. in., the convex area. Ans.

EXAMPLE.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

SOLUTION.—$\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the area of the convex surface.

Area of the upper base = $8^2 \times .7854 = 50.27$ sq. in.

Area of the lower base = $12^2 \times .7854 = 113.1$ sq. in.

The entire area of the frustum = 376.99 + 50.27 + 113.1 = 540.36 sq. in. Ans.

427. To find the volume of the frustum of a pyramid or cone:

Rule.—*Add together the areas of the upper and lower bases, and the square root of the product of the two areas; multiply the sum by one-third of the altitude.*

EXAMPLE.—Given, a frustum of a square pyramid (one whose base is a square); each edge of the lower base is 12 inches, each edge of the upper base is 5 inches, and its altitude is 16 inches; what is its volume?

SOLUTION.—Area of upper base = $5 \times 5 = 25$ sq. in.; area of lower base = $12 \times 12 = 144$ sq. in.; the square root of the product of the two areas = $\sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

EXAMPLE.—How many gallons of water will a round tank hold, which is 4 feet in diameter at the top, 5 feet in diameter at the bottom, and 8 feet deep?

SOLUTION.—There are 231 cubic inches in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of the frustum of a cone. The upper diameter $= 4 \times 12 = 48$ inches; the lower diameter $= 5 \times 12 = 60$ inches, and the depth $= 8 \times 12 = 96$ inches. Area of upper base $= 48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2{,}827.44$ sq. in.; $\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95$.

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$; $6{,}898.95 \times \frac{96}{3} =$ 220,766.4 cu. in. = contents. Now, since there are 231 cu. in. in one gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gallons, nearly. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of the frustum of a square pyramid, one edge of whose lower base is 15 inches long, one edge of whose upper base is 14 inches long, and whose slant height is one inch.

Ans. 58 sq. in.

2. Find the volume of the above frustum, supposing its altitude to be 3 inches. Ans. 631 cu. in.

3. Find the volume of the frustum of a cone whose altitude is 12 feet and the diameters of whose upper and lower bases are 8 and 10 feet, respectively. Ans. 766.55 cu. ft.

4. If a tank had the dimensions of example 3, how many gallons would it hold? Ans. 5,734.2 gallons, nearly.

THE SPHERE AND CYLINDRICAL RING.

428. A **sphere** (Fig. 59) is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

The word **ball,** or **globe,** is generally used instead of sphere.

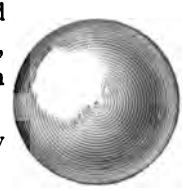

FIG. 59.

429. To find the area of a sphere:

Rule.—*Square the diameter and multiply the result by 3.1416.*

EXAMPLE.—What is the area of the surface of a sphere whose diameter is 14 inches?

SOLUTION.—Diameter squared $\times$ 3.1416 $= 14^2 \times 3.1416 = 14 \times 14 \times$ 3.1416 = 615.75 sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used; a *great circle* of a sphere is the intersection of its surface with a plane passing through its center; for instance, the *great circle* of a sphere 6 in. diameter is a circle of 6 in. diameter. Any number of *great circles* could be described on a given sphere.

430. To find the volume of a sphere:

Rule.—*Cube the diameter and multiply the result by .5236.*

EXAMPLE.—What is the weight of a lead ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Diameter cubed $\times$ .5236 = $12 \times 12 \times 12 \times .5236 = 904.78$ cu. in., or the volume of the ball. The weight, therefore, = $904.78 \times .41 = 370.96$ pounds. Ans.

431. To find the convex area of a cylindrical ring:

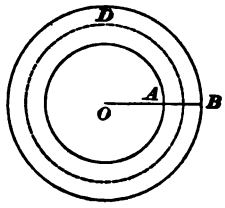

FIG. 60.

A **cylindrical ring** (Fig. 60) is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area:

Rule.—*Multiply the circumference of an imaginary cross-section on the line A B, by the length of the center line D.*

EXAMPLE.—If the outside diameter of the ring is 12 inches, and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$, and $10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the inner circle is 4 inches; of the outside circle, 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, $2 \times 3.1416 = 6.2832$ inches, and $6.2832 \times 31.416 = 197.4$ sq. in., the convex area. Ans.

432. To find the volume of a cylindrical ring:

Rule.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D, and whose base is the same as a cross-section of the ring on the line A B, drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on the line A B, by the length of the center line D.*

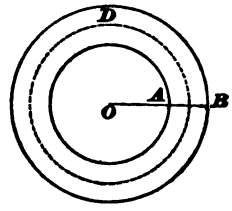

FIG. 61.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches, and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12+8}{2} = 10$.

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle = 6 inches; of the inside circle = 4 inches; therefore, the diameter of the cross-section on the line $A\ B$ = 2 inches.

Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cubic inches, the volume. Ans.

EXAMPLES FOR PRACTICE.

1. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in.

2. How many square inches in the surface of the above sphere?
Ans. 2,827.44 sq. in.

3. Required the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches, and the inside diameter 7½ inches. Ans. 107.95 sq. in.

4. Find the cubical contents of the ring in the last example.
Ans. 33.73 cu. in.

5. The surface of a sphere contains 314.16 square inches. What is the volume of the sphere? Ans. 523.6 cu. in.

MECHANICS.

433. **Mechanics** is that science which treats of the action of forces upon bodies, and the effects which they produce; it treats of the laws which govern the movement and equilibrium of bodies, and shows how they may be utilized.

MATTER AND ITS PROPERTIES.

434. **Matter** is anything that occupies space. It is the substance of which all bodies consist. Matter is composed of *molecules* and *atoms*.

435. A **molecule** is the smallest portion of matter that can exist without changing its nature.

436. An **atom** is an indivisible portion of matter.

Atoms unite to form molecules, and a collection of molecules forms a mass or body.

A drop of water may be divided and subdivided, until each particle is so small that it can only be seen by the most powerful microscope, but each particle will still be water.

Now, imagine the division to be carried on still further, until a limit is reached beyond which it is impossible to go without changing the nature of the particle. The particle of water is now so small that, if it be divided again, it will cease to be water, and will be something else; we will call this particle a *molecule*.

If a molecule of water be divided, it will yield two atoms of hydrogen gas, and one of oxygen gas. If a molecule of sulphuric acid be divided, it will yield two atoms of hydrogen, one of sulphur, and four of oxygen.

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It has been calculated that the diameter of a molecule is larger than $\frac{1}{125000000}$ of an inch, and smaller than $\frac{1}{50000000}$ of an inch.

437. Bodies are composed of collections of molecules. Matter exists in three conditions or forms: *solid*, *liquid*, and *gaseous*.

438. A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

439. A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the shape of vessels which contain them, and their upper surface always tends to become perfectly level. Water, mercury, molasses, etc., are liquids.

440. A **gaseous body,** or gas, is one whose molecules tend to separate from one another; as air, oxygen, hydrogen, etc.

Gaseous bodies are sometimes called **aeriform** (air-like) **bodies.** They are divided into two classes: the so-called "*permanent*" *gases*, and *vapors*.

441. A **permanent gas** is one which remains a gas at ordinary temperatures and pressures.

442. A **vapor** is a body which at ordinary temperatures is a liquid or solid, but when heat is applied, becomes a gas, as steam.

One body may be in all three states; as, for example, mercury, which at ordinary temperatures is a liquid, becomes a solid (freezes) at 40° below zero, and a vapor (gas) at 600° above zero. By means of great cold, all gases, even hydrogen, have been liquefied, and some solidified.

By means of heat, all solids have been liquefied, and a great many vaporized. It is probable that, if we had the means of producing sufficiently great extremes of heat and cold, all solids might be converted into gases, and all gases into solids.

443. Every portion of matter possesses certain qualities called *properties.* Properties of matter are divided into two classes: *general* and *special.*

444. General properties of matter are those which are common to all bodies. They are as follows: *Extension, impenetrability, weight, indestructibility, inertia, mobility, divisibility, porosity, compressibility, expansibility,* and *elasticity.*

445. Extension is the property of occupying space. Since all bodies must occupy space, it follows that extension is a general property.

446. By **impenetrability** we mean that no two bodies can occupy exactly the same space at the same time.

447. Weight is the measure of the earth's attraction upon a body. All bodies have weight. In former times it was supposed that gases had no weight, since, if unconfined, they tend to move away from the earth, but, nevertheless, they will finally reach a point beyond which they can not go, being held in suspension by the earth's attraction. Weight is measured by comparison with a standard. The standard is a bar of platinum owned and kept by the Government; it weighs one pound.

448. Inertia means that a body can not put itself in motion nor bring itself to rest. To do either it must be acted upon by some force.

449. Mobility means that a body can be changed in position by some force acting upon it.

450. Divisibility is that property of matter on account of which a body may be separated into parts.

451. Porosity is the term used to denote the fact that there is space between the molecules of a body. The molecules of a body are supposed to be spherical, and, hence, there is space between them, as there would be between peaches in a basket. The molecules of water are larger than those of salt; so that when salt is dissolved in water,

its molecules wedge themselves between the molecules of the water, and unless too much salt is added, the water will occupy no more space than it did before. This does not prove that water is penetrable, for the molecules of salt occupy the space that the molecules of water did not.

Water has been forced through iron by pressure, thus proving that iron is porous.

452. Compressibility.—This property is a natural consequence of the preceding one. Since there is space between the molecules, it is evident that by means of force (pressure) they can be brought closer together, and thus the body be made to occupy a smaller space.

453. Expansibility is the term used to denote the fact that the molecules of a body will, under certain conditions (when heated, for example), move farther apart, and so cause the body to *expand*, or occupy a greater space.

454. Elasticity is that property of matter which enables a body when distorted within certain limits to resume its original form when the distorting force is removed. Glass, ivory, and steel are very elastic, clay and putty in their natural state being very slightly so.

455. Indestructibility is the term used to denote the fact that we can not *destroy* matter. A body may undergo thousands of changes, be resolved into its molecules, and its molecules into atoms, which may unite with other atoms to form other molecules and bodies entirely different in appearance and properties from the original body, but the same number of atoms remain. The whole number of atoms in the universe is exactly the same now as it was millions of years ago, and will always be the same. *Matter is indestructible.*

456. Special properties are those which are not possessed by all bodies. Some of the most important are as follows: *hardness*, *tenacity*, *brittleness*, *malleability*, and *ductility*.

457. Hardness.—A piece of copper will scratch a piece of wood, steel will scratch copper, and tempered steel

will scratch steel in its ordinary state. We express all this by saying that steel is *harder* than copper, and so on. Emery and corundum are extremely hard, and the diamond is the hardest of all known substances. It can only be polished with its own powder.

458. Tenacity is the term applied to the power with which some bodies resist a force tending to pull them apart. Steel is very tenacious.

459. Brittleness.—Some bodies possess considerable power to resist either a pull or a pressure, but they are easily broken when subjected to shocks or jars; for example, good glass will bear a *greater* compressive force than most woods, but may be easily broken when dropped on to a hard floor; this property is called *brittleness.*

460. Malleability is that property which permits of some bodies being hammered or rolled into sheets. Gold is the most malleable of all substances.

461. Ductility is that property which enables some bodies to be drawn into wire. Platinum is the most ductile of all substances.

MOTION AND VELOCITY.

462. Motion is the opposite of rest, and indicates a changing of position in relation to some object which is for that purpose regarded as being fixed. If a large stone is rolled down hill, it is in motion in relation to the hill.

If a person is on a railway train, and walks in the opposite direction from that in which the train is moving, and with the same speed, he will be in motion as regards the train, but at rest with respect to the earth, since, until he gets to the end of the train, he will be directly over the spot at which he was when he started to walk.

463. The **path** of a body in motion is the line described by a certain point in the body called its center of gravity. No matter how irregular the shape of the body may be, nor how many turns and twists it may make, the line which

indicates the direction of this point for every instant that it is in motion is the path of the body.

464. Velocity is rate of motion. It is measured by a unit of space passed over in a unit of time. When equal spaces are passed over in equal times, the velocity is said to be **uniform.** In all other cases it is **variable.**

If the fly-wheel of an engine keeps up a constant speed of a certain number of revolutions per minute, the velocity of any point is uniform. A railway train having a constant speed of 40 miles per hour, moves 40 miles every hour, or $\frac{40}{60} = \frac{2}{3}$ of a mile every minute, and since equal spaces are passed over in equal times, the velocity is uniform.

465. To find the uniform velocity which a body must have to pass over a certain distance or space in a given time:

Rule.—*Divide the distance by the time.*

EXAMPLE.—The piston of a steam engine travels 3,000 feet in 5 minutes; what is its velocity in feet per minute?

SOLUTION.—Here 3,000 feet is the distance, and 5 minutes is the time. Applying the rule, $3{,}000 \div 5 = 600$ feet per minute. Ans.

CAUTION.—Before applying the above or any of the succeeding rules, care must be taken to reduce the values given to the denominations required in the answer. Thus, in the above example, had the velocity been required in feet per second instead of feet per minute, the 5 minutes would have been reduced to seconds before dividing. The operation would then have been, 5 min. $= 5 \times 60 = 300$ sec. Applying the rule, $3{,}000 \div 300 = 10$ ft. per sec. Ans.

466. Had the velocity been required in inches per second, it would have been necessary to reduce the 3,000 feet to inches and the 5 minutes to seconds, before dividing. Thus, 3,000 ft. $\times$ 12 = 36,000 in. 5 min. $\times$ 60 = 300 sec. Now, applying the rule, $36{,}000 \div 300 = 120$ in. per sec. Ans.

EXAMPLE.—A railroad train travels 50 miles in $1\frac{1}{2}$ hours; what is its average velocity in feet per second?

SOLUTION.—Reducing the miles to feet, and the hours to seconds, 50 miles $\times$ 5,280 = 264,000 ft. $1\frac{1}{2}$ hours $\times$ 60 $\times$ 60 = 5,400 sec. Applying the rule, 264,000 $\div$ 5,400 = $48\frac{8}{9}$ ft. per sec. Ans.

467. If the uniform velocity (or the average velocity) and the time are given, and it is required to find the distance which a body having the given velocity would travel in the given time:

Rule.—*Multiply the velocity by the time.*

EXAMPLE.—The velocity of sound in still air is 1,092 feet per second; how many miles will it travel in 16 seconds?

SOLUTION.—Reducing the 1,092 ft. to miles, $1{,}092 \div 5{,}280 = \frac{1{,}092}{5{,}280}$. Applying the rule, $\frac{1{,}092}{5{,}280} \times 16 = 3.31$ miles, nearly. Ans.

EXAMPLE.—The piston speed of an engine is 11 ft. per sec., how many miles does the piston travel in 1 hour and 15 minutes?

SOLUTION.—1 hour and 15 minutes reduced to seconds = 4,500 seconds = the time. 11 feet reduced to miles = $\frac{11}{5{,}280}$ mile = velocity in miles per second. Applying the rule, $\frac{11}{5{,}280} \times 4{,}500 = 9.375$ miles. Ans.

468. If the distance through which a body moves is given, and also its average or uniform velocity, and it is desired to know how long it takes the body to move through the given distance:

Rule.—*Divide the distance, or space passed over, by the velocity.*

EXAMPLE.—Suppose that the radius of the crank of a steam engine is 15", and that the shaft makes 120 revolutions per minute, how long will it take the crank-pin to travel 18,849.6 feet?

SOLUTION.—Since the radius, or distance from the center of the shaft to the center of the crank-pin, is 15", the diameter of the circle it moves in is 15" $\times$ 2 = 30" = 2.5 ft. The circumference of this circle is 2.5 $\times$ 3.1416 = 7.854 ft. 7.854 $\times$ 120 = 942.48 ft. — distance that the crank-pin travels in one minute = velocity in feet per minute. Applying the rule, 18,849.6 $\div$ 942.48 = 20 minutes. Ans.

EXAMPLE.—A point on the rim of an engine fly-wheel travels at the rate of 150 feet per second; how long will it take to travel 45,000 feet?

SOLUTION.—Applying the rule,

45,000 $\div$ 150 = 300 seconds = 5 minutes. Ans.

EXAMPLES FOR PRACTICE.

1. A fan 80" in diameter makes 293 revolutions per minute; what is the velocity of the points of the fan blades in (*a*) feet per second? (*b*) miles per hour?

Ans. { (*a*) 102.277 ft. per sec. (*b*) 69.734 mi. per hr.

2. Assuming the velocity of steam as it enters the cylinder to be 900 feet per second, how far could it travel, if unobstructed, during the time the fly-wheel of an engine revolved 7 times, if the number of revolutions per minute were 120? Ans. 3,150 ft.

3. The average velocity of water flowing through a pipe is 528 ft. per minute; how long will it take the water to flow 4 miles?

Ans. 40 min.

4. A speed of 40 miles per hour equals how many feet per second?

Ans. 58⅔ ft.

5. The earth turns around once in 24 hours. If the diameter be taken as 8,000 miles, what is the velocity of a point on the earth's surface in miles per minute? Ans. 17.45¼ mi. per min.

6. The stroke of a pump piston is 14 inches. If the piston makes 11,400 strokes per hour, (*a*) what is its speed in feet per minute? (*b*) How far will this piston travel in 11 minutes?

Ans. { (*a*) 221⅓ ft. per min. (*b*) 2,438 ft. 4 in.

FORCE.

469. Force is that which produces, or tends to produce or destroy, motion. Forces are called by various names, according to the effects which they produce upon a body, as *attraction*, *repulsion*, *cohesion*, *adhesion*, *accelerating* force, *retarding* force, *resisting* force, etc., but all are equivalent to a push or pull, according to the direction in which they act upon a body. That the effect of a force upon a body may be compared with another force, it is necessary that three conditions be fulfilled in regard to both bodies. They are as follows:

(1) *The point of application, or point at which the force acts upon the body, must be known.*

(2) *The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.*

(3) *The magnitude or value of the force, when compared with a given standard, must be known.*

470. *The unit of magnitude of forces will always be taken as one pound*, and all forces will be spoken of as a certain number of pounds.

In practice, force is always regarded as a pressure; that is, a force may always be replaced by an equivalent weight. Thus, a force of 20 lb. acting upon a body is regarded as a pressure of 20 lb. produced by a weight of 20 lb. The tendency of a force is always to produce motion in the direction in which it acts. The resistance may be too great for it to cause motion, but it *always tends* to produce it.

471. The fundamental principles of the relations between force and motion were first stated by Sir Isaac Newton. They are called "Newton's Three Laws of Motion," and are as follows:

(*1*) *All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted upon by some external force that compels a change.*

(*2*) *Every motion or change of motion is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*

(*3*) *To every action there is always opposed an equal and contrary reaction.*

472. In the first law of motion it is stated that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted upon by some other force which compels a change. It is not possible to actually verify this law, on account of the earth's attraction for all bodies, but, from astronomical observations, we are certain that the law is true. This law is often called *the law of inertia.*

473. The word **inertia** is so abused that a full understanding of its meaning is necessary. Inertia is not a force, although it is often so called. If a force acts upon a body and puts it in motion, the effect of the force is stored in the body, and a second body, in stopping the first, will receive a blow equal in every respect to the original force, assuming that there has been no resistance of any kind to the motion of the first body.

It is dangerous for a person to jump from a fast moving train, for the reason that, since his body has the same velocity as the train, it has the same force stored in it that would cause a body of the same weight to take the same velocity as the train, and the effect of a sudden stoppage is the same as the effect of a blow necessary to give the person that velocity.

By "bracing" himself and jumping in the same direction that the train is moving, and running, he brings himself gradually to rest, and thus reduces the danger. If a body is at rest, it must be acted upon by a force in order to be put in motion, and, no matter how great the force may be, it can not be *instantly* put in motion.

The resistance thus offered to being put in motion is commonly, but erroneously, called the "*Resistance of Inertia.*" It should be called the *Resistance due to Inertia.*

474. From the second law, we see that if two or more forces act upon a body, their final effect upon the body will be in proportion to their magnitudes, and to the directions in which they act.

Thus, if the wind be blowing due West, with a velocity of 50 miles per hour, and a ball be thrown due North with the same velocity, or 50 miles per hour, the wind will carry the ball West while the force of the throw is carrying it North, and the combined effect will be to cause it to move Northwest.

The amount of departure from due North will be proportional to the force of the wind, and independent of the velocity due to the force of the throw.

475. In Fig. 62 a ball *e* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm is actuated by the spring *d* in such a manner that, when drawn back, as shown, and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b* an impulse in a horizontal direction, and swings *o* so as to allow *e* to fall.

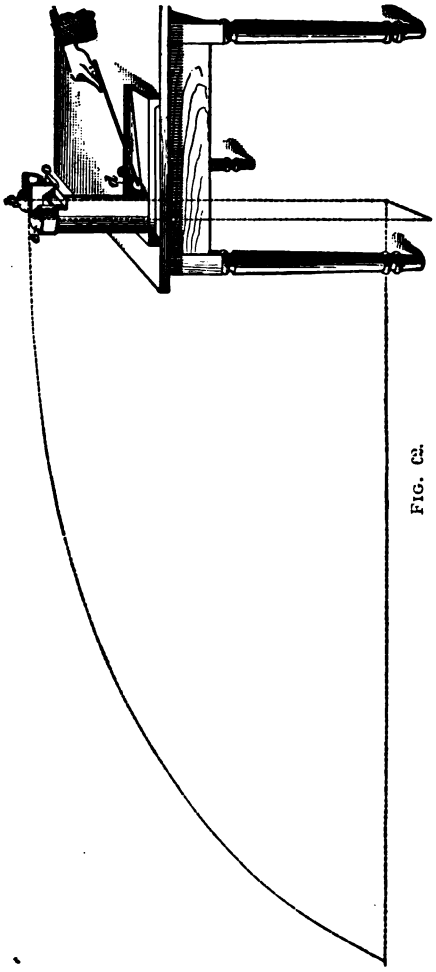

FIG. 62.

On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *e*, which drops vertically. This shows that the force which gave the first ball its horizontal movement had no effect on the vertical force which compelled both balls to fall to the floor; the vertical force producing the same effect as if the horizontal force had not acted. The second law may also be stated as follows: *A force has the same effect in producing motion, whether it acts upon a body at rest or in motion, and whether it acts alone or with other forces.*

476. The third law states that action and reaction are equal and opposite. A man can not lift himself by his bootstraps, for the reason that he presses downwards with the same force that he pulls upwards; the downward reaction equals the upward action, and is opposite to it.

In springing from a boat we must exercise caution or the reaction will drive the boat from the shore. When we jump from the ground, we tend to push the earth from us, while the earth reacts and pushes us from it.

EXAMPLE.—Two men pull on a rope in opposite directions, each exerting a force of 100 pounds; what is the force which the rope resists?

SOLUTION.—Imagine the rope to be fastened to a tree, and one man to pull with a force of 100 pounds. The rope evidently resists 100 pounds. According to Newton's third law, the reaction of the tree is also 100 pounds. Now, suppose the rope to be slackened, but that one end is still fastened to the tree, and the second man to take hold of the rope near the tree, and pull with a force of 100 pounds, the first man pulling as before. The resistance of the rope is 100 pounds, as before, since the second man merely takes the place of the tree. *He is obliged to exert a force of 100 pounds to keep the rope from slipping through his fingers.* If the rope be passed around the tree, and each man pulls an end with a force of 100 pounds in the same and parallel directions, the stress in the rope is 100 pounds, as before, but the tree must resist the pull of both men, or 200 pounds.

477. A **force** may be represented by a line; thus, in Fig. 63, let *A* be the *point of application* of the force; let the length of the line *A B* represent its *magnitude*, and let the arrow-head indicate the *direction* in which the force acts, then the line *A B* fulfils the three conditions (see Art. **469**), and the force is fully represented.

FIG. 63.

CENTER OF GRAVITY.

478. *The center of gravity of a body is that point at which the body may be balanced, or it is the point at which the whole weight of a body may be considered as concentrated.*

This point is not always *in* the body; in the case of a horseshoe or a ring it lies outside of the substance of, but within the space enclosed by, the body.

In a moving body, the line described by its center of gravity is always taken as the path of the body. In finding the distance that a body has moved, the distance that the center of gravity has moved is taken.

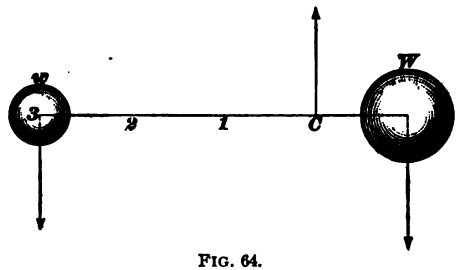

FIG. 64.

The definition of the center of gravity of a body may be applied to a system of bodies if they are considered as being connected at their centers of gravity.

If w and W, Fig. 64, be two bodies of known weight, their center of gravity will be at C. The point C may be readily determined as follows:

Rule.—*The distance of the common center of gravity, from the center of gravity of the large weight, is equal to the weight of the smaller body multiplied by the distance between the centers of gravity of the two bodies, and this product divided by the sum of the weights of the two bodies.*

EXAMPLE.—In Fig. 64, $w = 10$ pounds, $W = 30$ pounds, and the distance between their centers of gravity is 36 inches where is the center of gravity of both bodies situated?

SOLUTION.—Applying the rule, $10 \times 36 = 360$. $10 + 30 = 40$. $360 \div 40 = 9'' =$ distance of center of gravity from center of large weight.

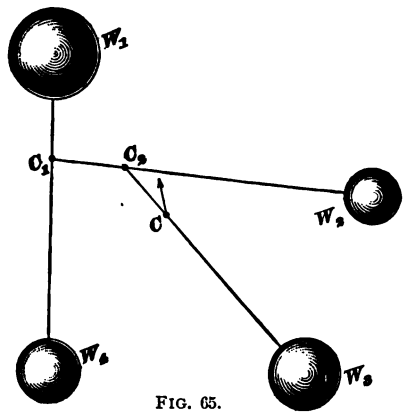

FIG. 65.

479. It is now very easy to extend this principle, to find the center of gravity of any number of bodies when their weights and the distances apart of their centers of gravity are known, by the following rule:

Rule.—*Find the center of gravity of two of the bodies as W_1 and W_4, in Fig. 65. Assume that the weight of both*

bodies is concentrated at C_1, and find the center of gravity of this combined weight at C_1, and the weight of W_2; let it be at C_2; then find the center of gravity of the combined weights of W_1, W_4, W_3 (concentrated at C_2), and W_5; let it be at C; then C will be the center of gravity of the four bodies.

480. To find the center of gravity of **any parallelogram:**

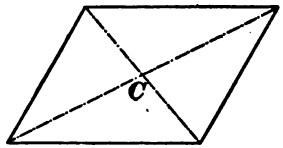

FIG. 66.

Rule.—*Draw the two diagonals, Fig. 66, and their point of intersection C will be the center of gravity.*

481. To find the center of gravity of a **triangle,** as *A B C*, Fig. 67:

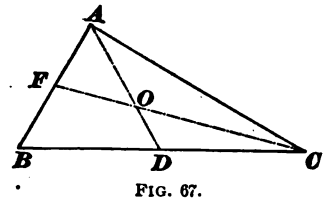

FIG. 67.

Rule.—*From any vertex, as A, draw a line to the middle point D of the opposite side B C. From one of the other vertexes, as C, draw a line to F, the middle point of the opposite side A B; the point of intersection O of these two lines is the center of gravity.*

It is also true that the distance $D\,O = \frac{1}{3}\,D\,A$, and that $F\,O = \frac{1}{3}\,F\,C$, and the center of gravity could have been found by drawing from any vertex a line to the middle point of the opposite side, and measuring back from that side $\frac{1}{3}$ of the length of the line.

The center of gravity of **any regular plane figure** is the same as the center of the inscribed or circumscribed circle.

482. To find the center of gravity of **any irregular plane figure,** but of uniform thickness throughout, divide one of the parallel surfaces into triangles, parallelograms, circles, ellipses, etc., according to the shape of the figure; find the area and center of gravity of each part separately, and combine the centers of gravity thus found in the same manner as in rule, Art. **479**; in this case, however,

dealing with the *area* of each part instead of its weight. See Fig. 68.

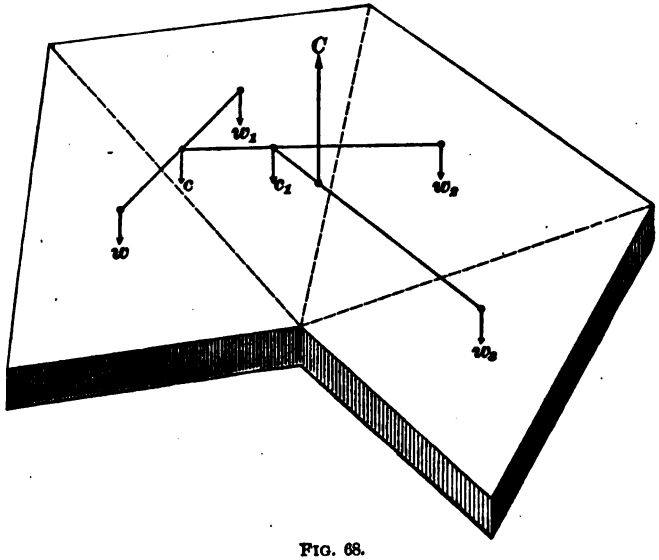

FIG. 68.

EXAMPLE.—Suppose that the two balls shown in Fig. 64 were 5″ and 10″ in diameter, and weighed 10 lb. and 80 lb., respectively. If the distance between their centers were 40″, and they were connected by a steel rod 1″ in diameter, where would the center of gravity be, taking the weight of a cubic inch of steel as .283 lb.?

SOLUTION.—The length of the rod $= 40 - \frac{5}{2} - \frac{10}{2} = 32\frac{1}{2}$″. Its volume is $1^2 \times .7854 \times 32\frac{1}{2} = 25.53$ cu. in. $25.53 \times .283 = 7.22$ lb. The rod being straight, its center of gravity is in the middle at a distance of $\frac{32.5}{2} + \frac{5}{2} = 18\frac{1}{4}$″ from the center of the smaller weight, and $\frac{32.5}{2} + \frac{10}{2} = 21\frac{1}{4}$″ from the center of the larger weight. Now, considering the weight of the rod to be concentrated at its center of gravity, we have three weights of 10, 7.22, and 80 lb., all in a straight line, and the distances between them given to find the center of gravity, or balancing point, of the combination, by rule, Art. **479.** We will first find the center of gravity of the two smaller weights by rule, Art. **478,** as follows: $7.22 \times 18\frac{1}{4} = 135.38$. $10 + 7.22 = 17.22$. $135.38 \div 17.22 = 7.86$″ = distance from the center of the 10-lb. weight. Considering both of the smaller weights to be concentrated at this point, we find the center of gravity of this combined weight and the large weight as follows: $40 - 7.86 =$

32.14″ = distance between the center of gravity of the two small weights and the center of gravity of the 80-lb. weight. Applying rule, Art. **478**, $17.22 \times 32.14 = 553.45$.

$17.22 + 80 = 97.22$. $553.45 \div 97.22 = 5.693''$ = distance from the center of the 80-lb. weight. Ans.

483. Center of Gravity of a Solid.—In a body free to move, the center of gravity will lie in a vertical plumb

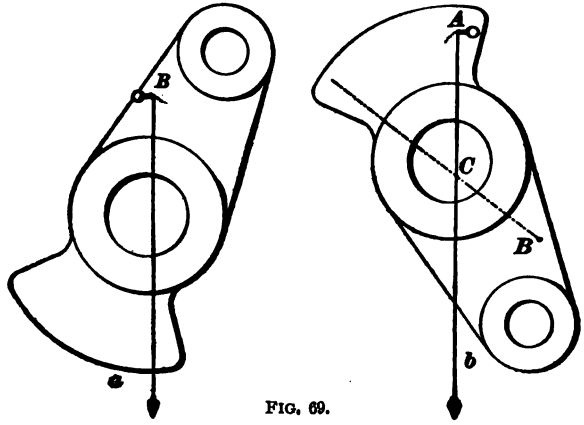

FIG. 69.

line drawn through the point of support. Therefore, to find the position of the center of gravity of an irregular solid, as the crank, Fig. 69, suspend it at some point, as *B*, so that it will move freely. Drop a plumb line from the point of suspension, and mark its direction. Suspend the body at another point, as *A*, and repeat the process. The intersection *C* of the two lines will be directly over the center of gravity.

Since the center of gravity depends wholly upon the shape and weight of a body, it may be without the body, as in the case of a circular ring whose center of gravity is the same as the center of the circumference of the ring.

EXAMPLES FOR PRACTICE.

1. A spherical shell has a wrought-iron handle attached to it. The shell is 10″ in diameter, and weighs 20 lb. The handle is $1\frac{1}{2}$″ in diameter, and the distance from the center of the shell to the end of the handle is 4 ft. Where is the center of gravity? Take the weight of a cubic inch of wrought iron as .278 lb. Ans. 13.612″ from center of shell.

2. The distance between the centers of two bodies is 51". The weights of the bodies being 20 and 73 lb., where is the center of gravity?

Ans. 10.968" from the center of large weight.

3. A hollow engine piston weighs 275 lb., and is $3\frac{1}{2}$" thick. Assuming the piston rod to be straight throughout its entire length, and to weigh 140 lb., at what point will the piston and rod balance if the length of the rod is 73" from the face of the piston? Consider the weight of the piston to be concentrated at its center.

Ans. 11.15", nearly, from face of piston.

4. Weights of 5, 9, and 13 lb. lie in one straight line in the order named. Distance from the 5-lb. weight to the 9-lb. weight is 22", and from the 9-lb. weight to the 12-lb. weight is 18". Where is the center of gravity?

Ans. 13.923" from 12-lb. weight.

SIMPLE MACHINES.

484. A **lever** is a bar capable of being turned about a pivot, or point, as in Figs. 70, 71, and 72.

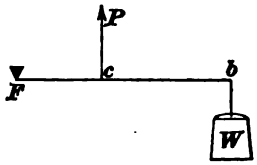

FIG. 70.

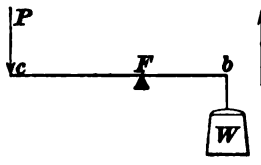

FIG. 71.

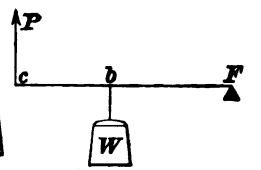

FIG. 72.

The object W to be lifted is called the **weight**; the force employed P is called the **power**; and the point, or pivot, F is called the **fulcrum.**

485. That part of the lever between the fulcrum and the weight, or $F\,b$, is called the **weight arm,** and the part between the fulcrum and the power, or $F\,c$, is called the **power arm.**

In order that the lever shall be in equilibrium (balance), *the power multiplied by the power arm must equal the weight multiplied by the weight arm;* that is, $P \times F\,c$ must equal $W \times F\,b$.

486. If F be taken as the center of a circle, and arcs be described through b and c, it will be seen that if the weight arm be moved through a certain angle, the power arm will move through the same angle. Since, in the same or equal angles, the lengths of the arcs are proportional to the radii

with which they were described, it is seen that the power arm is proportional to the distance through which the power moves, and the weight arm is proportional to the distance through which the weight moves.

Hence, instead of writing $P \times F c = W \times F b$, we might have written it $P \times$ distance through which P moves $=$ $W \times$ distance through which W moves. This is the general law of all machines, and can be applied to any mechanism from the simple lever up to the most complicated arrangement. Stated in the form of a rule, it is as follows:

Rule.—*The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.*

487. In the above rule, it will be noticed that there are four requirements necessary for a complete knowledge of the lever, viz.: the power (or force), the weight, the power arm (or distance through which the power moves), and the weight arm (or distance through which the weight moves). If any three are given, the fourth may be found by letting x represent the requirement which is to be found, and multiplying the power by the power arm and the weight by the weight arm; then, dividing the product of the two known numbers by the number by which x is multiplied; the result will be the requirement which was to be found.

EXAMPLE.—If the weight arm of a lever is 6 in. long, and the power arm is 4 ft. long, how great a weight can be raised by a force of 20 lb. at the end of the power arm?

SOLUTION.—In this example, the weight is unknown; hence, representing it by x, we have, after reducing the 4 ft. to inches, $20 \times 48 = 960 =$ power multiplied by the power arm, and $x \times 6 =$ weight multiplied by the weight arm. Dividing the 960 by 6, the result is 160 lb., the weight. Ans.

488. If the distance through which the power or weight moved had been given instead of the power arm or weight arm, and it were required to find the power or weight, the process would have been exactly the same, using the given distance instead of the power arm or weight arm.

EXAMPLE.—If, in the above example, the weight had moved 2½", how far would the power have moved?

SOLUTION.—In this example, the distance through which the power moves is required. Let x represent the distance. Then, $20 \times x =$ distance multiplied by power, and $2\frac{1}{2} \times 160 = 400 =$ distance multiplied by the weight. Hence, $x = \frac{400}{20} = 20$ in. = distance through which the power arm moves.

The ratio between the weights and the power is $160 \div 20 = 8$. The ratio between the distance through which the weight moves and the distance through which the power moves is $2\frac{1}{2} \div 20 = \frac{1}{8}$. This shows that while a force of 1 lb. can raise a weight of 8 lb., the 1-lb. weight must move through 8 times the distance that the 8-lb. weight does. It will also be noticed that the ratio of the lengths of the two arms of the lever is also 8, since $48 \div 6 = 8$.

489. The law which governs the straight lever also governs the bent lever, but care must be taken to determine the true lengths of the lever arms, which are in every case *the perpendicular distances from the fulcrum to the line of direction of the weight or power.*

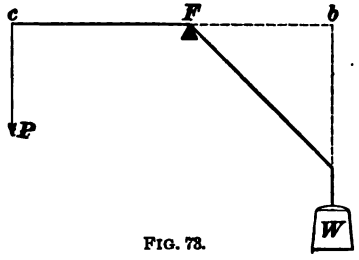

FIG. 73.

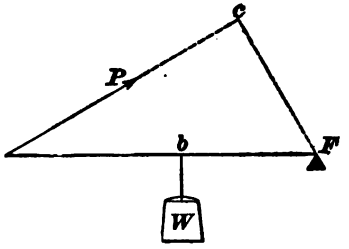

FIG. 74.

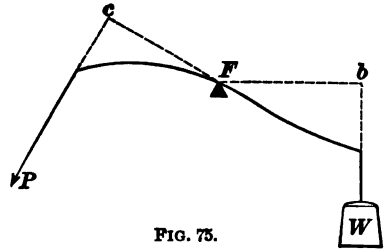

FIG. 75.

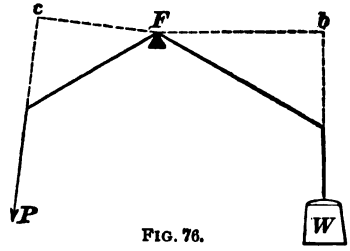

FIG. 76.

Thus, in Figs. 73, 74, 75, and 76, Fc in each case represents the power arm, and Fb the weight arm.

490. A **compound lever** is a series of single levers arranged in such a manner that when a power is applied to

the first it is communicated to the second, and from that to the third, and so on.

Fig. 77 shows a compound lever. It will be seen that when a power is applied to the first lever at P it will be

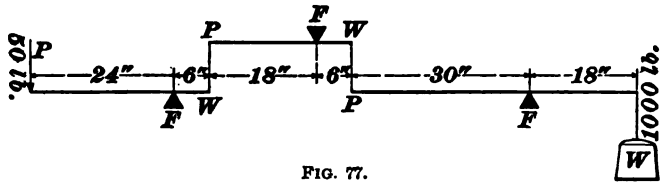

FIG. 77.

communicated to the second lever at P, from this to the third lever at P, and thus raise the weight W.

The weight which the power of the first lever could raise acts as the power of the second, and the weight which this could raise through the second lever acts as the power of the third lever, and so on, no matter how many single levers make up the compound lever.

In this case, as in every other, the power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.

Hence, if we move the P end of the lever, say 4 inches, and the end carrying the weight W moves $\frac{1}{5}$ of an inch, we know that the ratio between P and W is the same as the ratio between $\frac{1}{5}$ and 4; that is, 1 to 20, and, hence, that 10 pounds at P would balance 200 pounds at W, without measuring the lengths of the different lever arms. If the lengths of the lever arms are known, the ratio between P and W may be readily obtained from the following rule:

Rule.—*The continued product of the power and each power arm equals the continued product of the weight and each weight arm.*

EXAMPLE.—If, in Fig. 77, $PF = 24$ inches, 18 inches, and 30 inches, respectively, and $WF = 6$ inches, 6 inches, and 18 inches, respectively, how great a force at P would it require to raise 1,000 pounds at W? What is the ratio between W and P?

SOLUTION.—Let x represent the power; then, $x \times 24 \times 18 \times 30 = 12{,}960\,x$ = continued product of the power and each power arm. $1{,}000 \times$

$6 \times 6 \times 18 = 648{,}000$ = continued product of the weight and each weight arm, and, since $12{,}960\,x = 648{,}000$,

$$x = \frac{648{,}000}{12{,}960} = 50 \text{ lb.} = \text{the power.} \quad \text{Ans.}$$

$1{,}000 \div 50 = 20$ = ratio between W and P. **Ans.**

491. The **wheel and axle** consists of *two cylinders of different diameters, rigidly connected*, so that they turn together about a common axis, as in Fig. 78. Then, as

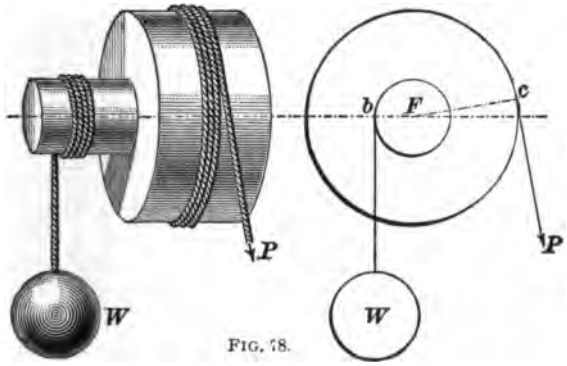

FIG. 78.

before, $P \times$ distance through which it moves $= W \times$ distance through which it moves; and, since these distances are proportional to the radii of the power cylinder and weight cylinder, $P \times F\,c = W \times F\,b$.

It is not necessary that an entire wheel be used; an arm, projection, radius, or anything which the power causes to revolve in a circle, may be considered as the wheel. Consequently, if it is desired to hoist a weight with a windlass, Fig. 79, the power is applied to the handle of the crank, and the distance between the center line of the crank handle and the axis of the drum corresponds to the radius of the wheel.

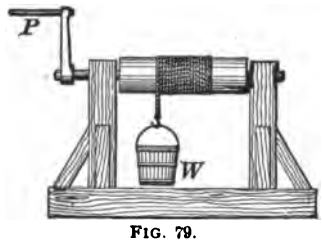

FIG. 79.

EXAMPLE.—If the distance between the center line of the handle and the axis of the drum, in Fig. 79, is 18 inches and the diameter of the drum is 6 inches, what force will be required at P to raise a load of 300 pounds?

SOLUTION.—$P \times (18 \times 2) = 300 \times 6$, or $P = 50$. **Ans.**

EXAMPLES FOR PRACTICE.

1. The lever of a safety valve is of the form shown in Fig. 70, where the force is applied at a point between the fulcrum and the weight lifted. If the distance from the fulcrum to the valve is $5\frac{1}{2}''$, and from the fulcrum to the weight is 42", what total force is necessary to raise the valve, the weight being 78 lb., and the weight of valve and lever being neglected? Ans. 595.64 lb.

2. If, in Fig. 77, $PF = 10''$, $12''$, $14''$, and $16''$ respectively, and $WF = 2''$, $3''$, $4''$, and $5''$, respectively, (*a*) how great a weight can a force of 20 lb. raise? (*b*) What is the ratio between W and P? (*c*) If P moves 4", how far will W move?

Ans. (*a*) 4,480 lb.; (*b*) 224; (*c*) $\frac{1}{56}''$.

3. A windlass is used to hoist a weight. If the diameter of the drum on which the rope winds is 4", and the distance from the center of the handle to the axis of the drum is 14", how great a weight can a force of 32 lb. applied to the handle raise? Ans. 224 lb.

4. In Fig. 72, if $cF = 42''$, and the weight $W = 60$ lb., where must it be hung so that a force of 15 lb. at P may just support it? Neglect the weight of the lever. Ans. $10\frac{1}{2}''$ from F.

5. In the last example, where must the weight be hung, if the lever weighs 20 lb., other conditions remaining the same?

Ans. $3\frac{1}{2}''$ from F.

6. In Fig. 79, the length of the handle, measured to the center of the drum, is 16", and the weight of the full bucket and the part of rope unwound is 80 lb.; a man exerting a force of 30 lb. on the handle can just sustain the weight. What is the diameter of the drum?

Ans. 12 inches.

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492. Wheelwork. — A combination of wheels and axles, as in Fig. 80, is called a **train.** The wheel in a train, to which motion is imparted from a wheel on another shaft by such means as a belt or gearing, is called the **driven wheel** or **follower;** the wheel which imparts the motion is called the **driver.**

493. It will be seen that the wheel and axle bears the same relation to the train that the simple lever does to the compound lever. Letting D_1, D_2, D_3, etc., represent the diameters of the driven wheels and d_1, d_2, d_3, etc., the diameters of the different drivers, we have the following

Rule.—*The continued product of the power and the radii of the driven wheels equals the continued product of the*

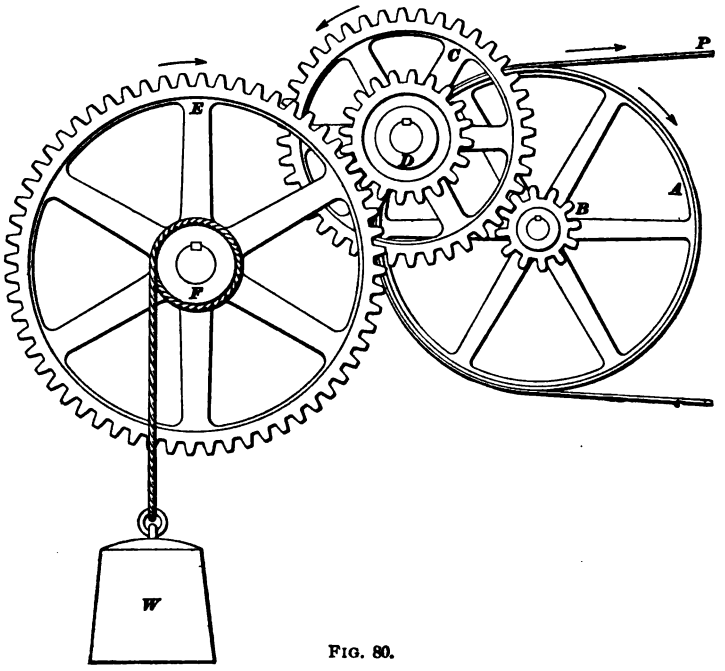

FIG. 80.

weight, the radius of the drum that moves the weight, and the radii of the drivers.

That is, $P \times D_1 \times D_2 \times D_3$, etc., $= W \times d_1 \times d_2 \times d_3$, etc.

EXAMPLE.—If the radius of the pulley A is 20 inches, of C, 15 inches, and of E, 24 inches, and the radius of the drum F is 4 inches, of the pinion D, 5 inches, and of the pinion B, 4 inches, how great a weight will a force of 1 pound at P raise?

SOLUTION.—Formula, $P \times D_1 \times D_2 \times D_3 = W \times d_1 \times d_2 \times d_3$.

Substituting, we have $1 \times 20 \times 15 \times 24 = W \times 4 \times 5 \times 4$, or

$$W = \frac{7,200}{80} = 90 \text{ lb.} \quad \text{Ans.}$$

Hence, also, if W were raised 1 inch, P would fall 90 inches, or P would have to move through 90 inches to raise W through 1 inch.

494. It is now clear that another great law has made itself manifest, and that is that *whenever there is a gain in power without a corresponding increase in the initial force, there is a loss in speed;* this is true of any machine.

In the last example, if P were to move the entire 90 inches in one second, W would move only 1 inch in one second.

Instead of using the diameter or radius of a gear, the number of teeth may be used when computing the weight which can be raised, or the velocity, as in the last example.

EXAMPLE.—The radius of the pulley A, Fig. 80, is 40″, and that of F is 12″. The number of teeth in B is 9; in C, 27; in D, 12, and in E, 36. If the weight to be lifted is 1,800 lb., how great a force at P is it necessary to apply to the belt?

SOLUTION.—Let P represent the force (power); then, by the last rule, $P \times 40 \times 27 \times 36 = 1{,}800 \times 12 \times 9 \times 12$, or $P \times 38{,}880 = 2{,}332{,}800$.

Hence, $P = \frac{2{,}332{,}800}{38{,}880} = 60$ lb. = force necessary to apply to the belt. Ans.

EXAMPLE.—The radius of pulley A, Fig. 80, is 28″, and that of the drum F is 10″; B has 8 teeth, C has 30, and E has 40. How many has D, if a force of 50 lb. at P will lift 1,500 lb. at W?

SOLUTION.—Using the same rule as in the preceding example, and letting x = the number of teeth required in D, we have

$$50 \times 28 \times 30 \times 40 = 1{,}500 \times 10 \times x \times 8;$$

whence, $$x = \frac{50 \times 28 \times 30 \times 40}{1{,}500 \times 10 \times 8} = 14.$$ Ans.

FIXED AND MOVABLE PULLEYS.

495. Pulleys are also used for hoisting or raising loads, in which case the frame which supports the axle of the pulley is called the **block.**

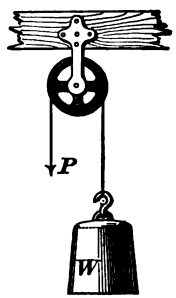

FIG. 81.

496. A **fixed pulley** is one whose block is not movable, as in Fig. 81. In this case if the weight W be lifted by pulling down P, the other end of the cord W will evidently move the same distance upwards that P moves downwards; hence, P must equal W.

497. A **movable pulley** is one whose block is movable, as in Fig. 82. One end of the cord is fastened to the beam, and the weight is suspended

from the pulley, the other end of the cord being drawn up by the application of a force P. A little consideration will show that if P moves through a certain distance, say 1 foot, W will move through *half* that distance, or 6 inches; hence, a pull of one pound at P will lift 2 pounds at W.

The same would also be true if the free end of the cord were passed over a *fixed pulley*, as in Fig. 83, in which case the fixed pulley merely changes the direction in which P acts, so that a weight of 1 pound hung on the free end of the cord will balance 2 pounds hung from the *movable pulley*.

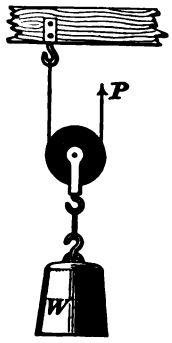

FIG. 82.

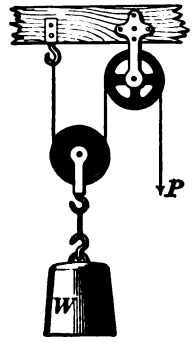

FIG. 83.

498. A **combination of pulleys,** as shown in Fig. 84, is sometimes used. In this case there are three movable and three fixed pulleys, and the amount of movement of W, owing to a certain movement of P, is readily found.

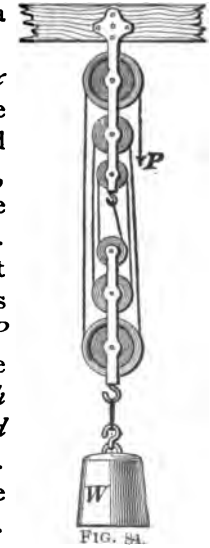

FIG. 84.

It will be noticed that there are *six parts* of the rope, not counting the free end; hence, if the movable block be lifted 1 foot, P remaining in the same position, there will be 1 foot of slack in each of the six parts of the rope, or *six feet* in all. Therefore, P would have to move 6 feet in order to take up this slack, or P moves 6 times as far as W. Hence, 1 pound at P will support 6 pounds at W, since the *power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.* It will also be noticed that there are three movable pulleys, and that $3 \times 2 = 6$.

499. Law of Combination of Pulleys:

Rule.—*In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as itself as there are parts of the rope supporting the load, not counting the free end.*

The above law is good, whether the pulleys are side by side, as in the ordinary block and tackle, or whether they are arranged as in the figure.

EXAMPLE.—In a block and tackle having five movable pulleys, how great a force must be applied to the free end of the rope to raise 1,250 pounds?

SOLUTION.—Since there are five movable pulleys, there must be 10 parts of the rope to support them. Hence, according to the above law, a force applied to the free end will support a load 10 times as great as itself, or the force $= \frac{1,250}{10} = 125$ lb. Ans.

THE INCLINED PLANE.

500. An **inclined plane** is a slope, or a flat surface, making an angle with a horizontal line.

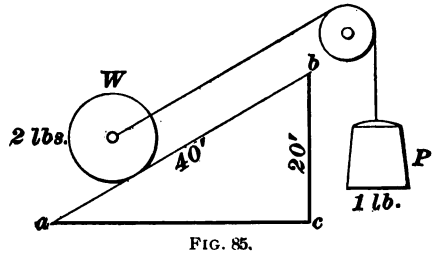

FIG. 85.

Three cases may arise in practice with the inclined plane:

1. Where the power acts parallel to the plane, as in Fig. 85.

2. Where the power acts parallel to the base, as in Fig. 86.

3. Where the power acts at an angle to the plane or to the base, as in Fig. 87.

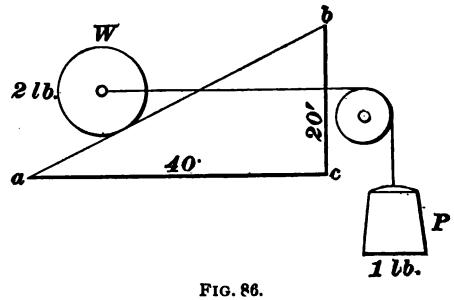

FIG. 86.

501. In Fig. 85 the relation existing between the power and the weight is easily

found. The weight ascends a distance equal to $c\,b$, or the height of the inclined plane, while the power descends through a distance equal to $a\,b$, or the length of the inclined plane. Therefore,

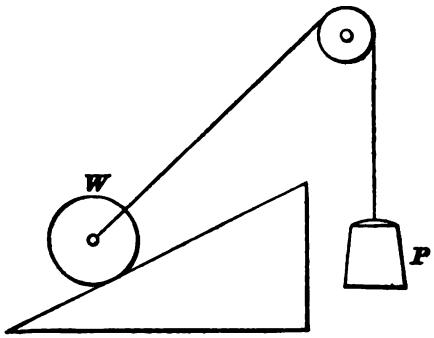

FIG. 87.

Rule I. — *The power multiplied by the length of the inclined plane equals the weight multiplied by the height of the inclined plane;* or, letting

$$\text{length of plane} = l;$$
$$\text{base of plane} = b;$$
$$\text{height of plane} = h,$$

we have

$$P = \frac{Wh}{l} \text{ and } W = \frac{Pl}{h}.$$

EXAMPLE.—The length of an inclined plane is 40 feet, and its height is 5 feet; what force P will sustain a weight W of 100 lb.?

SOLUTION.—$P = \frac{Wh}{l} = \frac{100 \times 5}{40} = 12\frac{1}{2}$ lb. Ans.

EXAMPLE.—The length of an inclined plane is 8 feet, and its height is 15 inches; what weight will 120 lb. support?

SOLUTION.— $W = \frac{Pl}{h} = \frac{120 \times 96}{15} = 768$ lb. Ans.

In Fig. 86 the power is supposed to act parallel to the base for any position of W; therefore, while W is moving from the level $a\ c$ to b, or through the height $c\ b$ of the inclined plane, P will move through a distance equal to the length of the base $a\ c$. Hence, when the power acts parallel to the base, we have

Rule II. — *The power multiplied by the base of the inclined plane equals the weight multiplied by the height of the plane,*

$$\text{whence, } P = \frac{Wh}{b} \text{ and } W = \frac{Pb}{h}.$$

EXAMPLE.—With a base 30 feet long, and a height of 6 feet, what power will sustain a weight of 75 lb.?

SOLUTION.—$P = \frac{75 \times 6}{30} = 15$ lb. Ans.

EXAMPLE.—A force of 12 lb. sustains a weight on a plane whose base is 6 ft. long, and height 18 inches. Find the weight.

SOLUTION.— $W = \frac{12 \times 6}{1\frac{1}{2}} = 48$ lb. Ans.

For Fig. 87 no rule can be given. The ratio of the power to the weight must be determined by trigonometry for every position of W; for, as W shifts its position, the angle that its cord makes with the face of the plane varies, and the magnitude of the force P depends on this angle; the smaller the angle the less is P required to be to support a given weight on a given plane.

502. The **wedge** is a movable inclined plane, and is used for moving a great weight a short distance. A common method of moving a heavy body is shown in Fig. 88.

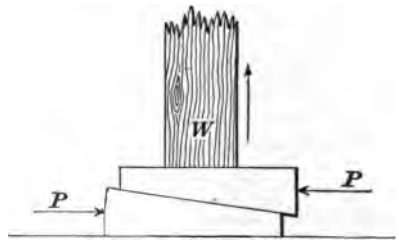

FIG. 88.

Simultaneous blows of equal force are struck on the heads of the wedges, thus raising the weight W. The laws for wedges are the same as for Case 2 of the inclined plane.

THE SCREW.

503. A **screw** is a cylinder with a helical projection winding around its circumference. This helix is called the *thread* of the screw. The distance that a point on the helix is drawn back or advanced in the direction of the length of the screw during one turn, is called the *pitch* of the screw.

The screw in Fig. 89 is turned in a *nut a*, by means of a force applied at the end of the handle *P*. For one complete revolution of the handle, the screw will be advanced lengthwise, *an amount equal to the pitch*. If the nut be fixed, and a weight be placed upon the end of the screw, as shown, it will be raised vertically a distance equal to the pitch by one revolution of the screw. During this revolution, the force at *P* will move through a distance equal to the circumference whose radius is *P F*. Therefore, *W* × pitch of thread = *P* × circumference of *P*.

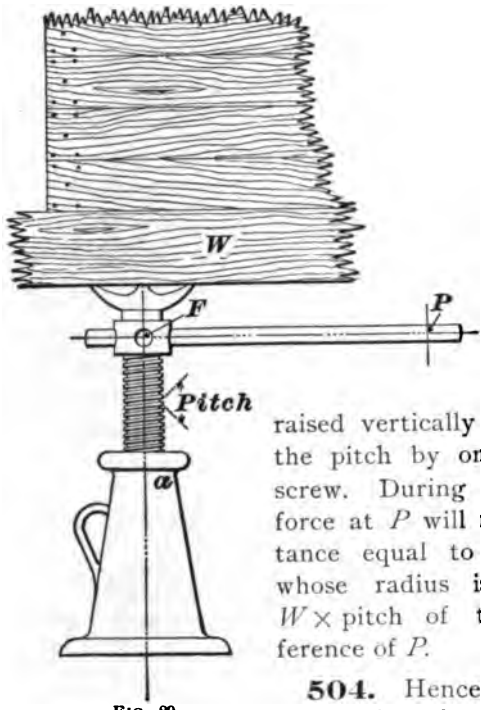

FIG. 89

504. Hence, to find the weight that a given force will lift, we have

Rule.—*Multiply the force by its distance from the axis, or center of screw, and by 6.2832; divide this product by the pitch, and the quotient will be the weight required;*

or $$W = \frac{P \times 6.2832 \times r}{p},$$

whence, $$P = \frac{W p}{6.2832 \times r}.$$

EXAMPLE.—It is desired to raise a weight by means of a screw having 5 threads per inch. The force is 40 pounds, and is applied at a distance of 14 inches from the center of the screw (*FP*, in Fig. 89); how great a weight can be raised?

SOLUTION.—Applying the formula,

$$W = \frac{40 \times 6.2832 \times 14}{\frac{1}{5}} = 17593 \text{ lb., very nearly.}$$ Ans.

EXAMPLE.—The weight to be lifted is 4,000 pounds; the pitch of the screw is $\frac{1}{4}$ (that is, there are 4 threads per inch). The force is applied at a distance of 12 inches from the center of screw; how great must it be?

SOLUTION.—Applying the formula,

$$P = \frac{4{,}000 \times \frac{1}{4}}{6.2832 \times 12} = \text{rather more than } 13\tfrac{1}{4} \text{ lb., nearly.}$$ **Ans.**

505. Single-threaded screws of less than 1 inch pitch are generally classified by the number of threads they have in 1 inch of their length. In such cases *one inch divided by the number of threads equals the pitch;* thus, the pitch of a screw that has 8 threads per inch is $\frac{1}{8}$; one of 32 threads per inch is $\frac{1}{32}$, etc.

506. Velocity Ratio.—The ratio of the distance through which the power moves to the corresponding distance through which the weight moves is called the **velocity ratio.**

Thus, if the power move through 12 inches while the weight moves through 1 inch, the velocity ratio is 12 to 1, or 12; that is, P moves 12 times as fast as W.

If the velocity ratio be known, the weight which any machine will raise can be found by multiplying the power *by the velocity ratio.* If the velocity ratio is 8.7 to 1, or 8.7, $W = 8.7 \times P$, since $W \times 1 = P \times 8.7$.

NOTE.—In all of the preceding cases, including the last, the effect of friction has been neglected.

EXAMPLES FOR PRACTICE.

1. If the distance from the point at which a force of 135 lb. is applied to a lever to the fulcrum is 4 feet, and the distance from the fulcrum to the weight is $1\frac{1}{2}$ inches, how great a weight will be lifted? Ans. 4,320 lb.

2. It is desired to raise a weight of 600 lb. by means of a block and tackle having two fixed and two movable pulleys; what force must be applied at the free end of the rope? Ans. 150 lb.

3. If, in the last example, the free end of the rope be attached to a windlass whose drum is 5 inches in diameter, and has a handle whose radius is 15 inches, what force will be necessary? Ans. 25 lb.

4. In Fig. 80, the radius of the wheel A is 32 inches, of C 18 inches, and of E 30 inches; the radius of the drum F is 5 inches, of the pinion D 4 inches, and of the pinion B 6 inches. If P moves 4 feet 6 inches, how far will the weight W move? Ans. $\frac{3}{4}$ in.

5. In the last example, how great a force must be applied at P to raise a weight of 2,160 lb.? Ans. 30 lb.

FRICTION.

507. Friction is the resistance that a body meets with from the surface on which it moves.

508. The **ratio** between the *resistance* to the motion of a body due to friction and the *perpendicular* pressure between the surfaces is called the **coefficient of friction.**

If a weight W, as in Fig. 90, rests upon a horizontal plane, and has a cord fastened to it passing over a pulley a, from which a weight P is suspended, then, if P is just sufficient to start W, the ratio of P to W, or $\frac{P}{W}$, is the *coefficient of friction* between W and the surface upon which it slides.

The weight W is the perpendicular pressure, and P is the

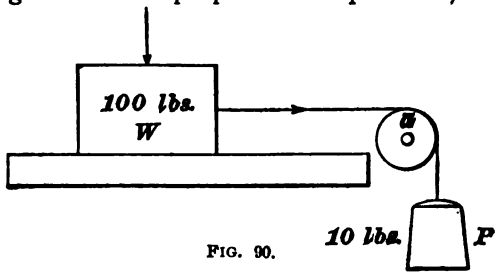

FIG. 90.

force necessary to overcome the resistance to the motion of W, due to friction.

If $W = 100$ pounds and $P = 10$ pounds, the coefficient of friction for this particular case would be $\frac{P}{W} = \frac{10}{100} = .1$.

509. Laws of Friction:

1. Friction is directly proportional to the perpendicular pressure between the two surfaces in contact.

2. Friction is independent of the extent of the surfaces in contact when the total perpendicular pressure remains the same.

3. Friction increases with the roughness of the surfaces.

4. Friction is greater between surfaces of the same material than between those of different materials.

5. Friction is greatest at the beginning of motion.

6. Friction is greater between soft bodies than between hard ones.

7. Rolling friction is less than sliding friction.

8. Friction is diminished by polishing or lubricating the surfaces.

510. Law 1 shows why the friction is so much greater on journals after they begin to heat than before. The heat causes the journal to expand, thus increasing the pressure between the journal and its bearing, and, consequently, increasing the friction.

Law 2 states that, no matter how small the surface may be which presses against another, if the perpendicular pressure is the same the friction will be the same. Therefore, large surfaces are used where possible, not to reduce the friction, but to reduce the wear and diminish the liability of heating.

TABLE OF COEFFICIENTS OF FRICTION.

TABLE 16.

Description of Surfaces in Contact.	Disposition of Fibers.	State of the Surfaces.	Coefficient of Friction.
Oak on oak.	Parallel	Dry	.48
Oak on oak.	Parallel	Soaped	.16
Wrought iron on oak.	Parallel	Dry	.62
Wrought iron on oak.	Parallel	Soaped	.21
Cast iron on oak.	Parallel	Dry	.49
Cast iron on oak.	Parallel	Soaped	.19
Wrought iron on cast iron.	—	Slightly Unctuous	.18
Wrought iron on bronze.	—	Slightly Unctuous	.18
Cast iron on cast iron.	—	Slightly Unctuous	.15

511. For instance, if the perpendicular pressure between a journal and its bearing is 10,000 pounds, and the coefficient of friction is .2, the amount of friction is $10,000 \times .2 = 2,000$ pounds.

Suppose that one-half the area of the surface of the journal is 80 square inches, then, the amount of friction for each square inch of bearing is $2,000 \div 80 = 25$ pounds.

If half the area of the surface had been 160 square inches, the friction would have been the same, that is, 2,000 pounds; but the friction per square inch would have been $2,000 \div 160 = 12\frac{1}{2}$ pounds, just one-half as much as before, and the wear and liability to heat would be one-half as great also.

512. The power which is required to raise a weight, or overcome an equal resistance in any machine, is thus always *greater than this weight or resistance, divided by the velocity ratio of the machine.*

Thus, if there were no friction, a machine whose velocity ratio was 5 would, by an application of 100 pounds of force, raise a weight of 500 pounds.

Now, suppose that the friction in the machine is equivalent to 10 pounds of force; then, it would take 110 pounds of force to raise 500 pounds.

If, in the above illustration, friction were neglected, 110 pounds $\times$ 5 = 550 pounds, or the weight that 110 pounds would raise; but, owing to the frictional resistance, it only raised 500 pounds. Therefore, we have for the ratio between the two, $\frac{500}{550} = .91$. That is, 500 : 550 :: .91 : 1.

513. Efficiency.—This ratio between the weight actually raised and the power multiplied by the velocity ratio, is called the **efficiency of the machine.**

For example, if the weight actually raised by a machine, say a screw, is 1,600 pounds, and the power multiplied by the velocity ratio is 2,400 pounds, the efficiency of this machine is

$$\frac{1,600}{2,400} = .66\frac{2}{3}, \text{ or } 66\frac{2}{3}\%.$$

EXAMPLE.—In a machine having a combination of pulleys and gears, the velocity ratio of the whole is 9.75. A force of 250 pounds just lifts a weight of 1,626 pounds. What is the efficiency of the machine?

SOLUTION.—Efficiency $= \frac{1,626}{250 \times 9.75} = .6671$, or 66.71%. Ans.

Since the total amount of friction varies with the load, it follows that the efficiency will also vary for different loads.

514. If the efficiency of a machine is known, the force actually required to raise a given load may be found by dividing the load by the product of the velocity ratio of the machine and the efficiency. Thus, if a certain machine has a velocity ratio of 10.6, and its efficiency is 60%, the force which must actually be applied to raise a load of 840 pounds is $840 \div 10.6 \times .60 = 840 \div 6.36 = 132.1$ pounds, nearly. If there had been no losses through friction, etc., the force required would have been $840 \div 10.6 = 79.25$ pounds, nearly.

If the efficiency is known, the weight which a certain force will raise may be found by multiplying together the force, velocity ratio, and the efficiency. Thus, if a certain machine has a velocity ratio of $6\frac{1}{2}$, and an efficiency of 78%, a force of 140 pounds will raise a weight of $140 \times 6\frac{1}{2} \times .78 = 709.8$ pounds.

515. When finding the force necessary to overcome the friction, the *perpendicular pressure* on the surface considered must always be taken. In order to find (approximately) the maximum force that is required to overcome the friction between cross-head and guides, we have

Rule.—*Multiply the total piston pressure by the length of crank and by the coefficient of friction and divide by the length of main rod;*

or, letting P = total piston pressure;
l = length of main rod;
r = length of crank;
c = coefficient of friction between cross-head and guides;
F = force required to overcome this friction,

we have $$F = \frac{Prc}{l}.$$

EXAMPLE.—An engine whose piston is 16″ in diameter carries a steam pressure of 80 lb. per sq. in. If the crank is 12″ long, and the connecting-rod is 66″ long, what is the perpendicular pressure on the guides? The coefficient of friction for this case being 12%, what force will be required to overcome the friction?

SOLUTION. — Pressure on piston $= 16^2 \times .7854 \times 80 = 16{,}085$ lb. $\frac{16{,}085 \times 12}{66} = 2{,}924.55$ lb. = perpendicular pressure. Ans. $2{,}924.55 \times .12 = 350.95$ lb. = force required to overcome the friction. Ans.

EXAMPLES FOR PRACTICE.

1. How great a force must be applied to the free end of the rope of a block and tackle which has four movable pulleys, to raise a weight of 746 lb.? Ans. 93½ lb.

2. An inclined plane is 30 ft. long and 7 ft. high; what force is required to roll a barrel of flour weighing 196 lb. up the plane, friction being neglected? Ans. 45.7½ lb.

3. The distance from the axis of a screw to the point on the handle where the force is applied is 12″. The screw has 8 threads per inch. What force is necessary to raise a weight of 1,248 lb.?

Ans. 2.07 lb., nearly.

4. In example 3, what should be the length of the handle to raise a weight of 5,400 lb. by the application of a force of 20 lb.?

Ans. 5.371″, nearly.

5. What is the velocity ratio (*a*) in example? 3 (*b*) in example 4?

Ans. (*a*) 603, nearly. (*b*) 270.

6. An engine piston is 24″ in diameter. If the steam pressure is 93 lb. per sq. in.; the length of the connecting-rod, 8 ft. 4″; the length of crank, 20″, and coefficient of friction, 14%, what is (*a*) the greatest perpendicular pressure on the guides? (*b*) the force required to overcome the friction?

Ans. (*a*) 8,414.46 lb. (*b*) 1,178 lb.

CENTRIFUGAL FORCE.

516. If a body be fastened to a string and whirled, so as to give it a circular motion, there will be a pull on the string which will be greater or less, according as the velocity increases or decreases. The cause of this pull on the string will now be explained.

Suppose that the body is revolved horizontally, so that the action of gravity upon it will always be the same. According to the first law of motion, a body put in motion tends to move in a straight line, unless acted upon by some other force, causing a change in the direction. When the body moves in a circle, the force that causes it to move in a circle instead of a straight line is exactly equal to the tension of the string. If the string were cut, the pulling force that drew it away from the straight line would be removed, and the body would then "fly off at a tangent;" that is, it would move in a straight line tangent to the circle, as shown in Fig. 91.

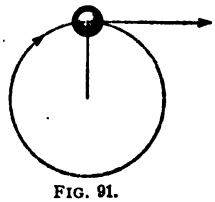

FIG. 91.

Since, according to the third law of motion, every action has an equal and opposite reaction, we call that force which acts as an equal and opposite force to the pull of the string the **centrifugal force,** and it acts *away* from the center of motion.

517. The other force, or tension, of the string is called the **centripetal force,** and it acts *towards* the center of motion. It is evident that these two forces, acting in opposite directions, tend to pull the string apart, and, if the velocity be increased sufficiently, the string will break. It is also evident that no body can revolve without generating centrifugal force.

518. The value of the centrifugal force, expressed in pounds, of any revolving body is calculated by the following rule:

Rule. — *The centrifugal force equals the continued product of .00034, the weight of the body in pounds, the radius in feet (taken as the distance between the center of gravity of the body and the center about which it revolves), and the square of the number of revolutions per minute.*

Let F = centrifugal force in pounds;

W = weight of revolving body in pounds;

R = radius in feet of circle described by center of gravity of revolving body;

N = revolutions per minute of revolving body; then

$$F = .00034\ W R N^2.$$

519. In calculating the centrifugal force of fly-wheels, it is the usual practice to consider the rim of the wheel only, and not take the arms and hub of the wheel into account. In this case R would be taken as the *distance between the center of the rim and the center of the shaft.*

EXAMPLE.—What would be the centrifugal force developed by a cast-iron fly-wheel, whose outside diameter was 10 feet, width of face 20 inches, and thickness of rim 6 inches, turning at the rate of 80 revolutions per minute? Take the weight of a cubic inch of cast iron as .261 lb.

SOLUTION.—First calculate the weight of the rim. The diameter of the rim $= 10 \times 12 = 120$ inches; the diameter of the circle midway between the inside and outside diameters of the rim $= 120 - 6 =$ 114 inches. The number of cubic inches in the rim $= 114 \times 3.1416 \times 20 \times 6 = 42{,}977$ cubic inches. $42{,}977 \times .261 = 11{,}217$ pounds = weight.

$$\text{Radius} = \frac{114}{2} \div 12 = 4\tfrac{1}{2} \text{ feet.} \quad \text{R. P. M.} = 80.$$

Hence, by rule, Art. **518**, centrifugal force $= .00034 \times 11{,}217 \times 4\tfrac{1}{2} \times 80^2 = 115{,}939$ pounds. Ans.

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520. Centrifugal Force in Liquids.—The effect of centrifugal force in a liquid contained in a vessel is noticeable by the central part of the surface of the liquid assuming a depression or a concave form. The outer parts of the surface of the liquid rise against the sides of the vessel and will attempt, by the extra head thus obtained, to counterbalance centrifugal force in the revolving liquid. Suppose that a common glass is partly filled with water, and the water stirred with a spoon is made to revolve, a difference in the surface of the water will be observed. The water tends to fly off at a tangent, but it can not, because the glass retains it. The pressure upon the sides of the glass is, however, increased, and a corresponding decrease in pressure is caused in the center of the volume of water. Hence, the difference in the water level.

Suppose that a pot of molten lead or solder is tied on the

end of a twisted rope for the purpose of being raised to the top of a building. It will be found that the weight of the pot and contents while being raised will untwist the rope; this will cause the pot to revolve; friction between the sides of the pot and the molten metal will cause the metal to revolve also, perhaps with such a velocity that a portion of the metal will overflow the pot. Plumbers should, therefore, be careful not to raise or lower a pot of solder or lead with a long rope, particularly if anybody is underneath.

SPECIFIC GRAVITY.

521. The **specific gravity of a body** *is the ratio between its weight and the weight of a like volume of water.*

522. Since gases are so much lighter than water, it is usual to take the specific gravity of a gas as the ratio between the weight of a certain volume of the gas and the weight of the same volume of air.

EXAMPLE.—A cubic foot of cast iron weighs 450 pounds; what is its specific gravity, a cubic foot of water weighing 62.5 pounds?

SOLUTION. $\frac{450}{62.5} = 7.2$. Ans.

523. The specific gravities of different bodies are given in printed tables; hence, if it is desired to know the weight of a solid or liquid body that can not be conveniently weighed, *calculate its cubical contents. Multiply the specific gravity of the body by the weight of a like volume of water, remembering that a cubic foot of water weighs 62.5 pounds. The weight of a cubic foot of any gas at atmospheric pressure and at 32° F. is found by multiplying its specific gravity by .08073 pound avoirdupois.*

EXAMPLE.—How much will 3,214 cubic inches of cast iron weigh? Take its specific gravity as 7.21.

SOLUTION.—Since 1 cubic foot of water weighs 62.5 pounds, 3,214 cubic inches weigh

$$\frac{3,214}{1,728} \times 62.5 = 116.25 \text{ pounds.}$$

$$116.25 \times 7.21 = 838.16 \text{ pounds.} \quad \text{Ans.}$$

EXAMPLE.—What is the weight of a cubic inch of cast iron?

SOLUTION.— $\frac{62.5}{1,728} \times 7.21 = .2608$ pound. Ans.

NOTE.—One cubic foot of pure distilled water at a temperature of 39.2° Fahrenheit weighs 62.42 pounds, but the value usually taken in making calculations is $62\frac{1}{2}$ pounds.

EXAMPLE.—What is the weight in pounds of 7 cubic feet of oxygen?

SOLUTION.—One cubic foot of air weighs .08073 lb., and the specific gravity of oxygen is 1.1056 compared with air; hence,

$$.08073 \times 1.1056 \times 7 = .62479 \text{ pound, nearly.}$$

EXAMPLES FOR PRACTICE.

1. The balls of a steam-engine governor each weigh 5 pounds. If they revolve in a circle whose diameter is 14″ at the rate of 80 revolutions per minute, what is the centrifugal force of each ball?

Ans. 6.347 lb., nearly.

2. The rim of a cast-iron engine fly-wheel has an outside diameter of 15 feet. If the rim is 8″ thick and 12″ wide, and the fly-wheel makes 40 R. P. M., what is the centrifugal force of the rim? Ans. 52,785 lb.

3. If a cubic foot of a certain alloy weighs 678 pounds, what is its specific gravity? Ans. 10.848.

4. What is the weight of (*a*) 12.4 cubic inches of lead? (*b*) of steel? (*c*) of aluminum?

Ans. { (*a*) 5.0964 lb. (*b*) 3.5216 lb. (*c*) 1.116 lb.

5. The specific gravity of an alloy of lead and zinc is 8.26; what is the weight of a cubic foot? Ans. 516.25 lb.

WORK.

524. **Work** *is the overcoming of resistance continually occurring along the path of motion.*

Mere motion is not work, but if a body in motion constantly overcomes a resistance, it does work.

525. The **measure of work** *is one pound raised vertically one foot*, and is called **one foot-pound.** All work is measured by this standard. A horse going up hill does an amount of work equal to his own weight plus the weight of the wagon and contents plus the frictional resistances reduced to an equivalent weight multiplied by the vertical height of the hill. Thus, if the horse weighs 1,200 pounds, the wagon and contents 1,200 pounds, and the frictional

resistances equal 400 pounds, then, if the vertical height of the hill is 100 feet, the work done is equal to $(1{,}200 + 1{,}200 + 400) \times 100 = 280{,}000$ foot-pounds.

Rule.—*In all cases the force (or resistance) multiplied by the distance through which it acts equals the work. If a weight be raised, the weight multiplied by the vertical height of the lift equals the work.*

526. The *total* amount of work is independent of time, whether it takes one minute or one year in which to do it; but in order to compare the work done by different machines with a common standard, time must be considered. If one machine does a certain amount of work in 10 minutes, and another machine does exactly the same amount of work in 5 minutes, the second machine can do twice as much work as the first in the same time.

527. The common standard to which all work is reduced is the **horsepower.**

One horsepower *is 33,000 foot-pounds per minute; in other words, it is 33,000 pounds raised vertically one foot in one minute, or 1 pound raised vertically 33,000 feet in one minute, or any combination that will, when multiplied together, give 33,000 foot-pounds in one minute.*

528. Thus, the work done in raising 110 pounds vertically 5 feet in one second is a horsepower; for, since in one minute there are 60 seconds, $110 \times 5 \times 60 = 33{,}000$ foot-pounds in one minute.

EXAMPLE.—If the coefficient of friction is .3, how many horsepower will it require to draw a load of 10,000 pounds on a level surface, a distance of one mile in one hour?

SOLUTION.—$10{,}000 \times .3 = 3{,}000$ pounds = the force necessary to overcome the resistance (resistance of the air is neglected). One mile = 5,280 feet; one hour = 60 minutes. Therefore, $\frac{5{,}280}{60} = 88$ feet per minute.

Work done = force multiplied by the space = $3{,}000 \times 88 = 264{,}000$ foot-pounds per minute.

Horsepower $= \frac{264{,}000}{33{,}000} = 8$ horsepower. Ans.

The abbreviation for horsepower is H. P.

529. Energy is a term used to express *the ability of an agent to do work.* Work can not be done without motion, and the work that a moving body is capable of doing in being brought to rest is called the **kinetic energy** of the body.

Kinetic energy means the actual visible energy of a body in motion. The work which a moving body is capable of doing in being brought to rest is exactly the same as the kinetic energy developed by it in falling in a vacuum through a height sufficient to give it the same velocity.

Rule.—*The kinetic energy of a moving body in foot-pounds equals its weight in pounds multiplied by the square of its velocity in feet per second, and divided by 64.32.*

If W represents the weight of the body in pounds, and v the velocity in feet per second,

$$\text{kinetic energy} = \frac{Wv^2}{64.32}.$$

If a weight is raised a certain height, a certain amount of work is done equal to the product of the weight and the vertical height. If a weight is suspended at a certain height and allowed to fall, it will do the same amount of work in foot-pounds that was required to raise the weight to the height through which it fell.

EXAMPLE.—If a body weighing 25 pounds falls from a height of 100 feet, how much work can it do?

SOLUTION.—Work $= Wh = 25 \times 100 = 2{,}500$ foot-pounds. Ans.

530. It requires the same amount of work or energy to stop a body in motion within a certain time as it does to give it that velocity in the same time.

EXAMPLE.—A body weighing 50 pounds has a velocity of 100 feet per second; what is its kinetic energy?

SOLUTION.—Kinetic energy $= \frac{Wv^2}{64.32} = \frac{50 \times 100^2}{64.32} = 7{,}773.63$ foot-pounds. Ans.

EXAMPLE.—In the last example, how many horsepower will be required to give the body this amount of kinetic energy in 3 seconds?

SOLUTION.—1 H. P. = 33,000 pounds raised 1 foot in one minute.

If 7,773.63 foot-pounds of work are done in 3 seconds, in one second

there would be done $\frac{7,773.65}{3} = 2,591.21$ foot-pounds of work. One horsepower = 33,000 ft.-lb. per min. = 33,000 ÷ 60 = 550 ft.-lb. per sec.

The number of horsepower developed will be

$$\frac{2,591.21}{550} = 4.7113 \text{ H. P.} \quad \text{Ans.}$$

531. Potential energy *is latent energy; it is the energy which a body at rest is capable of giving out under certain conditions.*

If a stone is suspended by a string from a high tower, it has potential energy. If the string is cut, the stone will fall to the ground, and during its fall its potential energy will change into kinetic energy, so that at the instant it strikes the ground its potential energy is wholly changed into kinetic energy.

At a point equal to one-half the height of the fall, the potential and kinetic energies are equal. At the end of the first quarter the potential energy was $\frac{3}{4}$, and the kinetic energy $\frac{1}{4}$; at the end of the third quarter the potential energy was $\frac{1}{4}$, and the kinetic energy $\frac{3}{4}$.

A pound of coal has a certain amount of potential energy. When the coal is burned, the potential energy is liberated and changed into kinetic energy in the form of heat. The kinetic energy of the heat changes water into steam, which thus has a certain amount of potential energy. The steam acting on the piston of an engine causes it to move through a certain space, thus overcoming a resistance, changing the potential energy of the steam into kinetic energy, and thus doing work.

Potential energy *then, is the energy stored within a body which may be liberated and produce motion, thus generating kinetic energy, and enabling work to be done.*

532. The principle of **conservation of energy** teaches that energy, like matter, can never be destroyed. If a clock is put in motion, the potential energy of the spring is changed into kinetic energy of motion, which turns the wheels, thus producing friction.

The friction produces heat, which dissipates into the surrounding air, but still the energy is not destroyed—it merely

exists in another form. The potential energy in coal was received from the sun, in the form of heat, ages ago, and has lain dormant for millions of years.

HYDROSTATICS.

533. **Hydrostatics** treats of liquids at rest under the action of forces.

534. Liquids are very nearly *incompressible*. A pressure of 15 pounds per square inch compresses water less than $\frac{1}{20000}$ of its volume.

535. Fig. 92 represents two cylindrical vessels of exactly the same size. The vessel *a* is fitted with a wooden block of the same size as the cylinder, and can move in it; the vessel *b* is filled with water, whose depth is the same as the length of the wooden block in *a*. Both vessels are fitted with air-tight pistons *P*, whose areas are each 10 sq. in.

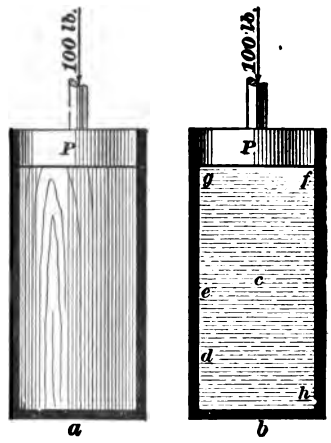

FIG. 92.

Suppose, for convenience, that the weights of the pistons, block, and water be neglected, and that a force of 100 pounds be applied to both pistons. The pressure per square inch will be $\frac{100}{10} = 10$ pounds. In the vessel *a* this pressure will be transmitted to the bottom of the vessel, and will be 10 pounds per square inch; it is easy to see that there will be no pressure on the sides. In the vessel *b* an entirely different result is obtained. The pressure on the bottom will be the same as in the other case, that is, 10 pounds per square inch, but, owing to the fact that the

molecules of the water are perfectly free to move, this pressure of 10 pounds per square inch is *transmitted in every direction with the same intensity;* that is to say, the pressure at any point *c*, *d*, *e*, *f*, *g*, *h*, etc., due to the force of 100 pounds, is exactly the same, and equals 10 pounds per square inch.

536. This may be easily proven experimentally by means of an apparatus like that shown in Fig. 93. Let the area of the piston *a* be 20 sq. in.; of *b*, 7 sq. in.; of *c*, 1 sq. in.; of *d*, 6 sq. in.; of *e*, 8 sq. in., and of *f*, 4 sq. in.

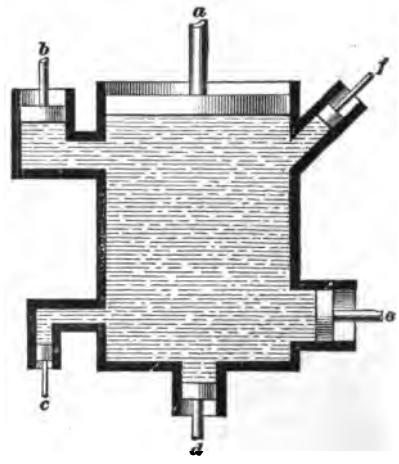

FIG. 93.

If the pressure due to the weight of the water be neglected, and a force of 5 pounds be applied at *c* (whose area is 1 sq. in.), a pressure of 5 pounds per square inch will be transmitted in all directions, and in order that there shall be no movement, a force of $6 \times 5 = 30$ pounds must be applied at *d*, 40 pounds at *e*, 20 pounds at *f*, 100 pounds at *a*, and 35 pounds at *b*.

If a force of 99 pounds were applied to *a*, instead of 100 pounds, the piston *a* would rise, and the other pistons *b*, *c*, *d*, *e*, and *f*, would move inwards; but, if the force applied to *a* were 100 pounds, they would all be in equilibrium. If 101 pounds were applied at *a*, the pressure per square inch would be $\frac{101}{20} = 5.05$ pounds, which would be transmitted in all directions; and, since the pressure due to the load on *c* is only 5 pounds per square inch, it is now evident that the piston *a* will move downwards, and the pistons *b*, *c*, *d*, *e*, and *f* will be forced outwards.

537. The whole of the preceding may be summed up as follows :

The pressure per unit of area exerted anywhere upon a mass of liquid is transmitted undiminished in all directions, and acts with the same force upon all surfaces, in a direction at **right angles** *to those surfaces.*

This law was first discovered by Pascal, and is the most important in **hydromechanics.** Its meaning should be thoroughly understood.

EXAMPLE.—If the area of the piston *e*, in Fig. 93, were 8.25 sq. in., and a force of 150 pounds were applied to it, what forces would have to be applied to the other pistons to keep the water in equilibrium, assuming that their areas were the same as given before?

SOLUTION.— $\frac{150}{8.25} = 18.182$ pounds per square inch, nearly.

$$\left.\begin{array}{l} 20 \times 18.182 = 363.64 \text{ lb.} = \text{force to balance } a. \\ 7 \times 18.182 = 127.274 \text{ lb.} = \text{force to balance } b. \\ 1 \times 18.182 = 18.182 \text{ lb.} = \text{force to balance } c. \\ 6 \times 18.182 = 109.092 \text{ lb.} = \text{force to balance } d. \\ 4 \times 18.182 = 72.728 \text{ lb.} = \text{force to balance } f. \end{array}\right\} \text{Ans.}$$

538. The pressure due to the weight of a liquid *may be downwards, upwards, or sideways.*

539. Downward Pressure.—In Fig. 94 the pressure on the bottom of the vessel *a* is, of course, equal to the weight of the water it contains. If the area of the bottom of the vessel *b*, and the depth of the liquid contained in it, are the same as in the vessel *a*, the pressure on the bottom of *b* will be the same as on the bottom of *a*. Suppose the bottoms of the vessels *a* and *b* are 6 inches square, and that the part *c d* in the vessel *b* is 2 inches square, and that they are filled with water. Then, the weight of one cubic inch of water is $\frac{62.5}{1,728}$ pound =

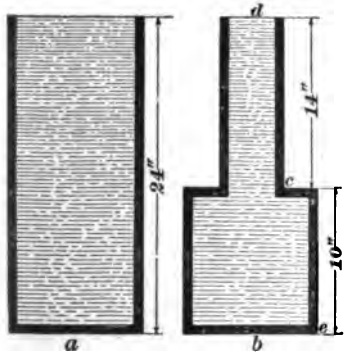

FIG. 94.

.03617 pound. The number of cubic inches in $a = 6 \times 6 \times 24 = 864$ cubic inches; and the weight of the water is $864 \times .03617 = 31.25$ pounds. Hence, the total pressure on the bottom of the vessel *a* is 31.25 pounds, or 0.868 pound per square inch.

The pressure in *b*, due to the weight contained in the part *b c* is $6 \times 6 \times 10 \times .03617 = 13.02$ pounds.

The weight of the part contained in *c d* is $2 \times 2 \times 14 \times .03617 = 2.0255$ pounds, and the weight per square inch of area in *c d* is $\frac{2.0255}{4} = .5064$ pound.

540. According to Pascal's law, this weight (pressure) is transmitted equally in all directions, therefore, an extra weight of .5064 pound is imposed on every square inch of the bottom of *b c*; the area of this is $6 \times 6 = 36$ square inches, and the pressure on it is, therefore, $36 \times .5064 = 18.23$ pounds, due to water contained in *c d*; thus, we have a total pressure on bottom of vessel *b* of $13.02 + 18.23 = 31.25$ pounds, the same as in vessel *a*. As a result of the above law, there is also an upward pressure of .5064 pound acting on every square inch of the *top* of the enlarged part *b c*.

If an additional pressure of 10 pounds per square inch were applied to the upper surface of both vessels, the total pressure on each bottom would be $31.25 + (6 \times 6 \times 10) = 31.25 + 360 = 391.25$ pounds.

If this pressure were to be obtained by means of a weight placed on each piston (as shown in Figs. 92 and 93), we should have to put a weight of $6 \times 6 \times 10 = 360$ pounds on the piston in vessel *a*, and one of $2 \times 2 \times 10 = 40$ pounds on the piston in vessel *b*.

541. The General Law for the Downward Pressure upon the Bottom of any Vessel:

Rule.—*The pressure upon the bottom of a vessel containing a fluid is independent of the shape of the vessel, and is equal to the weight of a column of the fluid, the area of whose base is equal to that of the bottom of the vessel, and whose altitude is the distance between the bottom and the upper surface of the fluid plus the pressure per unit of area*

upon the upper surface of the fluid multiplied by the area of the bottom of the vessel.

542. Suppose that the vessel *b*, in Fig. 94, were inverted, as shown in Fig. 95; the pressure upon the bottom will still be 0.868 pound per square inch, but it will require a weight of 3,490 pounds to be placed upon a piston at the upper surface to make the pressure on the bottom 391.25 pounds, instead of a weight of 40 pounds, as in the other case.

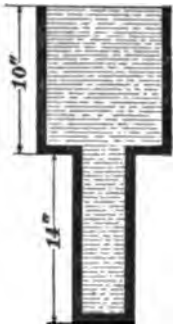

FIG. 95.

EXAMPLE.—A vessel filled with salt water, having a specific gravity of 1.03, has a circular bottom 13 inches in diameter. The top of the vessel is fitted with a piston 3 inches in diameter, on which is laid a weight of 75 pounds; what is the total pressure on the bottom, if the depth of the water is 18 inches?

SOLUTION.—Applying the rule, the weight of 1 cubic inch of the water is $\frac{62.5 \times 1.03}{1,728} = .037254$ lb.

$13 \times 13 \times .7854 \times 18 \times .037254 = 89.01$ pounds = the pressure due to the weight of the water.

$\frac{75}{3 \times 3 \times .7854} = 10.61$ pounds per square inch, due to the weight on the piston. $13 \times 13 \times .7854 \times 10.61 = 1,408.29$ pounds.

Total pressure = 1,408.29 + 89.01 = 1,497.3 pounds. Ans.

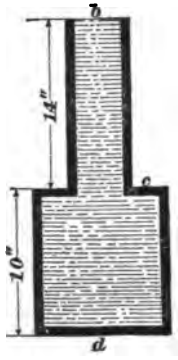

FIG. 96.

543. Upward Pressure.—In Fig. 96 is represented a vessel of exactly the same size as that shown in Fig. 95. There is no upward pressure on the surface *c*, due to the weight of the water in the large part *c d*, but there is an upward pressure on *c*, due to the weight of the water in the small part *b c*. The pressure per square inch, due to the weight of the water in *b c*, was found to be .5064 pound (see Art. **539**), the area of the upper surface *c* of the large part *c d* is $(6 \times 6) - (2 \times 2) = 36 - 4 = 32$ sq. in., and the total upward pressure, due to the weight of the water, is $.5064 \times 32 = 16.2$ pounds.

If an additional pressure of 10 pounds per square inch were applied to a piston fitting in the top of the vessel, the total upward pressure on the surface c would be $16.2 + (32 \times 10) = 336.2$ pounds.

544. General Law for Upward Pressure.

Rule.—*The upward pressure on any submerged horizontal surface equals the weight of a column of the liquid whose base has an area equal to the area of the submerged surface, and whose altitude is the distance between the submerged surface and the upper surface of the liquid plus the pressure per unit of area on the upper surface of the fluid multiplied by the area of the submerged surface.*

EXAMPLE.—A horizontal surface 6 inches by 4 inches is submerged in a vessel of water 26 inches below the upper surface; if the pressure on the water is 16 pounds per square inch, what is the total upward pressure on the horizontal surface?

SOLUTION.—Applying the rule, we get $6 \times 4 \times 26 \times .03617 = 22.57$ pounds for the upward pressure, due to the weight of the water, and $6 \times 4 \times 16 = 384$ pounds for the upward pressure, due to the outside pressure of 16 pounds per square inch.

Therefore, the total upward pressure $= 384 + 22.57 = 406.57$ pounds. Ans.

545. Lateral (Sideways) Pressure.—Suppose the top of the vessel shown in Fig. 97 is 10 inches square, and that the projections at a and b are 1 inch $\times$ 1 inch and 10 inches long.

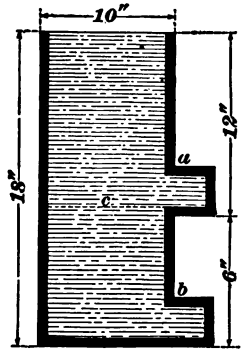

FIG. 97.

The pressure per square inch on the bottom of the vessel, due to the weight of the liquid, is $1 \times 1 \times 18 \times$ the weight of a cubic inch of the liquid.

The pressure at a depth equal to the distance of the upper surface b is $1 \times 1 \times 17 \times$ the weight of a cubic inch of the liquid.

Since both of these pressures are transmitted in every direction, they are also transmitted laterally (sideways),

and the *pressure per unit of area on the projection b is a mean between the two*, and equals $1 \times 1 \times 17\frac{1}{2} \times$ the weight of a cubic inch of the liquid.

To find the lateral pressure on the projection *a*, imagine that the dotted line *c* is the bottom of the vessel, then the conditions would be the same as in the preceding case, except that the depth is not so great.

The lateral pressure per sq. in. on *a* is thus seen to be $1 \times 1 \times 11\frac{1}{2} \times$ the weight of a cubic inch of the liquid.

546. General Law for Lateral Pressure:

Rule.—*The pressure upon any vertical surface, due to the weight of the liquid, is equal to the weight of a column of the liquid whose base has the same area as the vertical surface, and whose altitude is the depth of the center of gravity of the vertical surface below the upper surface of the liquid.*

Any additional pressure is to be added as in the previous cases.

EXAMPLE.—A well 3 feet in diameter, and 20 feet deep, is filled with water; what is the pressure on a strip of the wall 1 inch wide, the top of which is 1 foot from the bottom of the well? What is the pressure on the bottom? What is the upward pressure per square inch, 2 feet 6 inches from the bottom?

SOLUTION.—Applying the rule, the area of the strip is equal to its length (= circumference of well) multiplied by its height. The length $= 36 \times 3.1416 = 113.1''$; height $= 1''$; hence, area of strip $= 113.1 \times 1 = 113.1$ sq. in. Depth of center of gravity of strip $= (20 - 1)$ ft. $+$ $\frac{1}{2}$ inch, the half width of strip, $= 228\frac{1}{2}$ inches. Consequently, the total pressure on the strip $= 113.1 \times 228.5 \times .03617 = 934.75$ lb. Ans.

The pressure on each square inch of the strip would be $\frac{934.75}{113.1} = 8.265$ pounds, nearly.

$36 \times 36 \times .7854 \times 20 \times 12 \times .03617 = 8{,}836$ pounds = the pressure on the bottom. Ans.

$20 - 2.5 = 17.5$. $1 \times 17.5 \times 12 \times .03617 = 7.596$ pounds = the upward pressure per square inch, 2 feet 6 inches from the bottom. Ans.

547. The effects of lateral pressure are illustrated in Fig. 98. In the figure, *e* is a tall vessel, having a stop-cock near its base, and arranged to float upon the water, as shown. When this vessel is filled with water, the lateral pressures at any two points of the surface of the vessel opposite to each

other are equal. Being equal, and acting in opposite directions, they destroy each other, and no motion can result; but if the stop-cock be opened there will be no resistance to the pressure acting on that part, and the water will flow out; at the same time, the pressure on the corresponding part of the opposite side of the vessel remains, and this, being no longer balanced, causes the vessel to move backwards through the water in a direction opposite to that of the issuing jet.

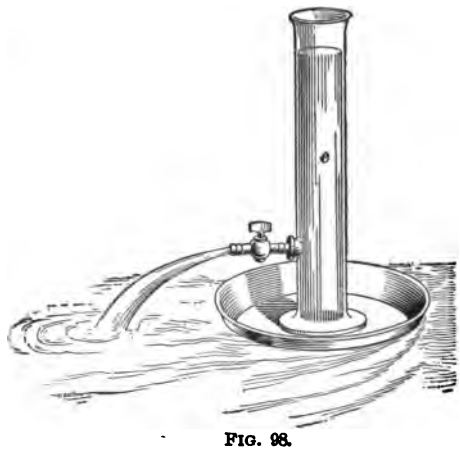

FIG. 98.

548. Since the pressure on the bottom of a vessel, due to the weight of the liquid, is dependent only upon the height of the liquid, and not upon the shape of the vessel, it follows

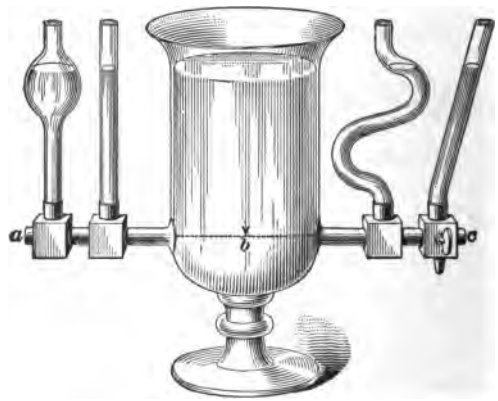

FIG. 99.

that if a vessel has a number of radiating tubes (see Fig. 99) the water in each tube will be on the same level, no matter

what may be the shape of the tubes. For, if the water were higher in one tube than in the others, the downward pressure on the bottom, due to the height of the water in this tube, would be greater than that due to the height of the water in the other tubes. Consequently, the upward pressure would also be greater; the equilibrium would be destroyed, and the water would flow from this tube into the vessel, and rise in the other tubes until it was at the same level in all, when it would be in equilibrium. This principle is expressed in the familiar saying, *water seeks its level.*

549. The above principle explains why city water reservoirs are located on high elevations, and why water on leaving the hose nozzle spouts so high.

If there were no resistance by friction and air, the water would spout to a height equal to the level of the water in the reservoirs. If a long pipe were attached to the nozzle whose length was equal to the vertical distance between the nozzle and the level of the water in the reservoir, the water would just reach the end of the pipe. If the pipe were lowered slightly, the water would trickle out.

Fountains, canal locks, and artesian wells are examples of the application of this principle.

EXAMPLE.—The water level in a city reservoir is 150 feet from the level of the street; what is the pressure of the water per square inch on the hydrant?

SOLUTION.—Apply rule, Art. **541**, $1 \times 150 \times 12 \times .03617 = 65.106$ pounds per square inch.

NOTE.—In measuring the height of the water to find the pressure which it produces, the ***vertical*** height, or distance, between the level of the water and the point considered is always taken. This vertical height is called the ***head.***

The weight of a column of water 1 in. square and 1 ft. high is $62.5 \div 144 = .434$ lb., very nearly. Hence, if the depth (head) be given, the pressure per square inch may be found by multiplying the depth in feet by .434. The constant .434 is the one ordinarily employed in practical calculations.

550. In Fig. 100, let the area of the piston *a* be 1 square inch, of *b* 40 square inches. According to Pascal's law, 1 pound placed upon *a* will balance 40 pounds placed upon *b.*

Suppose that a moves downwards 10 inches, then, 10 cubic inches of water will be forced into the tube b. This will be distributed over the entire area of the tube b, in the form of a cylinder whose cubical contents must be 10 cubic inches, whose base has an area of 40 square inches, and whose altitude must be $\frac{10}{40} = \frac{1}{4}$ of an inch; that is, a movement of 10 inches of the piston a will cause a movement of $\frac{1}{4}$ of an inch in the piston b.

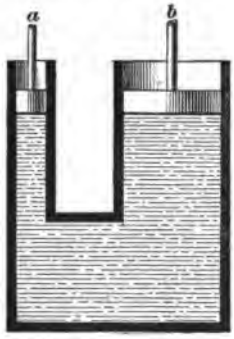

FIG. 100.

Here is the old principle of machines: *The power multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.*

Since, if 1 pound on the piston a represents the power P, the equivalent weight W on b may be obtained from the equation, $W \times \frac{1}{4} = P \times 10$, whence $W = 40\,P = 40$ lb.

551. Another familiar fact is also recognized, for the velocity ratio of P to W is $10 : \frac{1}{4}$, or 40; and, since in any machine the weight equals the power multiplied by the velocity ratio, $W = P \times 40$, and when $P = 1$, $W = 40$.

This principle is made use of in the hydraulic press represented in Fig. 101. As the man depresses the lever O, he forces down the piston a upon the water in the cylinder A. The water is forced through the bent tube d into the cylinder in which the large ram, or plunger, C works, and causes it to rise, thus lifting the platform K, and compressing the bales. If the area of a be 1 sq. in., and that of C be 100 sq. in., the velocity ratio will be 100.

If the length of the lever between the hand and the fulcrum is 10 times the length between the fulcrum and the piston a, the velocity ratio of the lever will be 10, and the total velocity ratio of the hand to the piston C will be $10 \times 100 = 1{,}000$.

Hence, a force of 100 pounds applied by the hand will raise $100 \times 1{,}000 = 100{,}000$ pounds. But, if the average move-

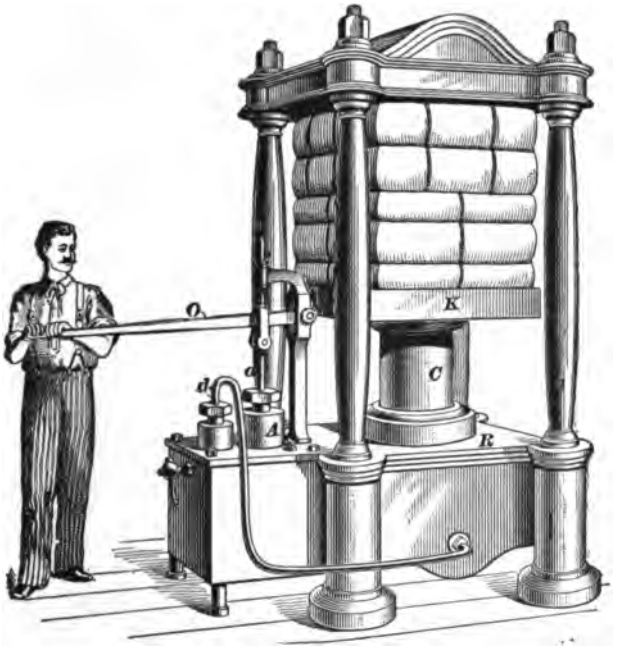

FIG. 101.

ment of the hand per stroke is 10 inches, it will require $\frac{1{,}000}{10} = 100$ strokes to raise the platform 1 inch, and it is again seen that what is gained in power is lost in speed.

552. Applications of this principle are seen in the hydraulic machines used for forcing locomotive drivers on their axles, etc., and for testing the strength of boiler shells.

EXAMPLE.—A suspended vertical cylinder is tested for the tightness of its heads by filling it with water. A pipe whose inside diameter is $\frac{1}{4}$ of an inch, and whose length is 20 feet, is screwed into a hole in the upper head, and then filled with water. What is the pressure per square inch on each head, if the cylinder is 40 inches in diameter and 60 inches long?

SOLUTION.—Area of heads $= 40^2 \times .7854 = 1{,}256.64$ sq. in.

Pressure per square inch on the bottom head, due to the weight of the water in the cylinder, $= 1 \times 60 \times .03617 = 2.17$ pounds.

$(\frac{1}{2})^2 \times .7854 = .04909$ sq. in., the area of the pipe.

$.04909 \times 20 \times 12 \times .03617 = .426$ pound = the weight of water in the pipe = the pressure on a surface area of .04909 sq. in.

The pressure per square inch, due to the water in the pipe, is $\frac{1}{.04909} \times .426 = 8.68$ pounds per square inch upon the upper head. Ans.

The total pressure per square inch on the lower head is $8.68 + 2.17 = 10.85$ pounds. Ans.

EXAMPLE.—In the last example, if the pipe be fitted with a piston weighing $\frac{1}{4}$ of a pound, and a 5-pound weight be laid upon it, what is the pressure per sq. in. upon the upper head?

SOLUTION.—In addition to the pressure of .426 pound on the area of .04909 sq. in., there is now an additional pressure upon this area of $5 + \frac{1}{4} = 5.25$ pounds, and the total pressure upon this area is $.426 + 5.25 = 5.676$ pounds. The pressure per square inch is $\frac{1}{.04909} \times 5.676 = 115.6$ pounds. Ans.

EXAMPLE.—In Fig. 102, the plunger A, is 10″ in diameter, and is forced

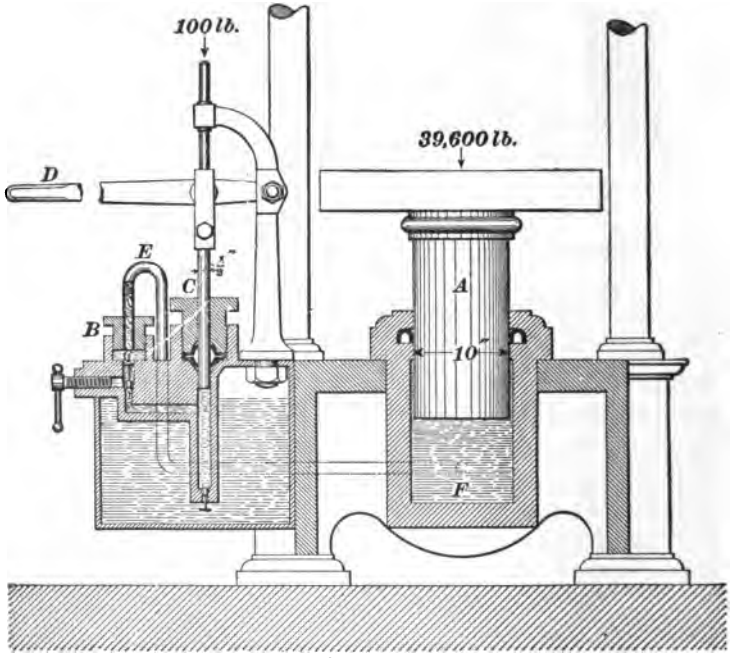

FIG. 102.

outwards by means of a small pump B, which supplies the press

cylinder with water. The plunger C of the pump B is $\frac{1}{2}$" in diameter. If a force of 100 lb. be applied to C by means of the lever D, how great a weight can the plunger A raise, if the plunger itself weighs 400 lb. ?

SOLUTION.—First find the pressure per sq. in. which the plunger C exerts upon the water. Area of $C = (\frac{1}{2})^2 \times .7854 = .19635$ sq. in. Since C exerts a pressure of 100 lb. upon an area of .19635 sq. in., it will exert a pressure $100 \times \frac{1}{.19635}$ lb. upon an area of 1 sq. in. This pressure is transmitted by the water in the tube E to the cylinder F, where it forces the plunger A upwards as the water is forced into F. The pressure per sq. in. on the bottom of A is the same as that exerted by C. Hence, the total pressure is $10^2 \times .7854 \times \frac{100}{.19635} = 40{,}000$ lb. This result, less the weight of A, equals the load lifted. Therefore, $40{,}000 - 400 = 39{,}600$ lb. Ans.

553. In these examples on the hydraulic press, no allowance has been made for the power lost in overcoming the friction between the cup leathers and the plunger; this varies according to the condition of the leathers, and, of course, the smoothness of the plunger; when the leathers are in good condition the loss is about 5% of the total pressure on the ram; when the leathers are old, stiff, and dirty the loss may amount to 15% or more.

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554. Pressure upon Oblique Surfaces and in Pipes and Cylinders.—According to the law of Pascal, the presssure which a fluid exerts upon a surface is at right angles to the surface; hence, the pressures exerted upon the oblique sides of the vessel shown in Fig. 103 are in the direction of the arrows.

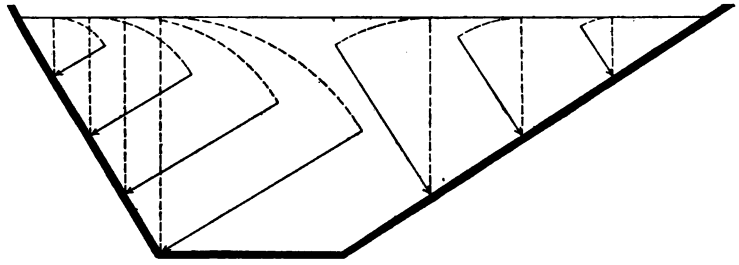

FIG. 103.

The lengths of the arrows also show the respective magnitudes of the pressures at the points of application.

A vertical line drawn from the point of pressure to the surface of the water is called the *vertical head* or *hydrostatic head.* The pressure per square inch at any point upon an oblique surface is found by the methods previously explained; that is, multiply the vertical head in inches by .03617, or multiply the vertical head in feet by .434.

555. The total pressure upon an oblique surface is found by multiplying the area of the surface by the pressure at the center of gravity of the surface.

Since the pressure is always exerted at right angles to the resisting surfaces, we find that the pressure in a pipe or cylinder is exerted in every direction, as shown by the arrows in Fig. 104, which is a cross-section through a kitchen double boiler; that is, a boiler, or cylindrical shell *A* within a boiler or cylindrical shell *B*.

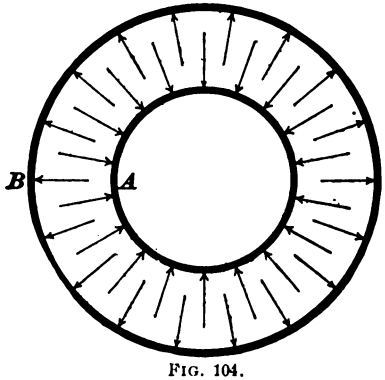

FIG. 104.

The arrows show how the pressure of the water in the outer boiler acts upon the outer and inner shells, and also shows how the internal boiler, which is generally made of copper, may collapse if it be emptied while the pressure is on the outer boiler.

556. To find the total pressure in a cylinder, multiply the area of its internal surface by the pressure of the water at its center of surface. The product will be the total pressure within the cylinder.

EXAMPLE.—A horizontal kitchen boiler is 5 ft. long and 14" in diameter, and is subjected to pressure due to a vertical head of 65 ft. Find the total pressure in the boiler.

SOLUTION.—Area of sides $= 14 \times 3.1416 \times 60 = 2,638.94$ sq. in. Area of top and bottom $= 14 \times 14 \times .7854 \times 2 = 307.88$ sq. in. Total area =

2,946.82 sq. in. Pressure per square inch at *center of boiler* = 65 × .434 = 28.21 lb. Total pressure = 2,946.82 × 28.21 = 83,129.79 lb. Ans.

557. Fig. 105 shows by arrows the direction of pressure upon the elliptical end of a kitchen boiler.

The total amount of pressure tending to force out the elliptical end is not equal to the amount of pressure upon it as indicated by the arrows (which would be the pressure multiplied by the area of the end), but is equal to the pressure per square inch multiplied by the *projected area* of the end as represented by the line *CB*, which in this case is equal to the sectional area of the cylinder.

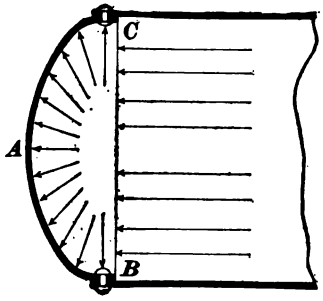

FIG. 105.

EXAMPLE.—If the diameter of the boiler shown in Fig. 105 is 20" and the pressure per square inch 80 lb., what is the force tending to blow out the end?

SOLUTION.—20 × 20 × .7854 × 80 = 9,424.8 lb. Ans.

558. The rule for finding the pressure exerted by a fluid in any direction upon a surface is:

Rule.—*The pressure exerted by a fluid in any direction upon* **any** *surface is equal to the weight of a prism of the fluid whose base is the projection of the surface at right angles to the direction considered, and whose height is the depth of the center of gravity of the surface below the level of the liquid.*

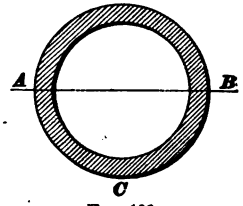

FIG. 106.

It can be proven by higher mathematics that rule, Art. **558**, holds good for any surface, no matter what its shape may be; that is, whether flat, spherical, cylindrical, or any shape whatever. Consequently, if a cylinder is filled with water and a pressure applied, the total pressure upon any half

section of the cylinder, as $A\ C\ B$, Fig. 106, tending to separate one half from the other is equal to the *projected area of the half cylinder (or the diameter A B times the length of the cylinder) multiplied by the depth of the center of gravity of the cylinder A C B (or $\frac{1}{2}$ d) multiplied by the weight of a cubic unit of water plus the diameter of the shell multiplied by the pressure per square inch multiplied by the length of the cylinder.*

If $d =$ the diameter $A\ B$ in inches and $l =$ the length of the cylinder in inches, the pressure due to the weight of the water when the cylinder is horizontal and filled with water upon the half cylinder $A\ C\ B = d \times l \times \frac{d}{2} \times$ the weight of a cubic inch of water $= l \times \frac{d^2}{2} \times$ the weight of a cubic inch of water.

The pressure due to an additional pressure of P_1 pounds per square inch $= l \times d \times P_1$. Thus, if a cylinder 60 inches long and 40 inches in diameter, lying horizontally, is filled with water, the pressure on any half section, as $A\ C\ B$, due to the weight of the water, will be found as follows:

$$60 \times \frac{40^2}{2} \times .03617 = 1{,}736.16 \text{ pounds.}$$

If there were an additional pressure per square inch of 50 pounds, the total pressure would be $60 \times 40 \times 50 + 1{,}736.16 = 121{,}736.16$ pounds.

559. If the cylinder were vertical instead of horizontal, the depth of the center of gravity would evidently be $\frac{1}{2}\ l$ below the surface, and the pressure tending to separate one half from the other, due to the weight of the water, would be $d \times l \times \frac{l}{2} \times$ weight of a cubic inch of water $= d \times \frac{l^2}{2} \times$ weight of a cubic inch of water.

Any additional pressure should be calculated as in the previous case.

EXAMPLE.—What would be the pressure due to the weight of the water, if the cylinder in the last example were vertical?

SOLUTION.— $40 \times \frac{60^2}{2} \times .03617 = 2{,}604.24$ lb. Ans.

560. In the case of a sphere, the projected area is evidently the area of a circle whose diameter is the same as the diameter of the sphere. Hence, the pressure upon a hemisphere, due to the weight of the water, will be $d^2 \times .7854 \times \frac{d}{2} \times$ the weight of a cubic inch of water $= \frac{d^3}{2} \times .7854 \times$ the weight of a cubic inch of water.

The pressure upon a hemisphere whose diameter is 20 inches, when filled with water, due to the weight of the water only, will be $\frac{20^3}{2} \times .7854 \times .03617 = 113.63$ pounds.

For an additional pressure P in pounds per square inch, the pressure on the hemisphere due to this will be $d^2 \times .7854 \times P$.

BUOYANT EFFECTS OF WATER.

561. In Fig. 107 is shown a 6-inch cube, entirely submerged in water. The lateral pressures are equal and in opposite directions. The upward pressure acting on the lower surface of the cube $= 6 \times 6 \times 21 \times .03617$, the downward pressure acting on the top of the cube $= 6 \times 6 \times 15 \times .03617$, and the difference $= 6 \times 6 \times 6 \times .03617 =$ the volume of the cube in cubic inches $\times$ the weight of one cubic inch of water. That is, the upward pressure exceeds the downward pressure by the weight of a volume of water equal to the volume of the body.

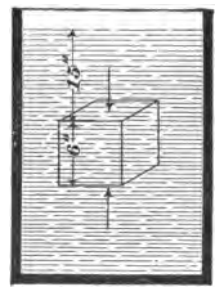

FIG. 107.

This excess of upward pressure over the downward pressure

acts against gravity; that is, the water presses the body *up* with a greater force than it presses it *down;* consequently, *if a body is immersed in a fluid, it will lose in weight an amount equal to the weight of the fluid it displaces.* This is called the **principle of Archimedes,** because it was first stated by him.

562. This principle may be experimentally demonstrated with the beam scales, as shown in Fig. 108.

From one scale pan suspend a hollow cylinder of metal *t*, and below that a solid cylinder *a* of the same size as the hollow part of the upper cylinder. Put weights in the other scale pan until they exactly balance the two cylinders. If *a* be immersed in water, the scale pan containing the weights will descend, showing that *a* has lost some of its weight. Now fill *t* with water, and the volume of water that can be poured into *t* will equal that displaced by *a*. The scale pan that contains the weights will gradually rise until *t* is filled, when the scales balance again.

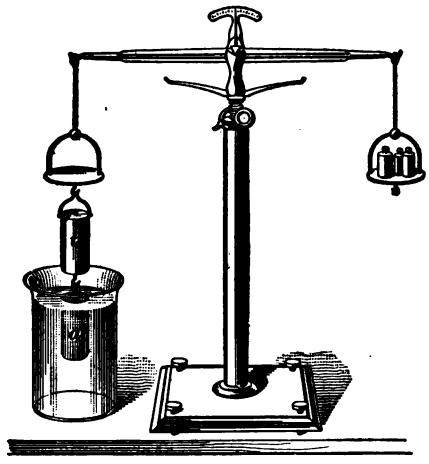

FIG. 108.

If a body be lighter than the liquid in which it is immersed, the upward pressure will cause it to rise, and project partly out of the liquid, until the weight of the body and the weight of the liquid displaced are equal. If the immersed body be heavier than the liquid, the downward pressure plus the weight of the body will be greater than the upward pressure, and the body will fall downwards until it touches bottom or meets an obstruction. If the weights of equal volumes of the liquid and the body are

equal, the body will remain stationary, and be in equilibrium in any position or depth beneath the surface of the liquid.

563. An interesting experiment in confirmation of the above facts may be performed as follows: Drop an egg into a glass jar filled with fresh water. The mean density of the egg being a little greater than that of water, it will fall to the bottom of the jar. Now, dissolve salt in the water, stirring it so as to mix the fresh and salt water. The salt water will presently become denser than the egg, and the egg will rise. Now, if fresh water be poured in until the egg and water have the same density, the egg will remain stationary in any position that it may be placed below the surface of the water.

EXAMPLES FOR PRACTICE.

1. Suppose a cylinder to be filled with water and placed in an upright position. If the diameter of the cylinder be 19", and its total length inside be 26", what will be the total pressure on the bottom when a pipe $\frac{1}{4}$" in diameter and 12 ft. long is screwed into the cylinder head and filled with water? The pipe is vertical. Ans. 1,743.2 lb.

2. In the last example, what is the total pressure against the upper head? Ans. 1,476.6 lb.

3. In example 1, a piston is fitted to the upper end of the pipe, and an additional force of 10 lb. is applied to the water in the pipe. (*a*) What is the total pressure on the bottom of the cylinder? (*b*) on the upper head?

Ans. { (*a*) 16,184 lb.
(*b*) 15,906 lb.

4. In example 3, what is the pressure per square inch in the pipe 2" from the upper cylinder head? Ans. 56.0656 lb. per sq. in.

5. A water tower 80 feet high is filled with water. A pipe 4" in diameter is so connected to the side of the tower that its center is 3 feet from the bottom. If the pipe is closed by a flat cover, what is the total pressure against the cover? Ans. 420 lb.

6. In the last example, what is the upward pressure per square inch 10 feet from the bottom of the tower?

Ans. 30.3828 lb. per sq. in.

7. A cube of wood, one edge of which measures 3 feet, is sunk until the upper surface is 40 feet below the level of the water; what is the total force which tends to move the cube upwards? Ans. 1,687.5 lb.

CAPILLARY ATTRACTION.

564. If a clean glass rod be placed vertically in water, the water will be drawn up around the tube. (See *a*, Fig. 109.) If the same rod be placed in mercury, the liquid will

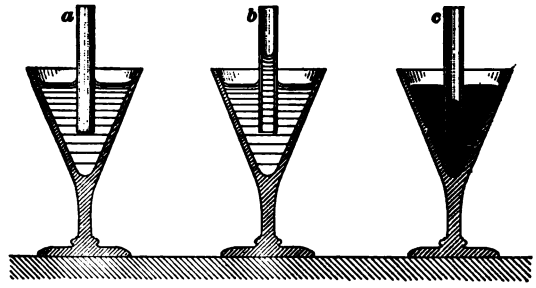

FIG. 109.

be depressed instead of raised. On examination, it will be found that water *wets the glass, while mercury does not.* If the rod be greased and placed in water, the surface of the water will be depressed about the rod. If a clean lead or zinc strip be placed in mercury, the surface of the mercury will be *raised* about the strip.

In the last two cases, the greased rod came out *dry*, no water adhering to it, while the mercury did adhere to the lead or zinc strip, which came out *wet.*

In general, *all liquids that will wet the solids placed in them, will be lifted, while those that do not, will be pushed down.*

These phenomena are called **capillary attraction,** because they are best shown in very fine or hair-like tubes. In Fig. 109, *b* is a glass tube in water and *c* is a glass tube in mercury. The surface of the water in the tube *b* is *concave*, while the surface of the mercury in the tube *c* is *convex.*

The amount which a liquid will ascend or be depressed varies inversely as the square of the diameter of the tube. Thus, water will rise four times as far in a tube $\frac{1}{32}$ of an inch in diameter as in one $\frac{1}{16}$ of an inch in diameter.

565. There are many illustrations of capillary action. It is capillary attraction that aids the ascent of sap in the pores of plants. It lifts the oil between the fibers of lamp wick to the place of combustion. It enables cloth and sponges to take up moisture. It causes blotting paper to absorb ink, but when paper is *sized*, its pores are filled, and the ink dries by evaporation. It is capable of exerting great force, as is shown in the effects produced by the swelling of wood and other substances when kept wet. Dry wooden wedges driven into a groove cut around a cylinder of stone, and occasionally wet, will cause it to break asunder. As the pores between the fibers of a rope run around it in spiral lines, the swelling produced by wetting a tight rope will cause the fibers to shorten, and to contract the rope with immense force.

MECHANICS.

(CONTINUED.)

HYDROKINETICS.

566. Hydrokinetics, also called **hydrodynamics,** and **hydraulics,** treats of water in motion. The velocity is not the same at all points of the flow, unless all cross-sections of the pipe or canal are equal. That *velocity*, which, being *multiplied by the area of the cross-section of the stream*, will equal the total quantity *discharged*, is called the **mean velocity.**

VELOCITY OF EFFLUX.

567. If a small aperture be made in a vessel containing water, the velocity with which the water issues from the vessel is the same as if it had fallen from the level of the surface to the level of the aperture, all resistance being neglected. This velocity is called the **velocity of efflux.**

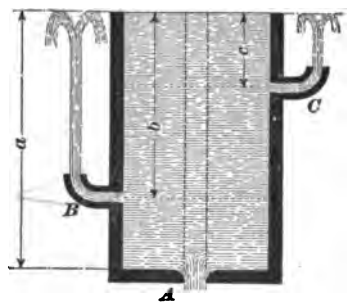

FIG. 110.

The *vertical height* of the level surface of the water above a horizontal line through the center of the aperture is called the **head.** In Fig. 110, *a* is the head for the aperture *A*; *b* is the head for the aperture *B*, and *c* is the head for the aperture *C*.

Were it not for the resistance of the air, friction, and the

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effect of the falling particles, the issuing water would spout to the level of the water in the vessel ; that is, to a height equal to its head.

THE CONTRACTED VEIN.

568. When water issues from an orifice in a thin plate (see Fig. 111), or from a square-edged orifice (see Fig. 112),

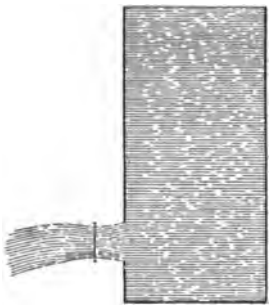


FIG. 111.

the stream is contracted at a short distance from the orifice, and expands again to the full size of the orifice. The point at which the contraction is greatest is at a distance from the orifice equal to about one-half of the diameter of the orifice. In consequence of this contraction, the *velocity of efflux* is slightly reduced from the theoretical value, and the *quantity discharged* is greatly reduced. This contraction is called the **contracted vein,** a name given to it by Sir Isaac Newton.

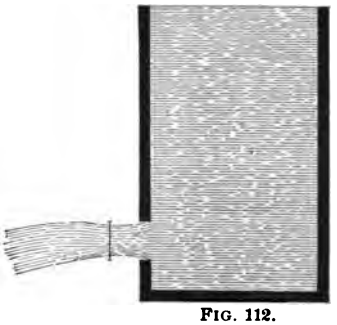


FIG. 112.

If water pipes be connected to a tank so that the orifice is of the same diameter as the pipe, the flow of water is considerably reduced by the presence of the contracted vein. To avoid this, the connections to the tank should be made funnel-mouthed.

A, Fig. 113, shows the contracted vein in an iron pipe connection to a tank. *B* is a lead pipe connection to the same tank, and is funnel-mouthed, so as to avoid the contracted vein. If the pipes *A* and *B* have the same inside diameter, the same length, and have the same vertical head,

the velocity of flow at the outlet of B will be considerably greater than at the outlet of A.

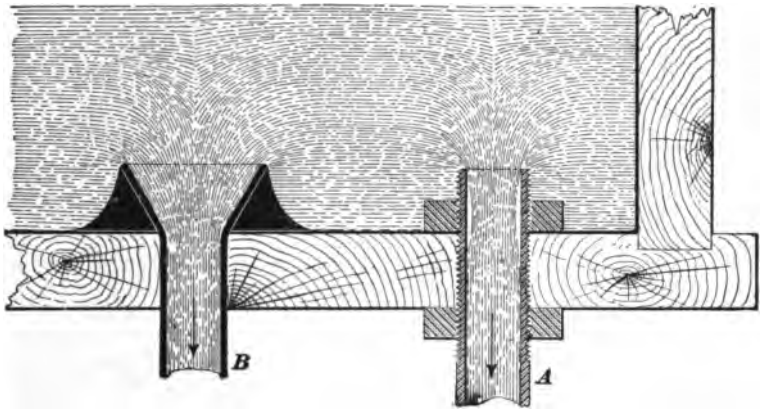


FIG. 113.

569. Water flowing through a pipe has different velocities at different points of the section. The **mean velocity** of discharge is that velocity which, multiplied by the area of the section, will give a product equal to the amount discharged in a given time. Hence, to find the mean velocity in feet per second,

Rule.—*Divide the number of cubic feet of water discharged in one second by the area of the orifice in square feet; the result will be the velocity in feet per second.*

That is, $$v_m = \frac{Q}{A},$$

when v_m = the mean velocity of efflux in feet per second;
Q = number of cubic feet discharged in one second;
A = area of cross-section of pipe in square feet.

EXAMPLE.—A pump delivers 1,000 cubic feet of water in 100 minutes through a 2″ pipe. What is the mean velocity of efflux in feet per second?

SOLUTION.—$\frac{1,000}{100 \times 60} = \frac{1}{6}$ cu. ft. per second. $2^2 \times .7854 = 3.1416$ sq. in. $= \frac{3.1416}{144}$ sq. ft. = area of pipe. Then, $v_m = \frac{1}{6} \div \frac{3.1416}{144} = \frac{144}{6 \times 3.1416} = 7.64$ ft. per second. Ans.

FLOW OF WATER IN PIPES.

570. When water flows from one reservoir to another through a pipe, the velocity of efflux is considerably less than the theoretical velocity due to the head. This loss is due to several causes, but is principally caused by the friction of the water against the inside surface of the pipe. *This friction varies directly as the length of the pipes, and inversely as the diameter;* that is, the friction in a pipe 200 feet long, is twice as much as in a pipe 100 feet long, and the friction in a pipe 4 inches in diameter is only half as much as in a pipe 2 inches in diameter, the velocity remaining the same in both cases. *The friction also varies nearly as the square of the velocity.*

Let v_m = mean velocity of discharge in feet per second;
h = total head in feet = the vertical distance between the level of the water in the reservoir and the point of discharge;
l = length of pipe in feet;
d = diameter of pipe in inches;
f = coefficient of friction.

Then, for straight cylindrical pipes of uniform diameter, the mean velocity of efflux may be calculated by the following rule:

Rule.—*The mean velocity of discharge equals 2.315 times the square root of the quotient obtained by dividing the head in feet multiplied by the diameter in inches, by the length in feet multiplied by the coefficient of friction plus .125 of the diameter of the pipe in inches.*

That is, $$v_m = 2.315\sqrt{\frac{h\,d}{f\,l + .125\,d}}.$$

NOTE.—The head is always taken as the vertical distance between an imaginary line through the point of discharge and the level of the water at the source or point from which the water is taken, and is always measured in feet. It matters not how long the pipe is, whether vertical or inclined, whether straight or curved, or whether any part of the pipe goes below the level of the point of discharge or not, the head is always measured as stated above.

EXAMPLE.—What is the mean velocity of efflux from a 6-inch pipe, 5,780 feet long, if the head is 170 feet? Take $f = .021$.

SOLUTION.— $$v_m = 2.315\sqrt{\frac{h\,d}{f\,l + .125\,d}} =$$

$$2.315\sqrt{\frac{170 \times 6}{.021 \times 5{,}780 + .125 \times 6}} = 6.69 \text{ ft. per sec.} \quad \text{Ans.}$$

571. When the pipe is very long, compared with the diameter as in the above example, the following rule may be used:

Rule.—*The mean velocity of the discharge equals 2.315 times the square root of the quotient obtained by dividing the product of the head in feet and diameter in inches by the product of the coefficient of friction and the length of the pipe in feet.*

That is, $$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}},$$

in which the letters have the same meaning as in the preceding rule.

EXAMPLE.—In the preceding example, calculate the value of v_m by the rule last given.

SOLUTION.—$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{170 \times 6}{.021 \times 5{,}780}} = 6.71$ feet per second. Ans.

The above rule may be used when the length of the pipe exceeds 10,000 times its diameter.

572. The actual head necessary to produce a certain velocity v_m, may be calculated by the following rule:

Rule.—*The total head in feet necessary to produce a velocity of efflux of v_m in a straight cylindrical pipe is equal to the coefficient of friction multiplied by the length of the pipe in feet multiplied by the square of the mean velocity of efflux in feet per second divided by 5.36 times the diameter of the pipe in inches plus .0233 times the square of the mean velocity.*

That is, $$h = \frac{f\, l\, v_m^2}{5.36\, d} + .0233\, v_m^2.$$

EXAMPLE.—A 7-inch pipe 6,000 feet long is required to deliver water with a velocity of 7 feet per second. What must be the necessary head? Assume $f = .026$.

SOLUTION.—$h = \frac{f\, l\, v_m^2}{5.36\, d} + .0233\, v_m^2 = \frac{.026 \times 6{,}000 \times 49}{5.36 \times 7} + .0253 \times 49 = 204.87$ feet, nearly. Ans.

The quantity discharged from pipes:

573. The rules just given are made use of in ascertaining the quantity of water that will be discharged from a pipe in a given time, with a given head. This is readily found by multiplying the area of the orifice by the mean velocity; thus, $Q = A\, v_m$.

Since $A = .7854\, d^2 =$ area of the inside of the pipe, the quantity discharged can be readily calculated as soon as v_m is known. This method gives the discharge in cubic feet per second, when the diameter d is taken in feet.

One cubic foot contains 7.48 gallons; hence, when d is taken in feet, $Q = .7854\, d^2\, v_m \times 7.48$ gallons. If d is taken in inches, $Q = \frac{.7854\, d^2}{144}\, v_m \times 7.48$, or $Q = .0408\, d^2\, v_m$.

Hence,

Rule.—*The discharge in gallons per second equals .0408 times the square of the diameter of the pipe in inches multiplied by the mean velocity of efflux in feet per second.*

EXAMPLE.—What is the discharge in gallons per minute from a 6-inch pipe, if the mean velocity of efflux is 5.6 feet per second?

SOLUTION.—$Q = .0408\, d^2\, v_m = .0408 \times 36 \times 5.6 = 8.225$ gallons per second. $8.225 \times 60 = 493.5$ gallons per minute. Ans.

574. If the diameter of the pipe and the discharge are known, the mean velocity can be readily found from the following rule:

Rule.—*The mean velocity of discharge equals 24.51 times the number of gallons discharged per second divided by the square of the diameter of the pipe in inches.*

That is, $$v_m = \frac{24.51\,Q}{d^2}.$$

EXAMPLE.—A 5-inch pipe is discharging 360 gallons per minute. What is the mean velocity of efflux?

SOLUTION.—$\frac{360}{60} = 6$ gallons discharged per second.

$$v_m = \frac{24.51 \times Q}{d^2} = \frac{24.51 \times 6}{25} = 5.882 \text{ feet per second.} \quad \text{Ans.}$$

575. If the head, the length of the pipe, and the diameter of the pipe are given, to find the discharge, use the following rule:

Rule.—*The discharge in gallons per second equals .09445 times the square of the diameter of the pipe in inches multiplied by the square root of the quotient obtained by dividing the head in feet times the diameter of the pipe in inches by the coefficient of friction times the length of the pipe in feet plus .125 times the diameter of the pipe in inches.*

That is, $$Q = .09445\,d^2\sqrt{\frac{h\,d}{f\,l + .125\,d}}.$$

To find the value of f, calculate v_m by rule, Art. **571**, assuming that $f = .025$, and get the final value of f from the following table:

$v_m =$	0.1	0.2	0.3	0.4	0.5	0.6
$f =$	.0686	.0527	.0457	.0415	.0387	.0365

$v_m =$	0.7	0.8	0.9	1	$1\frac{1}{4}$	$1\frac{1}{3}$
$f =$	.0349	.0336	.0325	.0315	.0297	.0284
$v_m =$	2	3	4	6	8	12
$f =$	.0265	.0243	.023	.0214	.0205	.0193

EXAMPLE.—The length of a pipe is 6,270 feet, its diameter is 8 inches, and the total head at the point of discharge is 215 feet. How many gallons are discharged per minute?

SOLUTION.—Using rule, Art. **571,**

$$v_m = 2.315\sqrt{\frac{h\,d}{f\,l}} = 2.315\sqrt{\frac{215 \times 8}{.025 \times 6{,}270}} = 7.67 \text{ feet per second, nearly.}$$

For $v_m = 6, f = .0214$, and for $v_m = 8, f = .0205$. $.0214 - .0205 = .0009$ = difference for a difference in the v_ms of $8 - 6 = 2$ ft. per sec.

$7.67 - 6 = 1.67$. Hence, $2 : 1.67 :: .0009 : x$, or $x = .0007515$. Using but four decimal places $x = .0008$. Since the value of f is smaller for $v_m = 8$, than for $v_m = 6$, the value of f for $v_m = 7.67$ is $.0214 - .0008 = .0206$. Hence, using rule, Art. **575,**

$$Q = .09445\,d^2\sqrt{\frac{h\,d}{f\,l + .125\,d}} = .09445 \times 8^2\sqrt{\frac{215 \times 8}{.0206 \times 6{,}270 + .125 \times 8}} =$$

21.974 gallons per second, nearly.

$21.974 \times 60 = 1{,}318.44$ gallons per minute. Ans.

576. If it is desired to find the head necessary to give a discharge of a certain number of gallons per second through a pipe whose length and diameter are known, calculate the mean velocity of efflux by using rule, Art. **574.** Find the value of f (from the table) corresponding to this value of v_m; substitute these values of f and v_m in rule, Art. **572,** and calculate the head.

EXAMPLE.—A 4-inch pipe 2,000 feet long is to discharge 24,000 gallons of water per hour. What must be the head?

SOLUTION.—$\frac{24{,}000}{60 \times 60} = 6\frac{2}{3}$ gallons per second. Using rule, Art. **574,**

$$v_m = \frac{24.51\,Q}{d^2} = \frac{24.51 \times 6\frac{2}{3}}{16} = 10.2 \text{ feet per second.}$$

In the table, $f = .0205$ for $v_m = 8$, and .0193 for $v_m = 12$.

$.0205 - .0193 = .0012$ = difference for a difference in the v_m s of $12 - 8 = 4$ ft. per sec. $10.2 - 8 = 2.2$. Hence, $4 : 2.2 :: .0012 : x$, or $x = .00066 = .0007$ when using but four decimal places. Then, $.0205 - .0007 = .0198 = f$ for $v_m = 10.2$.

Substituting in rule, Art. **572**,

$$h = \frac{f l v_m^2}{5.36 d} + .0233 v_m^2 = \frac{.0198 \times 2{,}000 \times 10.2^2}{5.36 \times 4} + .0233 \times 10.2^2 = 194.6 \text{ ft.},$$

nearly. Ans.

577. In laying pipes, all bends and elbows should be avoided as much as possible. When they are absolutely necessary, they should be as large as the circumstances will permit so as to change the direction gradually. Sudden changes in direction destroy the velocity very rapidly, and, consequently, reduce the discharge. A reduction or increase in the size of the pipe, owing to screwing on of branch pipes, smaller or larger than the main pipe, also reduces the velocity.

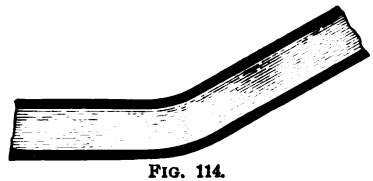


FIG. 114.

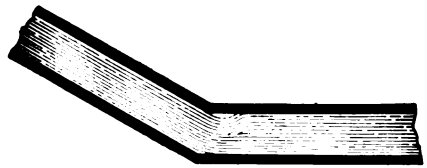


FIG. 115.

When bends are necessary, it is better to round them as shown in Fig. 114, than to have a sharp bend as shown in Fig. 115. A bend at right angles, as shown in Fig. 116, is very destructive to the velocity. A rounded elbow, as shown in Fig. 117, should be used, in which the radius should be made as large as possible.

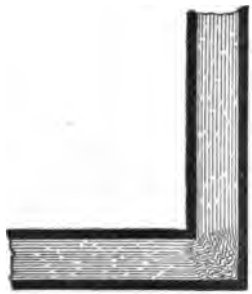


FIG. 116.

578. The foregoing examples have all been based on the assumption that all of the pipes were

straight, but in cases where the pipe changes its direction the following additions to its length may be made to compensate for the loss due to friction:

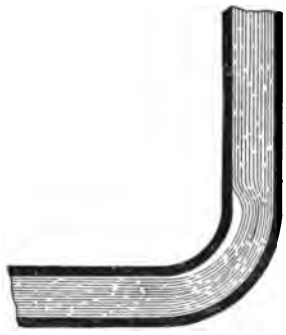


FIG. 117.

For every square elbow or T on an iron pipe or a very sharp bend or branch on a lead pipe allow an increase in length of 38 times the diameter of the pipe.

The friction of water passing through a lead pipe having a radius of five times the diameter of the pipe is very little in excess of that through a straight pipe of the same length as the bend.

Rust in iron pipes retards the velocity of the water passing through, not only by the roughened surface, but also by a contraction of the waterway.

579. When wrought-iron pipes are used for conveying water, the ends of the pipes should be reamed out before being screwed into the fittings, to take off the "burr" made by the pipe cutters; and when lead pipes are used both ends should be slightly opened before being joined together, so that no sharp edges shall be left inside for the water to impinge against.

EXAMPLES FOR PRACTICE.

1. A 6-inch pipe 2 miles long discharges water under a head of 78 feet. What is the mean velocity of discharge? Ans. 3.13 ft. per sec.

2. If the pipe in the above example had been $\frac{1}{4}$ mile long, what would have been the mean velocity? Ans. 6.63 ft. per sec.

3. A 4-inch pipe discharges 280 gallons of water per minute. What is the mean velocity of efflux? Ans. 7.15 ft. per sec.

4. A 7-inch pipe is required to discharge 51,200 gallons of water per hour. If the length of the pipe is 3,160 feet, what is the necessary head? Ans. 90.26 ft.

5. If 14,000 gallons per minute are discharged through a $16'' \times 18''$ rectangular orifice, what is the mean velocity of efflux in feet per second? Ans. 15.6 ft. per sec.

PNEUMATICS.

580. Pneumatics is that branch of mechanics which treats of the properties of gases.

581. The most striking feature of all gases is their extreme expansibility. *If we inject a portion of gas, however small it may be, into a vessel, it will expand and fill that vessel.* If a bladder or football be partly filled with air, and placed under a glass jar (called a receiver), from which the air has been exhausted, the bladder or football will immediately expand, as shown in Fig. 118. The force which a gas always exerts when confined in a limited space is called **tension.** The word *tension* in this case means pressure, and is only used in this sense in reference to gases.

FIG. 118.

582. As *water* is the most common type of fluids so *air* is the most common type of gases. It was supposed by the ancients that air had no weight, and it was not until about the year 1650 that it was proven that air really had weight. A cubic inch of air, under ordinary conditions, weighs .31 grain, nearly. At a temperature of 32° and a pressure of 14.7 pounds per square inch, the ratio of the weight of air to water is about 1 : 774; that is, air is only $\frac{1}{774}$ as heavy as water. In Art. **561,** it was shown that if a body were immersed in water, and weighed less than the volume of water displaced, the body would rise and project partly out of the water. The same is true to a certain extent of air. If a vessel made of light material be filled with a gas lighter than air, so that the total weight of the vessel and

gas is less than the air they displace, the vessel will rise. It is on this principle that balloons are made.

583. Since air has weight, it is evident that the enormous quantity of air that constitutes the atmosphere must exert a considerable pressure upon the earth. This is easily proven by taking a long glass tube closed at one end, and filling it with mercury. If the finger be placed over the open end so as to keep the mercury from running out, and the tube inverted and placed in a cup of mercury, as shown in Fig. 119, the mercury will fall, then rise, and after a few oscillations will come to rest at a height above the top of the mercury in the glass equal to about 30 inches. This height will always be the same under the same atmospheric conditions. Now, if the atmosphere has weight, it must press upon the upper surface of the mercury in the glass with equal intensity upon every square unit, except upon that part of the surface occupied by the tube. In order that there may be equilibrium, the weight of the mercury in the tube must be equal to the pressure of the air upon a portion of the surface of mercury in the glass, equal in area to the inside of the tube. Suppose that the area of the inside of the tube is 1 square inch, then, since mercury is 13.6 times as heavy as water, the weight of the mercury column is $.03617 \times 13.6 \times 30 = 14.7574$ lb. The actual height of the mercury is a little less than 30 inches, and the actual weight of a cubic inch of distilled water is a little less than .03617 lb. When these considerations are taken into account, the average weight of the mercurial

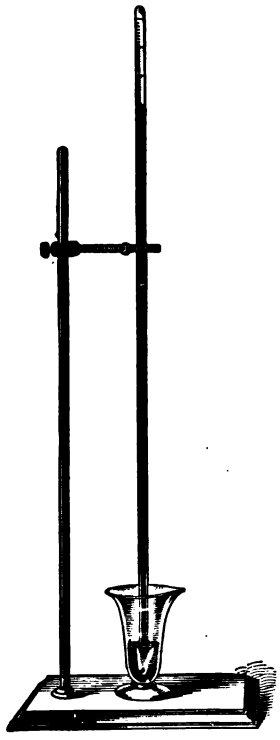


FIG. 119.

column at the level of the sea, when the temperature is 60°, is 14.69 lb., or practically 14.7 lb. Since this weight, when exerted upon 1 square inch of the liquid in the glass, just produces equilibrium, it is plain that the pressure of the outside air is 14.7 lb. upon every square inch of surface.

584. Vacuum.—The space between the upper end of the tube and the upper surface of the mercury is called a **vacuum,** meaning that it is an entirely empty space, and does not contain any substance, solid, liquid, or gaseous. If there were a gas of some kind there, no matter how small the quantity might be, it would expand, filling the space, and its tension would cause the column of mercury to fall and become shorter, according to the amount of gas or air present. The space is then called a **partial vacuum.** If the mercury fell 1 inch, so that the column was only 29 inches high, we should say, in ordinary language, that there were *29 inches of vacuum.* If it fell 8 inches, we should say that there were 22 inches of vacuum; if it fell 16 inches, we should say that there were 14 inches of vacuum, and so on. Hence, when the vacuum gauge of a condensing engine shows 26 inches of vacuum, there is enough air in the condenser to produce a pressure of $\frac{30-26}{30} \times 14.7 = \frac{4}{30} \times 14.7 = 1.96$ lb. per sq. in. In all cases where the mercury column is used to measure a vacuum, the height of the column in inches gives the number of inches of vacuum. Were the column only 5″ high, the vacuum would be 5″.

If the tube had been filled with water instead of mercury, the height of the column of water to balance the pressure of the atmosphere would have been $30 \times 13.6 = 408$ inches $= 34$ feet. This means that, if a tube be filled with water, inverted, and placed in a dish of water in a manner similar to the experiment made with the mercury, the height of the column of water will be 34 feet.

585. The **barometer** is an instrument used for measuring the pressure of the atmosphere. There are two kinds

in general use—the mercurial barometer and the aneroid barometer. The **mercurial barometer** is shown in Fig. 120. The principle is the same as the inverted tube shown in Fig. 119. In this case the tube and the cup at the bottom are protected by a brass or iron casing. At the top of the tube is a graduated scale which can be read to $\frac{1}{1000}$ of an inch by means of a vernier. Attached to the casing is an accurate thermometer for determining the temperature of the outside air at the time the barometric observation is taken. This is necessary, since mercury expands when the temperature is increased, and contracts when the temperature falls; for this reason a standard temperature is assumed, and all barometer readings are reduced to this temperature. This standard temperature is usually taken at 32° F., at which temperature the height of the mercurial column is 30 inches. Another correction is made for the altitude of the place above sea level, and a third correction for the effects of capillary attraction.

FIG. 120.

586. In Fig. 121 is an illustration of an **aneroid barometer.** These instruments are made in various sizes, from the size of a watch up to 8 or 10 inches in diameter. They consist of a cylindrical box of metal with a top of thin, elastic corrugated metal. The air is exhausted from the box. When the atmospheric pressure increases, the top is pressed inwards, and when it is diminished, the top is pressed outwards by its own elasticity, aided by a spring beneath. These movements of the cover are transmitted and multiplied by a combination of delicate levers, which act upon an index hand, and cause it to move either to the right or left, over a graduated scale. These barometers are self-correcting (compensated) for variations in temperature. They are very portable, occupying

but a small space, and are so delicate that they are said to show a difference in the atmospheric pressure when trans-

FIG. 121.

ferred from the table to the floor. The mercurial barometer is the standard.

587. With air, as with water, the lower we get the greater the pressure, and the higher we get the less the pressure. At the level of the sea, the height of the mercurial column is about 30 inches; at 5,000 feet above the sea, it is 24.7 inches; at 10,000 feet above the sea, it is 20.5 inches; at 15,000 feet, it is 16.9 inches; at 3 miles, it is 16.4 inches, and at 6 miles above the sea-level, it is 8.9 inches.

The density or weight of the atmosphere also varies with the altitude; that is, a cubic foot of air at an elevation of 5,000 feet above the sea-level will not weigh as much as a cubic foot at sea level. This is proven conclusively by the

fact that at a height of 3½ miles the mercurial column measures but 15 inches, indicating that half the weight of the entire atmosphere is below that. It is known that the height of the earth's atmosphere is at least 50 miles; hence, the air just before reaching the limit must be in an exceedingly rarefied state. It is by means of barometers that great heights are measured. The aneroid barometer has the heights marked on the dial, so that they can be read directly. With the mercurial barometer, the heights must be calculated from the reading.

588. The atmospheric pressure is everywhere present, and presses all objects in all directions with equal force. If a book is laid upon the table, the air presses upon it in every direction with an equal average force of 14.7 pounds per square inch. It would seem as though it would take considerable force to raise a book from the table, since, if the size of the book were 8 inches by 5 inches, the pressure upon it would be $8 \times 5 \times 14.7 = 588$ lb.; but there is an equal pressure *beneath* the book which counteracts the pressure on the top. It would now seem as though it would require a great force to open the book, since there are two pressures of 588 pounds each acting in opposite directions and tending to crush the book; so it would, but for the fact that there is a layer of air between each leaf acting upwards and downwards with a pressure of 14.7 pounds per square inch. If two metal plates be made as perfectly smooth and flat as it is possible to get them, and the edge of one be laid upon the edge of the other, so that one may be slid upon the other and thus exclude the air, it will take an immense force, compared to the weights of the plates, to separate them. This is because the full pressure of 14.7 pounds per square inch is then exerted upon each plate, with no counteracting equal pressure between them.

If a piece of flat glass be laid upon a flat surface that has been previously moistened with water, it will require considerable force to separate them; this is because the water helps to fill up the pores in the flat surface and glass, and

thus creates a partial vacuum between the glass and the surface, thereby reducing the counter pressure beneath the glass.

589. Tension of Gases.—In Fig. 119, the space above the column of mercury was said to be a vacuum, and it was also said that if any gas or air were present, it would expand, its tension forcing the column of mercury downwards. If sufficient gas be admitted to cause the mercury to stand at 15 inches, the tension of the gas is evidently $\frac{14.7}{2} = 7.35$ lb. per square inch, since the pressure of the outside air, 14.7 lb. per square inch, now balances only 15 instead of 30 inches of mercury; that is, it balances only half as much as it would if there were no gas in the tube; therefore, the pressure (tension) of the gas in the tube is 7.35 pounds If more gas be forced into the tube until the top of the mercurial column is just level with the mercury in the cup, the gas in the tube will then have a tension equal to the outside pressure of the atmosphere. Suppose that the bottom of the tube is fitted with a piston, and that the total length of the inside of the tube is 36 inches. If the piston be shoved upwards so that the space occupied by the gas is 18 inches long instead of 36 inches, the temperature remaining the same as before, it will be found that the tension of the gas within the tube is 29.4 lb. It will be noticed that the volume occupied by the gas is only half that in the tube before the piston was moved, while the pressure is twice as great, since $14.7 \times 2 = 29.4$ lb. If the piston be shoved up, so that the space occupied by the gas is only 9 inches, instead of 18 inches, the temperature still remaining the same, the pressure will be found to be 58.8 pounds per square inch. The volume has again been reduced one-half, and the pressure increased two-fold, since $29.4 \times 2 = 58.8$ lb. The space now occupied by the gas is 9 inches long, whereas, before the piston was moved, it was 36 inches long; as the tube was assumed to be of uniform diameter throughout its length, the volume is now $\frac{9}{36} = \frac{1}{4}$ of its original volume, and

its pressure is $\frac{58.8}{14.7} = 4$ times its original pressure. Moreover, if the temperature of the confined gas remains the same, the pressure and volume will always vary in a similar way.

590. The law which states these effects is called *Mariotte's law*, and is as follows:

Mariotte's Law.—*The temperature remaining the same, the volume of a given quantity of gas varies inversely as the pressure.*

The meaning of the law is this: If the volume of a gas be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its former volume, the tension will be increased 2, 3, 5, etc., times, or if the outside pressure be increased 2, 3, 5, etc., times, the volume of the gas will be diminished to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{5}$, etc., of its original volume, the temperature remaining constant. It also means that if a gas is under a certain pressure, and this pressure is diminished to $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{10}$, etc., of its original intensity, the volume of the confined gas will be increased 2, 4, 10, etc., times—its tension decreasing at the same rate.

Suppose 3 cubic feet of air to be under a pressure of 60 pounds per square inch in a cylinder fitted with a movable piston, then the product of the volume and pressure is $3 \times 60 = 180$. Let the volume be increased to 6 cubic feet, then the pressure will be 30 pounds per square inch, and $30 \times 6 = 180$, as before. Let the volume be increased to 24 cubic feet; it is then $\frac{24}{3} = 8$ times its original volume, and the pressure is $\frac{1}{8}$ of its original pressure, or $60 \times \frac{1}{8} = 7\frac{1}{2}$ lb., and $24 \times 7\frac{1}{2} = 180$, as in the two preceding cases. It will now be noticed that if a gas be enclosed within a confined space, and allowed to expand without losing any heat, *the product of the pressure and the corresponding volume for any one position of the piston is the same as for any other position.* If the piston were forced inwards so as to compress the air, the same results would be obtained.

591. If the volume of the vessel and the pressure of the gas are known, and it is desired to know the pressure after the first volume has been changed:

Rule.—*Divide the product of the first, or original, volume and pressure by the new volume; the result will be the new pressure.*

Or, let p = original pressure; p_1 = final pressure; v = volume corresponding to the pressure p; v_1 = volume corresponding to the pressure p_1;

Then, $$p_1 = \frac{pv}{v_1}.$$

EXAMPLE.—At the point of cut-off in a steam engine, the amount of steam in the cylinder is 862 cu. in. The pressure at this point is 120 lb. per sq. in. What will be the pressure of the steam when the piston has reached the end of its stroke, and the volume is 1,800 cu. in.?

SOLUTION.—Applying the rule, $\frac{862 \times 120}{1{,}800} = 57.47$ lb. per sq. in. Ans.

592. If it is required to determine the volume after a change in the pressure:

Rule.—*Divide the product of the original volume and pressure by the new pressure; the result will be the new volume.*

Or, using the same letters as before, $v_1 = \frac{pv}{p_1}$.

593. In all that has been said before, it has been stated that the temperature was constant; the reason for this will now be explained. Suppose 5 cubic feet of air to be confined in a cylinder whose area is 10 square inches, placed in a vacuum so that there will be no pressure due to the atmosphere, and let the cylinder be fitted with a piston weighing, say 100 pounds. The tension of the gas will be $\frac{100}{10} = 10$ lb. per square inch. Suppose that the temperature of the air is 32° F., and that it is heated until the temperature is 33° F., or the temperature is increased 1°, it will be found that the piston has risen a certain amount, and, consequently, the volume has increased, while the pressure is the same as before, or 10 pounds per square inch. If more heat is applied until the temperature of the gas is 34° F., it will be found that the piston has again risen, and the volume again increased, while the pressure still remains the same.

It will be found that for every increase of temperature there will be a corresponding increase of volume, the exact increase being $\frac{1}{491}$ of its volume at 32° for every degree Fahrenheit rise in temperature.

If the pressure remains constant, it will also be found that every decrease of temperature of 1° F., will cause a decrease of $\frac{1}{491}$ of the volume.

594. If two liquids which do not act chemically upon each other, are mixed together and allowed to stand, it will be found that after a time the two liquids have separated, and the heavier has fallen to the bottom. If two equal vessels containing gases of different densities be put in communication with each other, they will be found to have mixed in equal proportions after a short time. If one vessel be higher than the other, and the heavier gas be in the lower vessel, the same result will occur. The greater the difference of the densities of the two gases, the quicker they will mix. It is assumed that no chemical action takes place between the two gases. When the two gases have the same temperature and pressure, the pressure of the mixture will be the same; this is evident, since the total volume has not been changed, and unless the volume or temperature changes, the pressure can not change. This property of the mixing of gases is a very valuable one, since if they acted like liquids, carbonic acid gas (the result of combustion), which is $2\frac{1}{2}$ times as heavy as air, would remain next to the earth, instead of dispersing into the atmosphere, the result being that no animal life could exist. This peculiar property common to gases is called the *diffusion of gases.*

EXAMPLE.—At the commencement of compression, the volume of the steam is 380 cu. in., and the pressure is 18 lb. per sq. in. At the end of compression the pressure is 112 lb. per sq. in. What is the final volume?

SOLUTION.—Applying the rule, $\frac{380 \times 18}{112} = 61.07$ cu. in. Ans.

EXAMPLE.—A vessel contains 10 cu. ft. of air at a pressure of 15 lb. per sq. in., and has 25 cu. ft. of air of the same pressure forced into it; what is the resulting pressure?

SOLUTION.—The original volume $= 10 + 25 = 35$ cu. ft. The original pressure is 15 lb. per sq. in. The final volume is 10 cu. ft. Hence, applying rule, Art. **591**, $\frac{35 \times 15}{10} = 52.5$ lb. per sq. in. Ans.

595. As a necessary consequence of Mariotte's law, it may be stated that *the weight of a cubic foot of gas varies directly as the pressure, and inversely as the volume;* that is, *the density increases as the pressure increases, and decreases as the volume increases.* The word density here means the weight of a cubic foot.

EXAMPLES FOR PRACTICE.

1. A vessel contains 25 cubic feet of gas, at a pressure of 18 lb. per sq. in.; if 125 cu. ft. of gas having the same pressure are forced into the vessel, what will be the resulting pressure? Ans. 108 lb. per sq. in.

2. The volume of steam in the cylinder of a steam engine at cut-off is 1.35 cu. ft., and the pressure is 85 lb. per sq. in. The pressure at the end of the stroke is 25 lb. per sq. in. What is the new volume?

Ans. 4.59 cu. ft.

3. A receiver contains 180 cu. ft. of gas, at a pressure of 20 lb. per sq. in.; if a vessel holding 12 cu. ft. be filled from the larger vessel until its pressure is 20 lb. per sq. in., what will be the pressure in the larger vessel? Ans. 18½ lb. per sq. in.

PNEUMATIC MACHINES.

596. The Air Pump.—*The air pump is an instrument for removing air from a given space.* A section of the

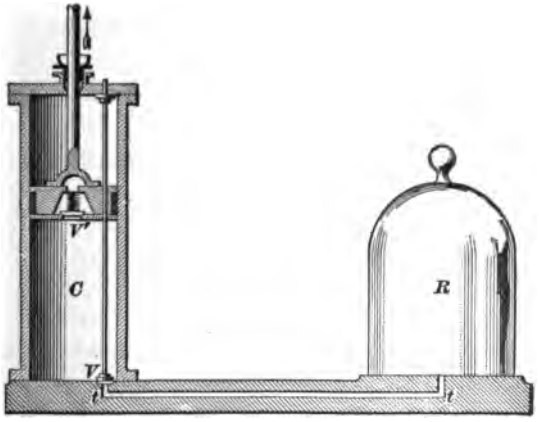


FIG. 122.

principal parts is shown in Fig. 122, and the complete instrument in Fig. 123. The closed vessel R is called the **receiver,** and the space which it encloses is that from which it is desired to remove the air. It is usually made of glass,

and the edges are ground so as to be perfectly air-tight. When made in the form shown, it is called a **bell-jar receiver.** The receiver rests upon a horizontal plate, in the center of which is an opening communicating with the pump cylinder *C* by means of the passage *t t*. The pump piston fits the cylinder accurately, and has a valve *V'* opening upwards. Where the passage *t t* joins the cylinder, is another valve *V*, also opening upwards. When the piston is raised, the valve *V'* closes, and, since no air can get into the cylinder from above, the piston leaves a vacuum behind it. The pressure upon *V* being now removed, the tension of the air in the receiver *R* causes *V* to rise; the air in the receiver and passage *t t* then expands so as to occupy the additional space provided by the upward movement of the piston.

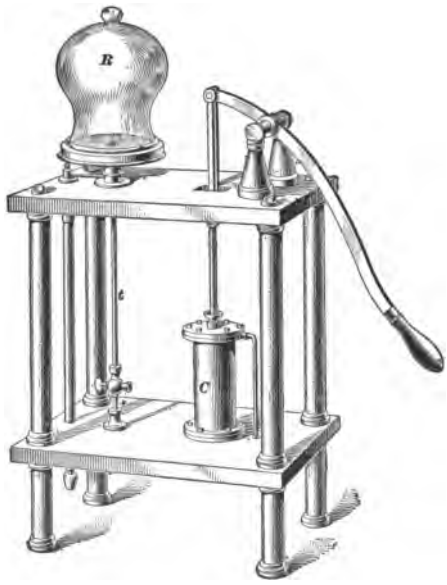


FIG. 123.

The piston is now pushed down, the valve *V* closes, the valve *V'* opens, and the air in *C* escapes. The lower valve *V* is sometimes supported, as shown in Fig. 122, by a metal rod passing through the piston, and fitting it somewhat tightly. When the piston is raised or lowered, this rod moves with it. A button near the upper end of the rod confines its motion within very narrow limits, the piston sliding upon the rod during the greater part of the journey.

In the complete form of the instrument shown in Fig. 123, communication between receiver and pump is made by means of the tube *t*.

597. Degrees and Limits of Exhaustion.—Suppose that the volume of R and t together is four times that of C, Fig. 123, and that there are say 200 grains of air in R and t, and 50 grains in C when the piston is at the top of the cylinder. At the end of the first stroke, when the piston is again at the top, 50 grains of air in the cylinder C will have been removed, and the 200 grains in R and t will occupy the space R, t, and C. The ratio between the sum of the spaces R and t, and the total space $R + t + C$ is $\frac{4}{5}$; hence, $200 \times \frac{4}{5} = 160$ grains = the weight of air in R and t after the first stroke. After the second stroke, the weight of the air in R and t would be $(200 \times \frac{4}{5}) \times \frac{4}{5} = 200 \times (\frac{4}{5})^2 = 200 \times \frac{16}{25} = 128$ grains. At the end of the third stroke the weight would be $[200 \times (\frac{4}{5})^2] \times \frac{4}{5} = 200 \times (\frac{4}{5})^3 = 200 \times \frac{64}{125} = 102.4$ grains. At the end of n strokes the weight would be $200 \times (\frac{4}{5})^n$. It is evident that *it is impossible to remove all of the air that is contained in R and t by this method.* It requires an exceedingly good air pump to reduce the tension of the air in R to $\frac{1}{600}$ of an inch of mercury. When the air has become so rarefied as this, the valve V' will not lift, and, consequently, no more air can be exhausted.

598. Magdeburg Hemispheres.—By means of the two hemispheres shown in Fig. 124, it can be proven that the atmosphere presses upon a body equally in all directions. They were invented by Otto Von Guericke, of Magdeburg, and are called the *Magdeburg hemispheres.* One of the hemispheres is provided with a stop-cock, by which it can be screwed on to an air pump. The edges fit accurately, and are well greased, so as to be air-tight. As long as the hemispheres contain air, they can be separated with no difficulty; but when the air in the interior is pumped out by means of an

FIG. 124.

air pump, they can be separated only with great difficulty. The force required to separate them will be equal to the area of the largest circle of the hemisphere in square inches multiplied by 14.7 pounds.

This force will be the same in whatever position the hemisphere may be held, thus proving that the pressure of air upon it is the same in all directions.

NOTE.—A theoretically perfect vacuum is sometimes called a ***Torricellian vacuum.***

599. The Weight Lifter.—The pressure of the atmosphere is very clearly shown by means of an apparatus like that illustrated in Fig. 125. Here a cylinder fitted with a piston is held in suspension by a chain. At the top of the cylinder is a plug *A* which can be taken out. This plug is removed, the piston pushed up (the force necessary being equal to the weight of the piston and rod *B*) until it touches the cylinder head. The plug is then screwed in, and the piston will remain at the top until a weight has been hung on the rod equal to the number of pounds obtained by multiplying the number of square inches in the piston by 14.7 pounds, and subtracting therefrom the weight of the piston and rod in pounds.

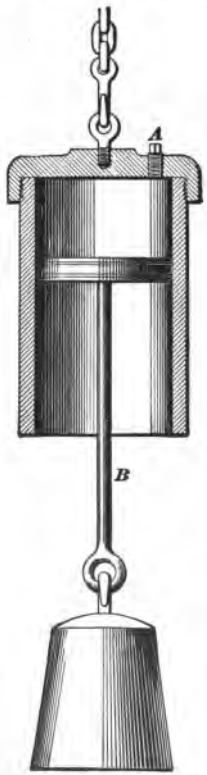


FIG. 125.

If a force sufficiently great were employed to pull the piston downwards, and then any less weight were attached, as shown in Fig. 125, the piston would ascend and carry the weight up with it.

600. Suppose the weight to be removed, and the piston to be supported, say midway of the length of the cylinder. Let the plug be removed, and air admitted above the piston, then screw the plug back into its place; if the piston be shoved upwards,

the further up it goes the greater will be the force necessary to push it, on account of the compression of the air. If the piston is of large diameter it will also require a great force to pull it out of the cylinder, as a little consideration will show. For example, let the diameter of the piston be 20 inches, the length of the cylinder 36 inches, and the weight of the piston and rod 100 pounds. If the piston is in the middle of the cylinder, there will be 18 inches of space above it, and 18 inches of space below it. The area of the piston is $20^2 \times .7854 = 314.16$ square inches, and the atmospheric pressure upon it is $314.16 \times 14.7 = 4{,}618$ lb., nearly. In order to shove the piston upwards 9 inches, the pressure upon it must be twice as great, or 9,236 pounds, and to this must be added the weight of the piston and rod, or $9{,}236 + 100 = 9{,}336$ lb. The force necessary to cause the piston to move upwards 9 inches would then be $9{,}336 - 4{,}618 = 4{,}718$ lb. Now, suppose the piston to be moved downwards until it is just on the point of being pulled out of the cylinder. The volume above it will then be twice as great as before, and the pressure one-half as great, or $4{,}618 \div 2 = 2{,}309$ lb. The total upward pressure will be the pressure of the atmosphere less the weight of the piston and rod, or $4{,}618 - 100 = 4{,}518$ lb., and the force necessary to pull it downwards to this point will be $4{,}518 - 2{,}309 = 2{,}209$ lb.

601. Air Compressors.—For many purposes compressed air is preferable to steam or other gas for use as a motive power. In such cases **air compressors** are used to compress the air. These are made in many forms, but the most common one is to place a cylinder, called the **air cylinder,** in front of the cross-head of a steam engine, so that the piston of the air cylinder can be driven by attaching its piston rod to the cross-head, in a manner similar to a steam pump. A cross-section of the air cylinder of a compressor of this kind is shown in Fig. 126, in which *A* is the piston, and *B* is the piston rod, driven by the cross-head of a steam engine, not shown in the figure. Both ends of the lower half of the cylinder are fitted with inlet valves *D*

and D' which allow the air to enter the cylinder, and both ends of the upper half are fitted with discharge valves F and

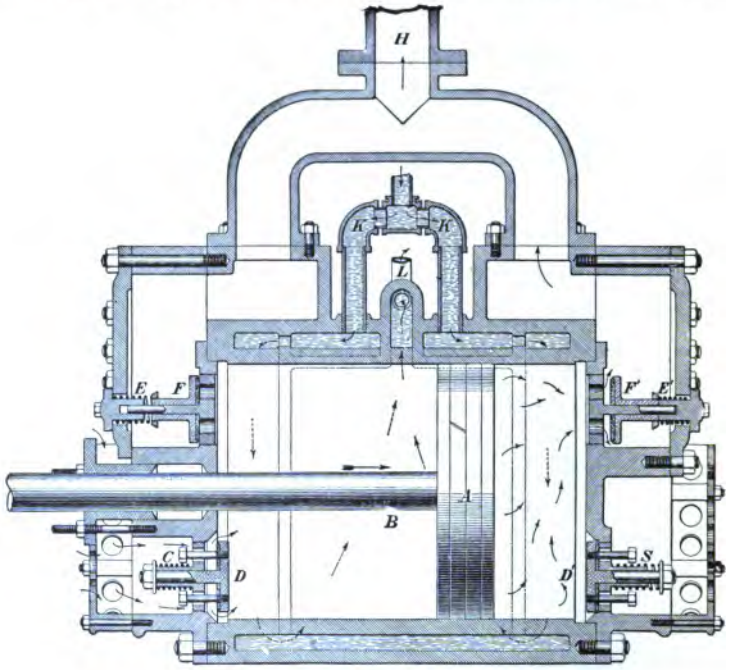


FIG. 126.

F' which allow the air to escape from the cylinder after it has been compressed to the required pressure.

602. Suppose the piston A to be moving in the direction of the arrow; then the inlet valves D in the left-hand end of the cylinder from which the piston is moving will be forced inwards by the pressure of the atmosphere, which overcomes the resistance of a light spring C, thus allowing the air to flow in and fill the cylinder. On the other side of the piston, the air is being compressed, and, consequently, it acts with the springs S to force the inlet valves D' in the right-hand end of the cylinder to their seats. In the right-hand end of the cylinder, the discharge valves F' are opened when the pressure of the air in the cylinder is great enough

to overcome the resistance of the light springs E' and the tension of the air in the passages leading to the discharge-pipe H, and in the left-hand end of the cylinder the discharge valves F are pressed against their seats by the springs E and the tension of the air in the passages. Suppose it is desired to compress the air to 59 pounds per square inch, and we wish to find at what point of the stroke the discharge valves will open. Now, 59 pounds per square inch = 4 atmospheres, very nearly; hence, the volume must be $\frac{1}{4}$ of the volume at atmospheric pressure, or the valves will open when the piston has traveled $\frac{3}{4}$ of its stroke.

Considerable heat is developed by the compression of the air; this heat increases the work required to compress the air. The loss of work is reduced by surrounding the cylinder with a **water-jacket,** a hollow space which receives cold water from the pipes K, K to be discharged when heated through the pipe L.

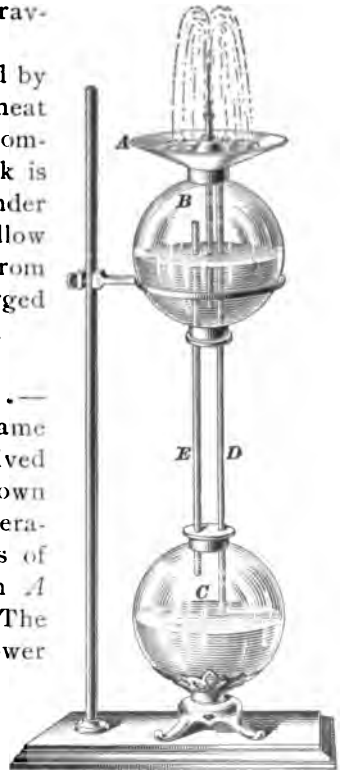


FIG. 127.

603. Hero's Fountain.— Hero's fountain derives its name from its inventor, Hero, who lived at Alexandria 120 B. C.; it is shown in Fig. 127. It depends for its operation upon the elastic properties of air. It consists of a brass dish A and two glass globes B and C. The dish communicates with the lower part of the globe C by means of a long tube D, and another tube E connects the two globes. A third tube passes through the dish A to the lower part of the globe B. This last tube being taken out, the globe B is partially filled with water, the tube is then replaced, and water is poured into the dish. The

water flows through the tube D into the lower globe, and expels the air, which is forced into the upper globe. The air thus compressed acts upon the water and makes it jet out as represented in the figure. Were it not for the resistance of the atmosphere and friction, the water would rise to a height above the water in the dish equal to the difference of the level of the water in the two globes.

604. The Siphon.—The action of the *siphon* illustrates the effect of atmospheric pressure. It is simply a bent tube of unequal branches, open at both ends, and is used to convey a liquid from a higher point to a lower, over an intermediate point higher than either. In Fig. 128, A and B are two vessels, B being lower than A, and $A\ C\ B$ is the bent tube, or siphon. Suppose this tube to be filled with water and placed in the vessels as shown, with the short branch $A\ C$ in the vessel A. The water will flow from the vessel A into B, as long as the level of the water in B is below the level of the water in A, and the level of the water in A is above the lower end of the tube $A\ C$. The atmospheric pressure upon the surfaces of A and B tends to force the water up the tubes $A\ C$ and $B\ C$. When the siphon is filled with water each of these pressures is counteracted in part by the pressure of the water in that branch of the siphon which is immersed in the water upon which the pressure is exerted. The atmospheric pressure opposed to the weight of the longer column of water will, therefore, be more resisted than that opposed to the weight of the shorter column; consequently, the pressure exerted upon the shorter column

FIG. 128.

will be greater than that upon the longer column, and this excess pressure will produce motion.

605. Let A = the area of the tube;
$h = DC$ = the vertical distance between the surface of the water in B and the highest point of the center line of the tube;
$h_1 = EC$ = the distance between the surface of the water in A and the highest point of the center line of the tube.

The weight of the water in the short column is $.03617 \times A\,h_1$, and the resultant atmospheric pressure tending to force the water up the short column is $14.7 \times A - .03617 \times A\,h_1$. The weight of the water in the long column is $.03617\,A\,h$, and the resultant atmospheric pressure tending to force the water up the long column is $14.7\,A - .03617\,A\,h$. The difference between these two is $(14.7\,A - .03617\,A\,h_1) - (14.7\,A - .03617\,A\,h) = .03617\,A\,(h - h_1)$. But $h - h_1 = ED$ = the difference between the levels of the water in the two vessels. In the above, h and h_1 were taken in inches and A in square inches.

It will be noticed that the short column must not be higher than 34 feet for water, or the siphon will not work, since the pressure of the atmosphere will not support a column of water that is higher than 34 feet.

606. The Injector.—A section of an injector is shown in Fig. 129. There are many different kinds of these instruments, but the principle is the same in all. When they are used for lifting water from a point below the discharge orifice and forcing it into the boiler of a steam engine or locomotive, they depend for their lifting action upon the creation of a partial vacuum by the action of steam. In the injector, Fig. 129, F is the connection for the steam pipe from the boiler, P is the connection for the pipe from the water supply, N is the connection to which the discharge pipe leading to the boiler is attached, and the waste water and steam are discharged through the **overflow** nozzle O.

607. The method of operation is as follows: The valve B is first opened by turning the wheel W; the primer valve R is then opened by the handle J, thus permitting steam to flow through the passage E and a connection, not shown in the figure, to the nozzle u. From u the jet of steam rushes out through O. A passage connects the chamber surrounding u with the space above the valve L. The jet of steam from u out through O carries with it the air in the chamber to which O is connected, thus forming a partial

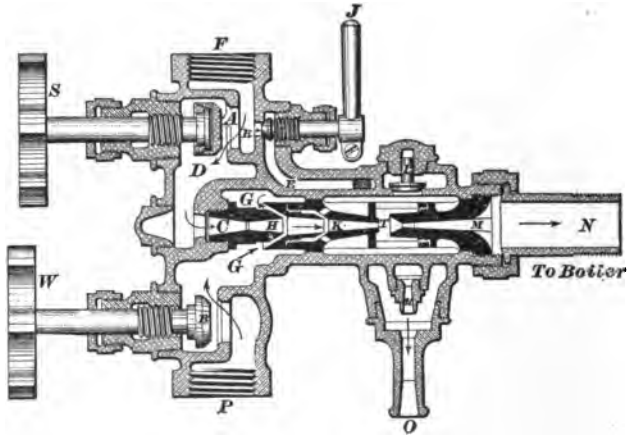


FIG. 129.

vacuum in the space above L; the air in the passages D, C, G, H, K, T, and in the water pipe connected at P is thus drawn out through the valve L, and a partial vacuum is formed which permits the pressure of the atmosphere to force water through P until it finally fills the passages and flows out through L and the overflow nozzle O. As soon as water appears at O, the valve R is closed and the main steam valve A is opened by the wheel S, thus admitting steam to the passages C, H, K. This steam draws water from G through the opening surrounding H and discharges it through K with such a high velocity that it rushes past the opening T into the nozzle M and thence into the boiler.

PUMPS.

608. The Suction Pump.—A section of an ordinary suction pump is shown in Fig. 130. Suppose the piston to be at the bottom of the cylinder, and to be just on the point of moving upwards in the direction of the arrow. As the piston rises it leaves a vacuum behind it, and the atmospheric pressure upon the surface of the water in the well causes it to rise in the pipe P for the same reason that the mercury rises in the barometer tube. The water rushes up the pipe and lifts the valve V, filling the empty space in the cylinder B displaced by the piston. When the piston has reached the end of its stroke the water entirely fills the space between the bottom of the piston and the bottom of the cylinder, and also the pipe P. The instant that the piston begins its down stroke, the water in the chamber B tends to fall back into the well, and its weight forces the valve V to its seat, thus preventing any downward flow of the water. As the piston descends the water must give way to it, and, since the valve V is closed, the valves u, u must open, and thus allow the water to pass through the piston, as shown in the right-hand figure. When the piston has reached the end of its downward stroke, the weight of the water above closes the valves u, u. All the water resting on the top of the piston is then lifted with the piston on its upward stroke, and discharged through the spout A, the valve V again opening, and the water filling the space below the piston as before.

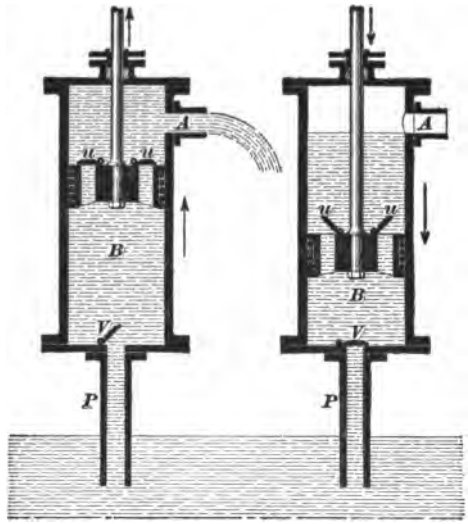


FIG. 130.

609. It is evident that the distance between the piston when at top of its stroke and the surface of the water in the well must not exceed 34 feet, the highest column of water which the pressure of the atmosphere will sustain, since, otherwise, the water in the pipe would not rise up and fill the cylinder as the piston ascended. In practice, this distance should not exceed 28 feet. This is due to the fact that there is a little air left between the bottom of the piston and the bottom of the cylinder, a little air leaks through the valves, which are not perfectly air-tight, and a pressure is needed to raise the valve against its weight, which, of course, acts downwards. There are many varieties of the suction pump, differing principally in the valves and piston, but the principle is the same in all.

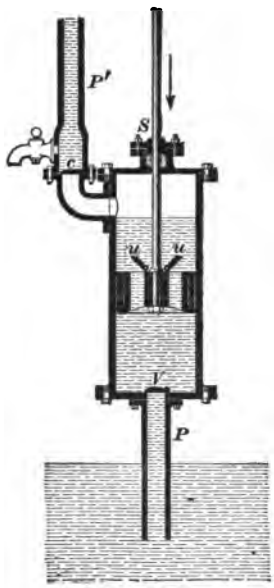


FIG. 131.

610. The Lifting Pump.—A section of a lifting pump is shown in Fig. 131. These pumps are used when water is to be raised to greater heights than can be done with the ordinary suction pump. As will be perceived, it is essentially the same as the pump previously described, except that the spout is fitted with a cock and has a pipe attached to it, leading to the point of discharge. If it is desired to discharge the water at the spout, the cock may be opened; otherwise, the cock is closed, and the water is lifted by the piston up through the pipe P' to the point of discharge, the valve C preventing it from falling back into the pump, and the valve V preventing the water in the pump from falling back into the well.

611. Force Pumps.—The force pump differs from the lifting pump in several important particulars, but chiefly in the fact that the piston is solid; that is, it has

no valves. A section of a suction and force pump is shown in Fig. 132. The water is drawn up the suction pipe as before, when the piston rises; but when the piston reverses, the pressure on the water caused by the descent of the piston opens the valve V' and *forces* the water up the delivery pipe P'. When the piston again begins its upward movement, the valve V' is closed by the pressure of the water above it, and the valve V is opened by the pressure of the atmosphere on the water below it, as in the previous cases. For an arrangement of this kind, it is not necessary to have a stuffing-box. The water may be forced to almost any desired height. The force pump differs again from the lifting pump in respect to its piston rod, which should not be longer than is absolutely necessary in order to prevent it from *buckling*, while in the lifting pump the length of the piston rod is a matter of indifference.

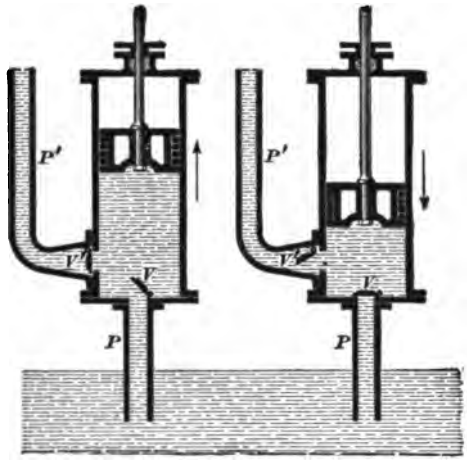


FIG. 132.

612. Double-Acting Pumps.—In the pumps previously described, the discharge was intermittent; that is, the pump could only discharge when the piston was moving in one direction. In some cases it is necessary that there should be a continuous discharge; in all cases it takes more power to run the pump with an intermittent discharge, as a little consideration will show. If the height that the water is to be raised is considerable, its weight will be very great, and the entire mass must be put in motion during one stroke of the piston.

In order to obtain the advantage of a more continuous dis-

charge, double-acting pumps are used. Fig. 133 shows a part sectional view of such a pump. Two pistons *a* and *b* are used, which are operated by one handle *c* in the manner shown. The pump has one suction pipe *s* and one discharge pipe *d*. The cylinders *e* and *f* are separated by a diaphragm *g*, so that they can not communicate with each other above the pistons. In the figure, the handle *c* is moving to the

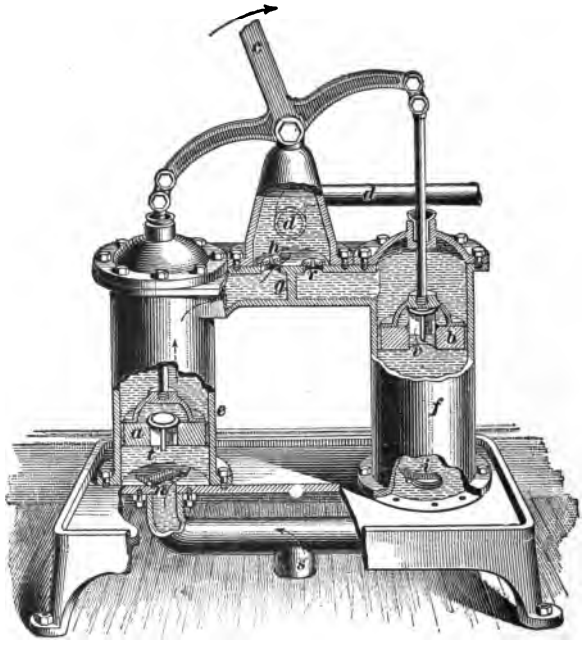


FIG. 133.

right, the piston *a* upwards, and the piston *b* downwards. As the piston *a* moves upwards, it lifts the water above it and causes it to flow through the delivery valve *h* into the discharge pipe *d*. This upward movement of the piston creates a partial vacuum below it in the cylinder *e*, and causes the water to rush up the suction pipe *s* into the cylinder, as shown by the arrows. In the cylinder *f*, the downward movement of the piston *b* raises the piston valve *v*, and the

weight of the water on the suction valve i keeps it closed. When the handle c has completed its movement to the right and begins its return, all of the valves on the right-hand side open except v, and those on the left-hand side close except t; water is then discharged into the delivery pipe by the cylinder f, and only at the instant of reversal is the flow into the delivery pipe d stopped.

613. Plunger Pumps.—When force pumps are used to convey water to great heights, the pressure of the water in the cylinder becomes so great that it becomes extremely difficult to keep the water from leaking past the piston, and the constant repairing of the piston packing becomes a nuisance. To obviate this difficulty the piston is made very long, as shown in Fig. 134, and is then called the **plunger.** The suction valve in this case consists of two clack valves inclined to each other and resting upon a square pin A; they are prevented from flying back too far during the up stroke of the plunger by the two uprights I, I. During the down stroke of the plunger, the valves at A are closed and the valve B in the delivery pipe is open. A little air is always carried into the cylinder of a pump with the entering water. In force pumps this fact becomes a serious consideration, since after repeated strokes the air accumulates, and during the down stroke of the plunger it is compressed. After a time it would become sufficient to entirely prevent the water from entering through the suction valve, the pressure on the top of the valve being greater than that of the atmosphere

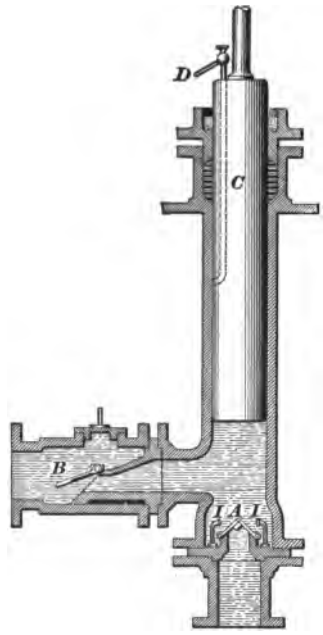


FIG. 134.

below. In the pump shown in the figure, the plunger is a trifle smaller than the cylinder, and the air collects around the plunger below the stuffing-box. To remove this air a narrow passage *C* (shown by the dotted lines), that can be closed at its upper end by the cock *D*, connects the interior of the pump with the atmosphere when the cock is open. It is evident that this cock must not be opened except during the down stroke of the plunger, for, if it were open during the up stroke, the pressure below the plunger being less than the pressure of the atmosphere above, the air would rush in instead of being expelled.

AIR CHAMBERS.

614. In order to obtain a continuous flow of water in the delivery pipe, with as nearly a uniform velocity as possible, an **air chamber** is usually placed on the delivery pipe of force pumps as near to the pump cylinder as the construction of the machine will allow. The air chambers are usually pear-shaped, with the small end connected to the pipe. They are filled with air, which the water compresses during the discharge. During the suction, the air thus compressed expands and acts as an accelerating force upon the moving column of water, a force which diminishes with the expansion of the air and helps to keep the velocity of the moving column more nearly uniform. An air chamber is sometimes placed upon the suction pipe. These air chambers not only tend to promote a uniform discharge, but also to equalize the stresses upon the pump and prevent shocks due to the incompressibility of water. They subserve the same purpose on pumps that a fly-wheel does on the steam engine. Unless the pump moves very slowly, it is absolutely necessary to have an air chamber on the delivery pipe.

615. The Hydraulic Ram.—The construction of a hydraulic ram is shown in Fig. 135. This machine is used for raising water from a point below the level of the water in a spring or reservoir to a point considerably higher, with

no power other than that afforded by the inertia of a moving column of water. In the figure, *a* is a pipe called the *drive pipe*, connecting the ram with the reservoir; the valve *b* slides freely in a guide, and is provided with locknuts to regulate the distance that the valve can fall below its seat. When the water is first turned on by opening the valve *n*, the valve *b* is already opened, and the water flows out through *c*, as shown. As the discharge continues, the velocity of the water in the drive pipe will increase until the

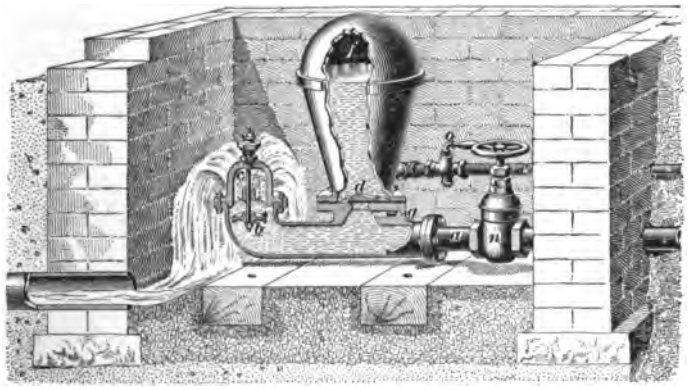


FIG. 135.

upward pressure against the valve *b* is sufficient to force the valve to its seat. The actual closing of the valve takes place very suddenly, and the momentum of the column of water, which was moving with an increasing velocity through the drive pipe *a*, will very rapidly force some water through the valve *d* into the air chamber *f*. Immediately after this a rebound takes place, and for a short interval of time the water flows back up the drive pipe *a* and tends to form a vacuum under the air chamber valve *d*; this opens the snifter valve *g* and admits a little air, which accumulates under the valve *d* and is forced into the air chamber with the next shock. This air keeps the air chamber constantly charged; otherwise, the water, being under a greater pressure in the air chamber than it is in the reservoir, would

soon absorb the air in the chamber and the ram would cease to work until the chamber was recharged with air. The rebound also takes the pressure off the under side of valve *b* and causes it to drop, and the above described operations are repeated. The delivery pipe is shown at *e*; a steady flow of water is maintained through it by the pressure of the air in the chamber *f*; this air also acts as a cushion when valve *b* suddenly closes, and prevents undue shock to the parts of the ram.

The height to which water can be raised by the hydraulic ram depends upon the weight of the valve *b* and the velocity of the water in *a*.

STEAM PUMPS.

616. Steam pumps are force pumps operated by steam acting upon the piston of a steam engine directly connected to the pump, and in many cases cast with the pump. A

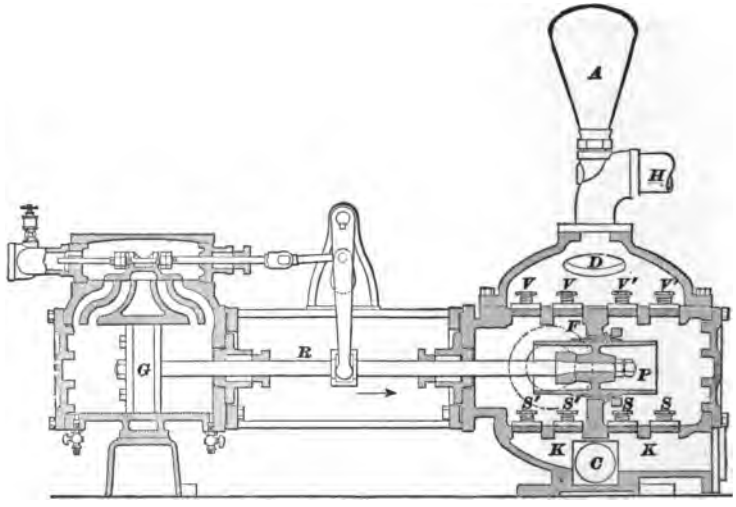


FIG. 136.

section of a double-acting steam pump, showing the steam and water cylinders, with other details, is illustrated in Fig. 136. Here *G* is the steam piston, and *R* the piston rod,

which is secured at its other end to the pump plunger *P*. *F* is a partition cast with the cylinder, which prevents the water in the left-hand half from communicating with that in the right-hand half of the cylinder. Suppose the piston to be moving in the direction of the arrow. When the piston has arrived at the end of its stroke, the water space in the left-hand half of the pump cylinder will have been increased by an amount equal to the area of the cross-section of the plunger, multiplied by the length of the stroke, and the volume of the right-hand half of the cylinder will have been diminished by a like amount. In consequence of this, a volume of water in the right-hand half of the cylinder equal to the volume displaced by the plunger in its forward movement will be forced through the valves *V*,' *V'*, through the orifice *D*, into the air chamber *A*, and then discharged through the delivery pipe *H*. By reason of the partial vacuum in the left-hand half of the pump cylinder, owing to this movement of the plunger, the water will be drawn from the reservoir through the suction pipe *C* into the chamber *K K*, lifting the valves *S'*, *S'* and filling the space displaced by the plunger. During the return stroke the water will be drawn through the valves *S*, *S* into the right-hand half of the pump cylinder, and at the same time water will be discharged from the left-hand half through the valves *V*, *V*, out through the pipe *H*, as before. Each one of the four suction and four discharge valves is kept to its seat, when not working, by light springs, as shown.

617. There are many varieties and makes of steam pumps, the majority of which are double-acting. In many cases two steam pumps are placed side by side, having a common delivery pipe. This arrangement is called a **duplex pump.** It is usual to so set the steam pistons of duplex pumps that when one is completing the stroke, the other is in the middle of its stroke. A double-acting duplex pump made to run in this manner, and having an air chamber of sufficient size, will deliver water with nearly a uniform velocity.

In mine pumps for forcing water to great heights, the plungers are made solid, and in most cases extend through the pump cylinder. In many steam pumps, pistons are used instead of plungers, but when very heavy duty is required, plungers are preferred.

CALCULATIONS RELATING TO PUMPS.

618. To find the size of the plunger cylinder to discharge a given number of gallons per minute:

Let G = number of gallons discharged per minute;
S = plunger speed in feet per minute;
d = diameter of cylinder in inches.

Then, the theoretical diameter equals 4.95 times the square root of the number of gallons discharged per minute divided by the plunger speed in feet per minute.

Since there is always more or less slip of the water past the plungers, it is usual to add $\frac{1}{4}$ of the required number of gallons to the value given to G in the above rule to allow for this slip. Doing so, we have the following rule:

Rule.—*The diameter of the cylinder in inches equals 5.535 times the square root of the quotient obtained by dividing the number of gallons discharged per minute by the plunger speed in feet per minute.*

That is, $$d = 5.535\sqrt{\frac{G}{S}}.$$

This rule should always be used when calculating the size of the plunger cylinder to discharge a certain number of gallons per minute. The piston speed is the number of feet traveled per minute by the plunger when forcing water; that is, it equals the length of the stroke in feet multiplied by the number of working strokes per minute. If the pump is double-acting, the number of working strokes is the same as the total number of plunger strokes, both forward and back; if single-acting, half of that number.

EXAMPLE.—What should be the diameter of a pump plunger required to discharge 130 gallons of water per minute, the speed of the plunger to be 115 feet per minute? If the pump is double-acting and the stroke is two times the diameter, how many strokes must it make per minute, and what is the length of the stroke?

SOLUTION.—Applying the rule, $d = 5.535\sqrt{\frac{G}{S}} = 5.535\sqrt{\frac{130}{115}} = 5.88''$, say $5\frac{7}{8}''$. Ans.

Since the stroke is twice the diameter, stroke $= 5\frac{7}{8} \times 2 = 11\frac{3}{4}'' = \frac{11.75}{12}$ ft. Ans.

Number of strokes $= 115 \div \frac{11.75}{12} = \frac{115 \times 12}{11.75} = 117.44$ strokes per minute, nearly. Ans.

This speed is rather high for a pump, and should be employed only when absolutely necessary.

619. To find the approximate discharge in gallons per minute of a pump when the diameter and plunger speed are known, use the following rule:

Rule.—*The number of gallons discharged per minute equals .03264 times the square of the diameter in inches multiplied by the plunger speed in feet per minute.*

That is, $G = .03264\ d^2\ S.$

The same allowance has been made for the slip in this rule that was allowed in rule, Art. **618.** If the theoretical discharge is required, the number of gallons discharged per minute equals .0408 times the square of the diameter in inches multiplied by the plunger speed in feet per minute.

That is, $G = .0408\ d^2\ S.$

EXAMPLE.—What is the probable discharge of a duplex double-acting pump whose plungers are 10″ in diameter, stroke 24″, and which makes 40 strokes per minute?

SOLUTION.—The discharge due to one side of pump is $G = .03264\ d^2\ S = .03264 \times 10^2 \times (2 \times 40) = 261.12$ gal. per min., since $24'' = 2$ ft. and the piston speed $= 2 \times 40$. The total discharge is twice this amount, or $261.12 \times 2 = 522.24$ gal. per min. Ans.

In the above example what is the theoretical discharge?

SOLUTION.—$G = .0408\ d^2 S = .0408 \times 10^2 \times (2 \times 40) = 326.4$ gal. per min. $326.4 \times 2 = 652.8$ gal. per min. Ans.

620. To find the horsepower of a steam or air cylinder to discharge a certain number of gallons of water per minute with a given lift, substitute in the following rule in which H = number of horsepower and h = vertical height in feet between the highest point of the center of the delivery or column pipe and the level of the water in the sump or place from which it was taken:

Rule.—*To find the horsepower of a pump, multiply .00038 times the number of gallons discharged per minute by the vertical height in feet that the water is to be raised.*

That is, $H = .00038\ G\,h.$

The theoretical horsepower will be two-thirds of the above result.

EXAMPLE.—How many horsepower should the steam cylinder of a pump be designed for which is required to discharge 350 gallons per minute, the total lift being 320 feet?

SOLUTION.—$H = .00038\ G\,h = .00038 \times 350 \times 320 = 42.56$ horsepower. Ans.

In the above example, the theoretical horsepower is $42.56 \times \frac{2}{3} = 28.37$ horsepower. In this rule allowance has been made for friction of water in the pipe, engine friction, pump friction, etc.

621. If it is desired to know the height to which a pump will raise water when the horsepower of the steam cylinder and the discharge of the pump have been determined, use the following rule, in which the letters have the same meaning as in the preceding one.

Rule.—*The vertical height in feet that the water is to be raised equals the horsepower divided by .00038 times the number of gallons discharged per minute.*

That is, $$h = \frac{H}{.00038G}.$$

EXAMPLE.—To what height will a 40 horsepower pump force 280 gallons of water per minute?

SOLUTION.— $h = \frac{H}{.00038\,G} = \frac{40}{.00038 \times 280} = 376$ feet.

622. To find the size of the steam or air cylinder of a pump, first calculate the horsepower by rule, Art. **620,** then proceed as follows: It is customary to design pumps on a basis of 100 feet plunger speed per minute. For pumps working continuously this is about right, although in some cases as high as 240 feet per minute have been attained. Nevertheless, for continuous working 100 feet per minute is a fair allowance, and does not bring excessive strains on the pump. If a single pump is to be used, the mean pressure of the steam or air will be the same as the gauge pressure at the pump, since the pressure is carried full stroke. If the pump be also direct-acting, the steam piston speed will be the same as the pump piston or plunger speed.

Let S = piston speed per minute;
d = diameter of cylinder in inches;
r = ratio between the length of stroke and diameter of cylinder;
l = length of stroke in feet;
n = number of strokes per minute;
H = horsepower;
P = steam or air pressure per sq. in.

Rule I.—*The diameter of the cylinder in inches equals the cube root of the quotient obtained by dividing 504,201.6 times the horsepower by the ratio between the length of stroke and diameter of cylinder times the steam or air pressure per sq. in. times the number of strokes per minute.*

That is, $$d = \sqrt[3]{\frac{504{,}201.6 \times H}{r\,P\,n}}.$$

Rule II.—*The diameter of the cylinder in inches equals the square root of the quotient obtained by dividing 42,016.8*

multiplied by the horsepower by the steam or air pressure per square inch times the piston speed.

That is, $$d = \sqrt{\frac{42{,}016.8 \times H}{PS}}.$$

Having obtained the diameter by means of either of the last two rules, the stroke can be found by multiplying the diameter by the value of the ratio r. When rule **II** is used, the number of strokes can be found by dividing the piston speed by the length of the stroke in feet.

EXAMPLE.—A pump to be driven by compressed air at a pressure of 45 pounds per square inch, is to have a piston speed of 100 feet per minute. If 32 horsepower are required to operate it, what should be the size of the air cylinder and the number of strokes?

SOLUTION.—Using rule **II**,

$$d = \sqrt{\frac{42{,}016.8 \times H}{PS}} = \sqrt{\frac{42{,}016.8 \times 32}{45 \times 100}} = 17.285'', \text{ or, say } 17\tfrac{1}{4}''.$$

For this case let the stroke be, say 22", thus making r a little over $1\frac{1}{4}$. The number of strokes will then be $100 \div \frac{22}{12} = 54\frac{6}{11}$. To make even figures all around, let the number of strokes be 55 per minute.

Since it is easier to extract the square root than the cube root, rule **II** is to be preferred to rule **I**.

EXAMPLES FOR PRACTICE.

1. What should be the diameter of the plunger of a pump that is required to discharge 240 gallons of water at a piston speed of 80 feet per minute? Give result to nearest $\frac{1}{8}''$. Ans. $9\frac{1}{8}''$.

2. In the above example, if the pump is double-acting and the stroke is about $2\frac{1}{2}$ times the diameter, (*a*) how many strokes must the plunger make per minute, and (*b*) what is the length of the stroke?

Ans $\begin{cases} (a)\ 40. \\ (b)\ 24''. \end{cases}$

3. What is the probable discharge of a single double-acting pump whose plunger is 6" in diameter, stroke 14", and which makes 70 strokes per minute? Ans. 96 gal. per min., nearly.

4. How many horsepower are required to operate a pump which discharges 180 gallons of water per minute under a head of 160 feet? Ans. 10.944 horsepower.

5. To what height will a 25 horsepower pump lift 300 gallons of water per minute? Ans. 219.3 ft.

6. What should be the diameter of the steam cylinder of a direct-acting single steam pump working under the conditions stated in example 4? The steam pressure is to be taken as 65 pounds per square inch, and the piston speed as 90 feet per minute. Give result to nearest $\frac{1}{4}$". Ans. $8\frac{1}{4}$".

STRENGTH OF MATERIALS.

623. When a force is applied to a body, it changes either its form or its volume. A force, when considered with reference to the internal changes it tends to produce in any solid, is called a **stress.**

Thus, if we suspend a weight of 2 tons by a rod, the stress in the rod is two tons. This stress is accompanied by a lengthening of the rod, which increases until the internal stress or resistance is in equilibrium with the external weight.

Stresses may be classified as follows:

Tensile, or pulling stress.	Transverse, or bending stress.
Compressive, or pushing stress.	Shearing, or cutting stress.
	Torsional, or twisting stress.

A **unit stress** is the amount of stress on a unit of area, and may be expressed either in pounds per square inch, or in tons per square foot; or, it is the load per square inch or square foot on any body.

Thus, if 10 tons are suspended by a wrought-iron bar which has an area of 5 square inches, the unit stress is 2 tons per square inch, because $\frac{10}{5} = 2$ tons.

624. **Strain** is the deformation or change of shape of a body resulting from stress.

For example, if a rod 100 feet long is pulled in the direction of its length, and if it is lengthened one foot, it is strained $\frac{1}{100}$th of its length, or 1 per cent.

625. **Elasticity** is the power which a body has of returning to its original form after the external force on

it is withdrawn, providing the stress has not exceeded the elastic limit.

Consequently, we see from this that all material is lengthened or shortened when subjected to either tensile or compressive stress, and the change of the length is directly proportional to the stress, within the elastic limit.

For stresses within the elastic limits, materials are perfectly elastic, and return to their original length on removal of the stresses; but, when their elastic limits are exceeded, the changes of their lengths are no longer regular, and a permanent **set** takes place; the destruction of the material has then begun.

The **measure of elasticity** of any material is the change of length under stress within the elastic limit.

The **elastic limit** is that unit stress under which the permanent set becomes visible.

The elasticity of wrought iron is practically the same as that of steel; that is, each material will change an equal amount of length under the same stress within the elastic limits.

The elastic limit of steel is higher than that of wrought iron; consequently, the former will lengthen or shorten more than the latter before its elasticity is injured.

TENSILE STRENGTH OF MATERIALS.

626. The tensile strength of any material is the resistance offered by its fibers to being pulled apart.

The tensile strength of any material is proportional to the area of its cross-section.

Consequently, when it is required to find the safe tensile strength of any material, we have only to find the area at the minimum cross-section of the body, and multiply it by its strength per square inch, as given in the following table under the heading "Working Stress."

NOTE.—The minimum cross-section referred to in the above paragraph is that section of the material which is pierced with holes; such as bolt or rivet holes in iron, or knots in wood, if there are any.

In the following table is given the average breaking and working tensile stress of different materials:

TABLE 17.

Material.	Breaking Stress in Pounds per Square Inch.	Working Stress in Pounds per Square Inch.
Cast Iron.....	16,000	1,500 to 3,500
Wrought Iron.....	50,000	5,000 to 12,000
Steel.....	70,000	6,000 to 13,000
Copper.....	11,200	1,100 to 2,200
Brass.....	18,000	1,800 to 3,500
Lead.....	2,200	200 to 400

The above table shows that the tensile breaking strength of cast iron is 16,000 pounds per square inch of cross-section, and that the working strength is from 1,500 to 3,500 pounds per square inch of cross-section.

627. In machinery, such as steam engines, where the parts are subjected to shocks, or are alternately compressed and extended, it is not safe to subject cast iron to a stress of more than 1,500 pounds per square inch of section, wrought iron to more than 5,000 pounds per square inch of section, or steel to more than 6,000 pounds per square inch of section.

But in structures in which the strains are constantly in one direction, as is the case with steam boilers, wrought iron may be strained with from 6,000 to 8,000 pounds per square inch of section, or steel with from 8,000 to 10,000 pounds per square inch of section.

Consequently, strict attention must be given to the *nature of the load* the given structure has to bear, and fix the working stress accordingly.

NOTE.—For structures on which the load is applied suddenly, use the smaller working stresses given in the table, and for those on which the load is applied gradually, use the larger working stresses.

RULES AND FORMULAS FOR TENSILE STRENGTH.

628. Let W = safe load in pounds;
A = area of minimum cross-section;
S = working stress in pounds per square inch, as given in the foregoing table.

Rule I.—*The working load in pounds for any bar subjected to a tensile stress is equal to the minimum sectional area of the bar, multiplied by the working stress in pounds per square inch, as given in the table.*

That is, $W = A\,S$.

EXAMPLE.—A bar of good wrought iron which is 3" square is to be subjected to a steady tensile stress; what is the maximum load that it should carry?

SOLUTION.—From what has been said above in regard to the materials and to the nature of the load, it will be safe in this case to use a working stress of 12,000 pounds per square inch.

Applying the rule, we have

$$W = 3 \times 3 \times 12{,}000 = 108{,}000 \text{ pounds.} \quad \text{Ans.}$$

Rule II. — *The minimum sectional area of any bar subjected to a tensile stress should be equal to the load in pounds, divided by the working stress in pounds per square inch, as given in the table.*

That is, $A = \frac{W}{S}$.

EXAMPLE.—What should be the area of a wrought-iron bar to carry a steady load of 108,000 pounds, if it is to resist a tensile stress of 12,000 pounds per square inch?

SOLUTION.—Applying the rule,

$$A = \frac{108{,}000}{12{,}000} = 9 \text{ sq. in.} \quad \text{Ans.}$$

Rule III.—*The working stress in pounds per square inch is equal to the load in pounds divided by the minimum sectional area of the bar.*

That is, $S = \frac{W}{A}$.

EXAMPLE.—A bar of wrought iron 3″ square, subjected to tensile stress, carries a load of 108,000 pounds; what is the stress per square inch?

SOLUTION.—Applying the rule,

$$S = \frac{108{,}000}{3 \times 3} = 12{,}000 \text{ lb. per sq. in. Ans.}$$

CHAINS.

629. Chains made of the same size iron vary in strength, owing to the different kinds of links from which they are made.

It is a good practice to anneal old chains which have become brittle by overstraining. This renders them less liable to snap from sudden jerks. It reduces their tensile strength, but increases their toughness and ductility, which are sometimes more important qualities.

When annealing, care should be taken that a sufficient heat be applied, otherwise no benefit will be gained; the chains ought to be heated to a cherry red, say 1300° F. at the least.

Rules and Formulas for the Strength of Chains:

Let W = safe load in pounds;

D = diameter of the iron, in inches, from which the links are made.

Rule I. — *The safe load in pounds of a stud-link wrought-iron chain is equal to 18,000, multiplied by the square of the diameter of the iron from which the links are made.*

That is, $W = 18{,}000\ D^2$.

EXAMPLE.—What is the maximum load that should be carried by a stud-link wrought-iron chain, if its links are made from ¾-inch round iron?

SOLUTION.—Applying the rule, $W = 18{,}000\ D^2$.

Substituting the value of D^2, we have $W = 18{,}000 \times \frac{3}{4} \times \frac{3}{4} = 10{,}125$ pounds. Ans.

Rule II. — *The safe load in pounds of a close-link wrought-iron chain is equal to 12,000, multiplied by the square of the diameter of the iron from which the links are made.*

That is, $W = 12{,}000\ D^2$.

EXAMPLE.—What is the maximum load that should be carried by a close-link wrought-iron chain, if its links are made from ¾-inch round iron?

SOLUTION.—Applying the rule, $W = 12{,}000\ D^2$.

Substituting the value of D^2, we have $W = 12{,}000 \times \frac{3}{4} \times \frac{3}{4} = 6{,}750$ pounds. Ans.

HEMP ROPES.

630. The strength of hemp ropes does not depend so much upon the quality of the material and the cross-section of the rope, as upon the method of manufacture and the amount of twisting.

The ropes in common use are three-strand, shroud-laid rope, and hawser or cable-laid rope.

The strongest ropes are three-strand shroud-laid, made without tar. Ropes made with tar are less flexible, and are reduced in strength about 25 per cent., but have better wearing qualities.

Rules and Formulas for the Strength of Hemp Ropes:

Let W = maximum working load in pounds;
C = circumference of rope in inches.

Rule I.—*The maximum working load in pounds that should be allowed on any hemp rope is equal to the square of the circumference of the rope, multiplied by 100.*

That is, $W = 100\ C^2$.

EXAMPLE.—What is the maximum load in pounds that should be carried by a hemp rope which has a circumference of 8 inches?

SOLUTION.—Substituting the value of C in the formula, $W = 100 \times 8^2 = 6{,}400$ lb. Ans.

Rule II. — *The circumference of any hemp rope is equal to the square root of the maximum working load in pounds which it is capable of carrying, multiplied by .1.*

That is, $C = .1\sqrt{W}$.

EXAMPLE.—A maximum working load of 1,000 pounds is to be carried by a hemp rope; what should be the circumference of the rope?

SOLUTION.—Applying the rule, $C = .1\sqrt{1{,}000} = 3.16''$. Ans.

When measuring ropes, the circumference is sought instead of the diameter, because the ropes are not round and the circumference therefore is not 3.1416 times the diameter. For three strands the circumference is about 2.86 d; for seven strands, 3 d.

631. The above formulas are very convenient for use, and easily remembered, but it is well to remark that the values thus given apply to ropes of a *good average* quality.

WIRE ROPES.

632. Wire rope is made of iron and steel wire. It is stronger than hemp rope, and, to carry the same load, is of smaller diameter.

In substituting steel for iron rope, the object in view should be to gain an increase of wear from the rope, rather than to reduce the size.

A steel rope to be serviceable should be of the best obtainable quality, because ropes made from low grades of steel are inferior to good iron ropes.

633. Formulas for the Strength of Wire Ropes:

Let W = maximum of working load in pounds;

C = circumference of rope in inches.

Rule I. — *The maximum working load in pounds that should be allowed on any iron wire rope is equal to the square of the circumference of the rope in inches, multiplied by 600.*

That is, $W = 600\ C^2$.

EXAMPLE.—What is the maximum load in pounds that should be carried by an iron wire rope whose circumference is 4½ inches?

SOLUTION.—Applying the formula,

$W = 600 \times 4.5^2 = 12{,}150$ lb. Ans.

their ends into which pins are fitted which are fastened to some other piece; as, for example, the connecting-rod of an engine. In such cases, it has been found that *a* is two times as strong as *c*, and that *b* is one and one-half times as strong as *c*. That is, in actual practice, a column fixed as at *c* is really $\frac{1}{2}$ as strong as one fixed as at *a*, instead of being only $\frac{1}{4}$ as strong, as given above.

Green or wet timber has only one-half the strength of dry and seasoned timber; consequently, its crushing strength is only one-half of that given in the table below.

636. In the following table is given the mean crushing strength of some short specimens of materials in tons (of 2,000 pounds) per square inch:

TABLE 18.

Materials.	Crushing Strength in Tons per Square Inch.
Cast Iron	40
Wrought Iron	18
Mild Steel	26
Cast Copper	5
Cast Brass	4.5
Timber (dry) endwise	3.5
Brick	1
Stone	3

637. Formula for the Strength of Pillars:

The following formula is applicable to pillars which are commonly used in practice, the lengths of which are about from 10 to 40 times their least diameter, or, if rectangular, their least thickness as indicated by *d*.

Let C = crushing strength in tons per square inch of a short specimen of the material, as given in the above table;

S = sectional area in square inches;

L = length in inches;

d = least thickness of rectangular pillar, or diameter of round pillar in inches;
W = breaking load in tons;
A = the area of the two flanges;
B = the area of the web;
a = constant, given in one of the following three tables:

TABLE 19.

CONSTANTS FOR WROUGHT-IRON PILLARS.

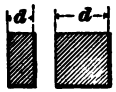
Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
Round.	2,250	1,500	1,125
 Square or Rectangle.	3,000	2,000	1,500
Thin Square Tube.	6,000	4,000	3,000
Thin Round Tube.	4,500	3,000	2,250
Angle with Equal Sides.	1,500	1,000	750
Cross with Equal Arms	1,500	1,000	750
I Beam.	$3{,}000 \times \frac{A}{A+B}$	$2{,}000 \times \frac{A}{A+B}$	$1{,}500 \times \frac{A}{A+B}$

TABLE 20.

CONSTANTS FOR CAST-IRON PILLARS.

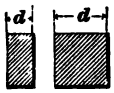
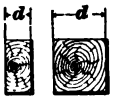
Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
Round.	281.25	187.5	140.625
 Square or Rectangle.	375.	250.	187.5
Thin Square Tube.	750.	500.	375.
Thin Round Tube.	562.5	375.	281.25
Angle with Equal Sides.	187.5	125.	93.75
Cross with Equal Arms.	187.5	125.	93.75
I Beam.	$375 \times \frac{A}{A+B}$	$250 \times \frac{A}{A+B}$	$125 \times \frac{A}{A+B}$

TABLE 21.

CONSTANTS FOR WOODEN PILLARS.

Cross-section of Pillar.	When Both Ends of the Pillar are Flat or Fixed.	When One End of the Pillar is Flat or Fixed, and the Other Round or Movable.	When Both Ends of the Pillar are Round or Movable.
Round.	187.5	125.	93.75
 Square or Rectangle.	250.	166.66	125.
Hollow Square Made of Boards.	500.	333.33	250.

638. Rule.—*The breaking load of a pillar in tons is equal to the crushing strength of a short specimen of the material as given in Table 18, Art.* **636,** *multiplied by the sectional area of the pillar in square inches, and the product divided by 1 plus the quotient obtained by dividing the square of the length of the pillar in inches by the square of the diameter (or least thickness, if rectangular) multiplied by the value of a.*

That is,
$$W = \frac{CS}{1 + \frac{L^2}{a\,d^2}}.$$

The result obtained by the formula must be divided by 6 to get the safe working load.

NOTE.—If the length of the pillar is given in feet, be sure to reduce it to inches before substituting in the formula.

EXAMPLE.—A wooden pillar, 6 inches square and 144 inches long, is fixed at both ends; what load will it sustain with safety?

SOLUTION.—By the rule, $W = \frac{CS}{1 + \frac{L^2}{a\,d^2}}$.

Substituting the values of C, S, L^2, a, and d^2, we have

$$W = \frac{3.5 \times 6 \times 6}{1 + \frac{144 \times 144}{250 \times 6 \times 6}} = 38.14 \text{ tons, nearly.}$$

Which, divided by 6, gives $\frac{38.14}{6} = 6.357$ tons, or the load it is capable of sustaining with safety. Ans.

EXAMPLE.—A wrought-iron pillar, 4 inches in diameter and 60 inches long, is fixed at one end and movable at the other; what load will it sustain with safety?

SOLUTION.—By the rule,

$$W = \frac{18 \times 4 \times 4 \times .7854}{1 + \frac{60 \times 60}{1{,}500 \times 4 \times 4}} = 196.69 \text{ tons.}$$

Which, divided by 6, gives $\frac{196.69}{6} = 32.78$ tons, nearly, or the load it is capable of sustaining with safety. Ans.

EXAMPLE.—What load will a cast-iron pillar sustain with safety, if it is 20 feet long, and if its cross-section is a cross with equal arms, two of which are equal to 10 inches in length (as d, see table), and whose arms are 1 inch thick; both ends of the pillar movable.

SOLUTION.—Area of cross-section $= (10 \times 1) + 2(4.5 \times 1) = 19$ square inches; 20 feet $= 240$ inches.

$$W = \frac{40 \times 19}{1 + \frac{240 \times 240}{93.75 \times 10 \times 10}} = 106.38 \text{ tons.}$$

Which, divided by 6, gives $\frac{106.38}{6} = 17.73$ tons, the load it is capable of sustaining with safety. Ans.

639. When using this formula, first obtain the value of C from Table 18, Art. **636.** Next, calculate the area of the cross-section of the pillar. Then, find the value of a from one of the last four tables. Finally, be sure that the length of the pillar has been reduced to inches before substituting in the formula.

To find the proper value of a in any example, first turn to the table dealing with the material in question, and find the figure corresponding to the given cross-section; in the horizontal line containing this are three numbers corresponding to the different conditions of the ends of the column. From these numbers select the one corresponding to the given conditions of the column to be calculated, and this will be the required value of a.

EXAMPLES FOR PRACTICE.

1. What load may be safely carried by a hollow cylindrical cast-iron pillar, 20 ft. long, inside diameter 8", and outside diameter 10"? Both ends of the pillar are fixed. Ans. 93.18 tons.

2. A rectangular wooden column is 14 ft. long, and has one end rounded; if the cross-section is 12" × 8", what load will be required to break it? Ans. 92.15 tons.

3. A solid wrought-iron column, which has both ends movable, is 3" in diameter and 8 ft. long; what load will it safely support? Ans. 11.1 tons.

TRANSVERSE STRENGTH OF MATERIALS.

640. The transverse strength of any material is the resistance offered by its fibers to being broken by bending. As, for example, when a beam, bar, rod, etc., which is supported at its ends, is broken by a force applied between its supports.

The transverse strength of any beam, bar, rod, etc., is proportional to the product of the square of its depth multiplied by its width; consequently, it is more economical to increase the depth than the width.

TABLE 22.

CONSTANTS FOR TRANSVERSE STRENGTH.

Material.	Constant in Pounds.	Material.	Constant in Pounds.
Metals:		Woods:	
Cast Iron.....	100	Birch	35
Wrought Iron...	150	Elm	25
Structural Steel.	100	Ash.....	45
Copper.....	50	Beech.....	30
Brass	55	Hickory.....	50
		Maple.....	60
		Oak (American).	45
		Pine (Pitch)....	40
		Pine (White).. .	30

641. A **cantilever** is a beam, bar, rod, etc., fixed at one end and subjected to a transverse stress, as shown in

Fig. 138. It has a tendency to overthrow the wall or structure to which it is attached.

The strength of a cantilever varies inversely as the distance of the load from the point of fixing; and the stress upon any section varies directly as the distance of the load from that section.

The strength of a beam, bar, rod, etc., which has both its ends supported, but not fixed, and which carries a load midway between its supports, is four times that of a beam of the same length, fixed into a wall at one end, and carrying a load at the other, as in Fig. 138.

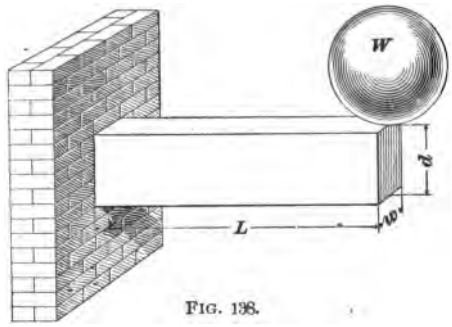


FIG. 138.

A cantilever uniformly loaded will sustain twice as great a load as one in which the load is applied at the free end; and a beam resting on two supports and uniformly loaded will sustain twice as great a load as it would if the load were all applied at the middle.

In Table 22 is given the safe transverse strength of bars of different kinds of material, one inch square and one foot long, with the load suspended from one end, the other being fixed, as shown in Fig. 138.

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642. Rules and Formulas for the Transverse Strength of Beams:

Let d = the depth of beam in inches;
w = the width of the beam in inches;
L = the length of the beam between its supports, in feet, or, for cantilever, the distance between load and fixed end;
S = the safe transverse strength, as given in Table 22;
W = the safe load in pounds.

For a rectangular or square cantilever to which the load is applied at one end, as shown in Fig. 138:

Rule.—*The maximum safe load in pounds that should be allowed at the end of any rectangular or square cantilever is equal to the square of the depth in inches multiplied by its width in inches multiplied by the constant given in the table, and the product divided by its length in feet.*

That is, $$W = \frac{d^2 w S}{L}.$$

EXAMPLE.—What is the maximum safe load that can be placed at one end of a cast-iron bar which projects four feet, the depth being 6 inches, and the width 3 inches?

SOLUTION.—By the rule, $W = \frac{d^2 w S}{L}$.

Substituting the values of d, w, S, and L, we have

$$W = \frac{6 \times 6 \times 3 \times 100}{4} = 2{,}700 \text{ pounds.} \quad \text{Ans.}$$

643. For a cylindrical cantilever to which the load is applied at one end, as shown in Fig. 139:

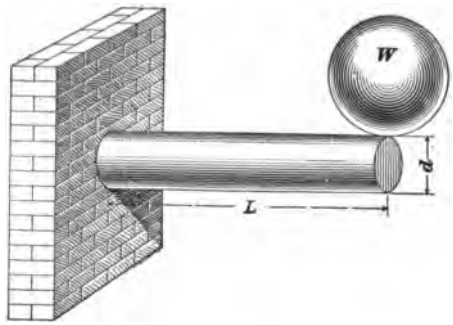

FIG. 139.

Rule.—*The maximum safe load in pounds that should be allowed at the end of any cylindrical cantilever is equal to the cube of its diameter in inches multiplied by .6 of the constant given in the table, and the product divided by its length in feet.*

That is, $$W = \frac{d^3 \times .6S}{L}.$$

EXAMPLE.—What is the maximum load that can be placed with safety at one end of a cast-iron bar 4 inches in diameter that projects 3 feet?

SOLUTION.—By the rule, $W = \frac{d^3 \times .6S}{L}$.

Substituting the values of d, S, and L, we have

$$W = \frac{4 \times 4 \times 4 \times .6 \times 100}{3} = 1{,}280 \text{ pounds.} \quad \textbf{Ans.}$$

644. When the load is uniformly distributed on a cantilever of any cross-section, as shown in Fig. 140, it will sustain a load twice as great as when the load is applied at one end. For example, if the cantilevers in the two examples above were to carry a uniformly distributed load, they would sustain $2{,}700 \times 2 = 5{,}400$ pounds, and $1{,}280 \times 2 = 2{,}560$ pounds respectively.

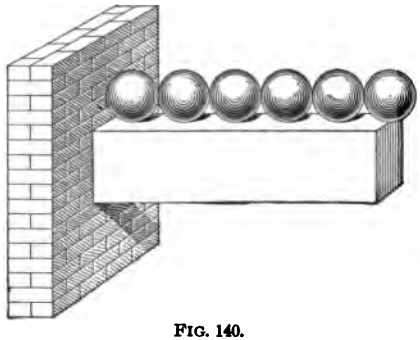

FIG. 140.

For a rectangular or square beam the ends of which merely rest upon supports, and loaded in the middle, as shown in Fig. 141:

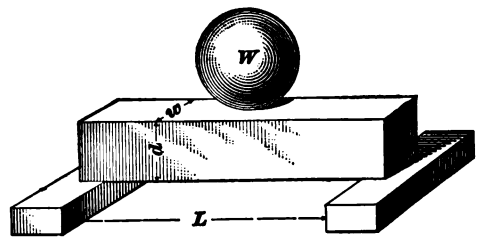

FIG. 141.

Rule.—*The maximum safe load in pounds that any rectangular or square beam is capable of sustaining at the middle, when its ends merely rest upon supports, is equal to four times the square of its depth in inches multiplied by its width in inches multiplied by the constant given in the table, and the product divided by the distance between its supports in feet.*

That is, $$W = \frac{4\, d^2\, w\, S}{L}.$$

EXAMPLE.—What maximum safe load is a bar of cast iron capable of sustaining in the middle between the supports on which its ends

merely rest, if its depth is 6 inches, its width 3 inches, and the distance between the supports is 4 feet?

SOLUTION.—By the rule, $W = \frac{4 \times 6^2 \times 3 \times 100}{4} = 10{,}800$ lb. Ans.

645. For a cylindrical beam supported at its ends and loaded in the middle, as shown in Fig. 142:

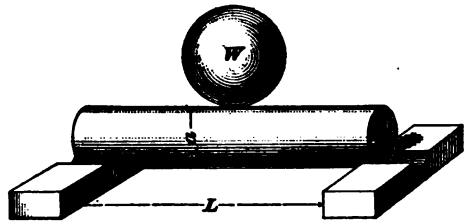

FIG. 142.

Rule.—*The maximum safe load in pounds that any cylindrical beam is capable of sustaining at the middle when its ends merely rest upon supports, is equal to four times the cube of its diameter multiplied by .6 of the constant given in the table, and the product divided by the distance between its supports in feet.*

That is, $$W = \frac{4\,d^3 \times .6\,S}{L}.$$

EXAMPLE.—What maximum safe load is a bar of cast iron capable of sustaining in the middle between the supports on which its ends merely rest, if it is 4 inches in diameter, and if the distance between its supports is 3 feet?

SOLUTION.—By the rule, $W = \frac{4 \times 4^3 \times .6 \times 100}{3} = 5{,}120$ lb. Ans.

646. When the load is uniformly distributed on a beam of any cross-section, as shown in Fig. 143, it will sustain a load twice as great as when the load is applied in the middle between the supports.

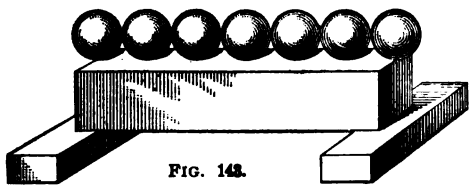
FIG. 143.

For example, if the beams in the last two examples were to carry a uniformly distributed load, they would sustain $10{,}800 \times 2 = 21{,}600$ pounds, and $5\,120 \times 2 = 10{,}240$ pounds, respectively.

SHEARING, OR CUTTING, STRENGTH OF MATERIALS.

647. The shearing strength of any material is the resistance offered by its fibers to being cut in two.

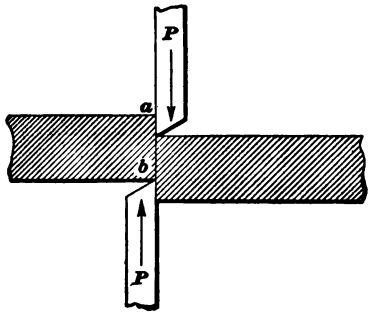

FIG. 144.

Thus, the pressure of the cutting edges of an ordinary shearing machine, Fig. 144, causes a shearing stress in the plane *a b*. The unit shearing force may be found by dividing the force *P* by the area of the plane *a b*.

648. Fig. 145 shows a piece in **double shear**; here the central piece *c d* is forced out while the ends remain on their supports *M* and *N*.

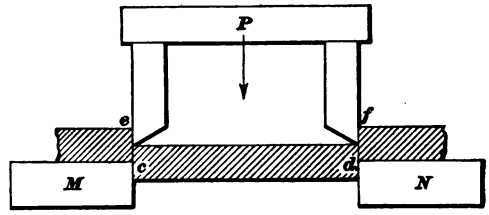

FIG. 145.

The shearing strength of any body is directly proportional to its area.

In the following table are given the greatest and safe shearing strengths per square inch of different kinds of materials:

TABLE 23.

Material.	Greatest Shearing Stress in Pounds per Square Inch.	Safe Shearing Stress in Pounds per Square Inch.
Cast Iron.....	18,000	1,500 to 3,000
Wrought Iron.....	40,000	4,000 to 10,000
Steel	60,000	5,000 to 12,000

649. Formula for the Shearing Strength of Materials:

Let a = area of cross-section in inches;
S = safe shearing stress as given in the table;
W = safe load in pounds.

Rule.—*The safe load that any body which is subjected to a shearing stress is capable of sustaining is equal to the area of its cross-section in inches multiplied by its safe shearing stress, as given in the table.*

That is, $W = a\,S$.

EXAMPLE.—If the beam in Fig. 145 is made of wrought iron, 4 inches in depth and 2 inches in width, what *steady* shearing stress is it capable of sustaining with safety?

SOLUTION.—Applying the rule, $W = 4 \times 2 \times 10{,}000 = 80{,}000$ lb. This result must be multiplied by 2, since the beam is sheared in two places, along the lines $e\,c$ and $f\,d$. Hence, the stress which the beam will safely sustain is $80{,}000 \times 2 = 160{,}000$ lb.

EXAMPLE.—What force is required to punch a hole $\frac{1}{8}$″ in diameter through a steel plate $\frac{1}{4}$″ thick?

SOLUTION.—It is evident that punching is but shearing in a circle instead of a straight line. The area punched (sheared) is equal to the thickness of the plate multiplied by the circumference of a circle having the same diameter as the punched hole. For, if the plate were cut through one of the diameters of the punched hole, and the two semicircles were straightened out, the punched surface would be a rectangle, which would have a length equal to the circumference of a circle whose diameter was equal to that of the hole, and a breadth equal to the thickness of the plate. In this case, the area $= \frac{1}{8} \times 3.1416 \times \frac{1}{4} = .98175$ sq. in. Table 23 gives the ultimate shearing strength of steel as 60,000 lb. per sq. in. Hence, the total force required is $.98175 \times 60{,}000 = 58{,}905$ lb. Ans.

EXAMPLES FOR PRACTICE.

1. What is the greatest load that can be safely carried by a steel rectangular cantilever at its extreme end, if the bar is 2″ wide, 3″ deep, and 2 ft. 6″ long? Ans. 1,152 lb.

2. What is the greatest uniform load that can be safely carried by a white-pine girder, 6″ wide, 8″ deep, 16 ft. long, and supported at its ends? Ans. 5,760 lb.

3. A cast-iron bar, $1\frac{1}{2}''$ in diameter and 5 ft. 3″ long, is supported at its ends; what load will it safely sustain in the middle? Ans. 245 lb.

4. What force is required to punch a $1\frac{1}{4}''$ hole through a wrought-iron plate $\frac{7}{16}''$ thick? Ans. 68,723 lb.

5. What force is required to cut off the end of a cast-iron bar whose diameter is $2\frac{1}{4}''$? Ans. 88,357 lb.

STRENGTH OF CYLINDERS SUBJECTED TO INTERNAL FLUID PRESSURES.

650. When a cylinder is subjected to internal fluid pressure, its strength is the same as that of an imaginary ring whose width is one inch and whose diameter is the same as the maximum diameter of the cylinder.

The pressure which comes upon this ring tends to separate one half from the other, and is equal in all cases to the width in inches times the diameter in inches multiplied by the pressure in pounds per square inch as indicated by the gauge.

651. For example, suppose that a wrought-iron cylinder, having a maximum diameter of 24 inches, is to be subjected to a pressure of 75 pounds per square inch. The pressure on each half of a ring one inch wide, tending to separate one half from the other, is $1 \times 24 \times 75 = 1{,}800$ pounds; this is resisted by twice the thickness, times the width multiplied by the tensile strength of the material.

Let D = maximum internal diameter in inches;
t = thickness of the metal in inches;
P = internal pressure per square inch;
S = safe tensile strength of material in pounds per square inch; then,

Rule I.—*The internal pressure per square inch that any cylinder is capable of sustaining is equal to twice the thickness of its metal in inches multiplied by the working tensile strength of the material in pounds per square inch, and the product divided by its maximum internal diameter in inches.*

That is, $$P = \frac{2tS}{D}.$$

EXAMPLE.—Find the steam pressure in pounds per square inch which can be sustained by a wrought-iron cylindrical boiler 40 inches in diameter, the plates of which are $\frac{5}{16}$ of an inch thick, taking the working strength of the plates at 8,000 pounds per square inch.

SOLUTION.—Applying the rule, we have

$$P = \frac{2 \times \frac{5}{16} \times 8,000}{40} = 125 \text{ lb. per sq. in.} \quad \text{Ans.}$$

Rule II.—*The maximum internal diameter of a cylinder in inches is equal to twice the thickness of its metal in inches multiplied by the working tensile strength of the material in pounds per square inch, and this product divided by the internal pressure in pounds per square inch.*

That is, $$D = \frac{2tS}{P}.$$

EXAMPLE.—The steam entering the cylinder of an engine has a pressure of 100 pounds per square inch, and it has been found that the metal of the cylinder, which is $\frac{1}{4}$ of an inch thick, is under a stress of 1,500 pounds per square inch. What is the diameter of the cylinder?

SOLUTION.—Applying the rule,

$$D = \frac{2tS}{P}.$$

Substituting the values of t, S, and P, we have

$$D = \frac{2 \times \frac{1}{4} \times 1,500}{100} = 22\frac{1}{2} \text{ inches diameter.} \quad \text{Ans}$$

Rule III.—*The thickness in inches of the metal of a cylinder is equal to its maximum internal diameter in inches multiplied by the internal pressure in pounds per square inch that it is to sustain, and this product divided by twice the working strength of the material in pounds per square inch.*

That is, $$t = \frac{DP}{2S}.$$

EXAMPLE.—Find the thickness of the plates of a wrought-iron cylindrical boiler 40 inches in diameter, to sustain a pressure of 100 pounds per square inch; take the working strength of the plates at 8,000 pounds per square inch.

SOLUTION.—Applying the rule, we have

$$t = \frac{40 \times 100}{2 \times 8,000} = \frac{1}{4} \text{ in.} \quad \text{Ans.}$$

Rule IV.—*The stress in pounds per square inch of the material of any cylinder is equal to the maximum internal diameter in inches multiplied by the internal pressure in pounds per square inch, and this product divided by twice the thickness of the metal.*

That is, $$S = \frac{DP}{2t}.$$

EXAMPLE.—A lead pipe whose internal diameter is 2" and which has a thickness of $\frac{5}{32}$" is subjected to a pressure of 40 lb. per sq. in. What is the stress in pounds per square inch of the material?

SOLUTION.—Applying the rule, we have

$$S = \frac{2 \times 40}{2 \times \frac{5}{32}} = 256 \text{ lb. per sq. in.} \quad \text{Ans.}$$

652. The working tensile strengths of the different materials which are to be used in calculations pertaining to the strength of cylinders subjected to internal fluid pressure (by fluid is meant any gas or liquid) may be obtained from Table 17, Art. **626**; the highest values given in the column headed "Working Stress" should be used.

EXAMPLE.—What should be the thickness of a brass cylinder 20" in diameter, subjected to a pressure of 25 lb. per sq. in.?

SOLUTION.—Applying the rule, and substituting for S the value 3,500 obtained from the table just mentioned, $t = \frac{20 \times 25}{2 \times 3{,}500} = \frac{1}{14}$", say $\frac{5}{64}$" Ans.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the questions and examples contained in the following pages are divided into sections corresponding to the sections of the text of the preceding pages, and that each section has a headline which is the same as the headline of the section to which the questions refer. No attempt should be made to answer any questions or to work any examples until the corresponding part of the text has been carefully studied.

ARITHMETIC.

(ARTS. 1-181.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words:

(*a*) 980; (*b*) 605; (*c*) 28,284; (*d*) 9,006,042; (*e*) 850,317,002; (*f*) 700,004.

(6) Represent in figures the following expressions:

(*a*) Seven thousand, six hundred. (*b*) Eighty-one thousand, four hundred, two. (*c*) Five million, four thousand, seven. (*d*) One hundred eight million, ten thousand, one. (*e*) Eighteen million, six. (*f*) Thirty thousand, ten.

(7) What is the sum of $3,290 + 504 + 865,403 + 2,074 + 81 + 7$? Ans. 871,359.

(8) $709 + 8,304,725 + 391 + 100,302 + 300 + 909 =$ what? Ans. 8,407,336.

(9) Find the difference between the following:

(*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

Ans. (*a*) 47,624. (*b*) 5,338.

(10) (*a*) $70,968 - 32,975 =$? (*b*) $100,000 - 98,735 =$?

Ans. (*a*) 37,993. (*b*) 1,265.

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(11) The greater of two numbers is 1,004 and their difference is 49; what is their sum? Ans. 1,959.

(12) From 5,962 + 8,471 + 9,023 take 3,874 + 2,039.
Ans. 17,543.

(13) A man willed $125,000 to his wife and two children; to his son he gave $44,675, to his daughter $26,380, and to his wife the remainder. What was his wife's share?
Ans. $53,945.

(14) Find the products of the following:
(*a*) 526,387 × 7; (*b*) 700,298 × 17; (*c*) 217 × 103 × 67.
Ans. { (*a*) 3,684,709. (*b*) 11,905,066. (*c*) 1,497,517.

(15) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800 times.

(16) If a monthly publication contains 24 pages in each issue, how many pages will there be in eight yearly volumes?
Ans. 2,304.

(17) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?
Ans. { (*a*) $34,689. (*b*) 37,935.

(18) Solve the following by cancelation:
(*a*) $\frac{72 \times 48 \times 28 \times 5}{96 \times 15 \times 7 \times 6} = ?$ (*b*) $\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$
Ans. { (*a*) 8. (*b*) 32.

(19) If a mechanic earns $1,500 a year for his labor, and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land, at $133 an acre?
Ans. 7 years.

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 miles.

(21) If the driving wheel of a locomotive is 16 ft. in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance of which is 354 miles, there being 5,280 feet in one mile? Ans. 116,820 rev.

(22) What is the quotient of
(*a*) 589,824 ÷ 576? (*b*) 369,730,620 ÷ 43,911? (*c*) 2,527,525 ÷ 505? (*d*) 4,961,794,302 ÷ 1,234?

Ans. { (*a*) 1,024. (*b*) 8,420. (*c*) 5,005. (*d*) 4,020,903.

(23) A man paid $444 for a horse, wagon, and harness. If the horse cost $264 and the wagon $153, how much did the harness cost? Ans. $27.

(24) What is the product of
(*a*) 1,024 × 576? (*b*) 5,005 × 505? (*c*) 43,911 × 8,420?

Ans. { (*a*) 589,824. (*b*) 2,527,525. (*c*) 369,730,620.

(25) If a man receives 30 cents an hour for his wages, how much will he earn in a year, working 10 hours a day and averaging 25 days per month? Ans. $900.

(26) What is a fraction?

(27) What are the terms of a fraction?

(28) What does the denominator show?

(29) What does the numerator show?

(30) How do you find the value of a fraction?

(31) Is $\frac{13}{8}$ a proper or an improper fraction, and why?

(32) Write three mixed numbers.

(33) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(34) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(35) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions.
Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(36) What is the value of each of the following: $1\frac{3}{2}$, $1\frac{7}{4}$, $\frac{69}{16}$, $\frac{16}{8}$, $\frac{67}{64}$? Ans. $6\frac{1}{2}$, $4\frac{1}{4}$, $4\frac{5}{16}$, 2, $1\frac{3}{64}$.

(37) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $1\frac{7}{8} \div 9$; (*d*) $1\frac{13}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans. (*a*) 112. (*b*) $\frac{3}{16}$. (*c*) $\frac{17}{18}$. (*d*) $4\frac{1}{28}$. (*e*) $3\frac{3}{5}$.

(38) $\frac{1}{8} + \frac{3}{8} + \frac{5}{8} = ?$ Ans. 1.

(39) $\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ Ans. $\frac{15}{16}$.

(40) $42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Ans. $83\frac{1}{16}$.

(41) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches, and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?
Ans. $190\frac{9}{16}$ sq. in.

(42) Find the value of each of the following:

$$(a)\ \frac{7}{\frac{3}{16}};\quad (b)\ \frac{\frac{15}{32}}{\frac{5}{8}};\quad (c)\ \frac{\frac{4+3}{2+6}}{5}$$

Ans. (*a*) $37\frac{1}{3}$. (*b*) $\frac{3}{4}$. (*c*) $\frac{7}{40}$.

(43) The numerator of a fraction is 28, and the value of the fraction $\frac{7}{8}$; what is the denominator? Ans. 32.

(44) What is the difference between (*a*) $\frac{7}{8}$ and $\frac{7}{16}$? (*b*) 13 and $7\frac{7}{16}$? (*c*) $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. (*a*) $\frac{7}{16}$. (*b*) $5\frac{9}{16}$. (*c*) $83\frac{13}{32}$.

(45) If a man travels $85\frac{5}{12}$ miles in one day, $78\frac{9}{15}$ miles in another day, and $125\frac{7}{3}$ miles in another day, how far did he travel in the three days? Ans. $289\frac{21}{420}$ miles.

(46) From $573\frac{1}{3}$ tons take $216\frac{3}{8}$ tons. Ans. $357\frac{7}{240}$.

(47) At $\frac{3}{8}$ of a dollar a yard, what will be the cost of $9\frac{1}{2}$ yards of cloth? Ans. $3\frac{15}{16}$ dollars.

(48) Multiply $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{11}$ of $\frac{11}{20}$ of 11 by $\frac{1}{8}$ of $\frac{5}{6}$ of 45.
Ans. $109\frac{13}{128}$.

(49) How many times are $\frac{3}{8}$ contained in $\frac{3}{4}$ of 16?
Ans. 18 times.

(50) Bought $211\frac{1}{4}$ pounds of old lead for $1\frac{7}{8}$ cents per pound. Sold a part of it for $2\frac{1}{2}$ cents per pound, receiving for it the same amount as I paid for the whole. How many pounds did I have left? Ans. $52\frac{3}{16}$ pounds.

(51) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(52) How do you place decimals for addition and subtraction?

(53) Give a rule for multiplication of decimals.

(54) Give a rule for division of decimals.

(55) State the difference between a fraction and a decimal.

(56) State how to reduce a fraction to a decimal.

(57) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

Ans. .5.
.875.
.15625
.65.
.125.

(58) Solve the following:

(*a*) $\frac{32.5 + .29 + 1.5}{4.7 + 9}$; (*b*) $\frac{1.283 \times \overline{8 + 5}}{2.63}$;

(*c*) $\frac{\overline{589 + 27} \times \overline{163 - 8}}{25 + 39}$; (*d*) $\frac{\overline{40.6 + 7.1} \times (3.029 - 1.874)}{6.27 + 8.53 - 8.01}$

Ans. { (*a*) 2.5029. (*b*) 6.3418. (*c*) 1,491.875. (*d*) 8.1139.

(59) How many inches in .875 of a foot? Ans. 10½ in.

(60) What decimal part of a foot is $\frac{3}{16}$ of an inch?
Ans. .015625.

(61) A cubic inch of water weighs .03617 of a pound. What is the weight of a body of water whose volume is 1,500 cubic inches? Ans. 54.255 lb.

(62) If by selling a carload of coal for $82.50, at a profit of $1.65 per ton, I make enough to pay for 72.6 ft. of fencing at $.50 a foot, how many tons of coal were in the car?
Ans. 22 tons.

(63) Divide 17,892 by 231, and carry the result to four decimal places. Ans. 77.4545 +.

(64) Find the value of the following expression when the result is carried to three decimal places:

$\frac{74.26 \times 24 \times 3.1416 \times 19 \times 19 \times 350}{33{,}000 \times 12 \times 4} = ?$ Ans. 446.619—.

(65) Express (*a*) .7928 in 64ths; (*b*) .1416 in 32ds; (*c*) .47915 in 16ths.

Ans. { (*a*) $\frac{51}{64}$. (*b*) $\frac{5}{32}$. (*c*) $\frac{8}{16}$

(66) Work out the following examples:

(*a*) 709.63 − .8514; (*b*) 81.963 − 1.7; (*c*) 18 − .18; (*d*) 1 − .001; (*e*) 872.1 − (.8721 + .008); (*f*) (5.028 + .0073) − (6.704 − 2.38).

Ans. (*a*) 708.7786. (*b*) 80.263. (*c*) 17.82. (*d*) .999. (*e*) 871.2199. (*f*) .7113.

(67) Work out the following:

(*a*) $\frac{7}{8}$ − .807; (*b*) .875 − $\frac{3}{8}$; (*c*) ($\frac{5}{32}$ + .435) − ($\frac{21}{100}$ − .07); (*d*) What is the difference between the sum of 33-millionths and 17-thousandths, and the sum of 53-hundredths and 274-thousandths?

Ans. (*a*) .068. (*b*) .5. (*c*) .45125. (*d*) .786967.

(68) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(69) 927.416 + 8.274 + 372.6 + 62.07938 = ?

Ans. 1,370.36938.

(70) Add 17-thousandths, 2-tenths, and 47-millionths.

Ans. .217047.

(71) Find the products of the following expressions:

(*a*) .013 × .107; (*b*) 203 × 2.03 × .203; (*c*) 2.7 × 31.85 × (3.16 − .316); (*d*) (107.8 + 6.541 − 31.96) × 1.742.

Ans. (*a*) .001391. (*b*) 83.65427. (*c*) 244.56978. (*d*) 143.507702.

(72) Solve the following:

(*a*) ($\frac{7}{16}$ − .13) × $\overline{.625 + \frac{5}{8}}$; (*b*) ($\frac{19}{32}$ × .21) − (.02 × $\frac{3}{16}$); (*c*) ($\frac{13}{4}$ + .013 − 2.17) × $\overline{13\frac{1}{4} - 7\frac{5}{16}}$.

Ans. (*a*) .384375. (*b*) .1209375. (*c*) 6.4896875.

(73) Solve the following:

(*a*) $.875 \div \frac{1}{2}$; (*b*) $\frac{7}{8} \div .5$; (*c*) $\frac{.375 \times \frac{1}{4}}{\frac{5}{16} - .125}$.

Ans. { (*a*) 1.75. (*b*) 1.75. (*c*) .5.

(74) Find the value of the following expression:

$$\frac{\frac{1.25 \times 20 \times 3}{87 + (11 \times 8)}}{459 + 32}$$

Ans. $210\frac{3}{7}$.

(75) From 1 plus .001 take .01 plus .000001.

Ans. .990999.

(76) Nos. 0000, 1, 5, 10, 14, and 17 of the American wire gauge are respectively .46, .2893, .1819, .1018, .0640, and .045 inches thick. Find the sum of the thickness of plates representing these numbers. Ans. 1.142 in.

(77) The areas of 4 steam pipes are respectively 927.416, 8.274, 372.6, and 62.07938 square feet. What is their total area? Ans. 1,370.36938 sq. ft.

(78) A line of gas-pipes 3,012 yards long is laid along an avenue, with branches leading off to various streets, as follows: Street A, 875 yards of pipe; street B, 630 yards of pipe; street C, 961 yards of pipe; street D, two branches, each 618 yards long, and street E, 3 branches, each 723 yards long. What is the total length of the pipe?

Ans. 8,883 yd.

(79) A gas-pipe job is estimated as follows: Cellar—187 feet of 1-inch pipe, 230 feet of $\frac{3}{4}$-inch pipe, 97 feet of $\frac{1}{2}$-inch pipe, 109 lb. of fittings; first floor—39 feet of 1-inch pipe, 77 feet of $\frac{3}{4}$-inch pipe, 879 feet of $\frac{1}{2}$-inch pipe, 1,036 feet of $\frac{3}{8}$-inch pipe, 62 pounds of fittings; second floor—103 feet of $\frac{3}{4}$-inch pipe, 927 feet of $\frac{1}{2}$-inch pipe, 2,193 feet of $\frac{3}{8}$-inch pipe, and 27 pounds of fittings. Find (*a*) the amount of pipe of various sizes and the weight of fittings; (*b*) the total length of the pipe.

Ans. { (*a*) { 226 ft. of 1-in. pipe. 410 ft. of $\frac{3}{4}$-in. pipe. 1,903 ft. of $\frac{1}{2}$-in. pipe. 3,229 ft. of $\frac{3}{8}$-in. pipe. 198 lb. of fittings. (*b*) 5,768 ft.

(80) If 1,007 feet of pipe are delivered at a building for plumbing purposes, and 326 feet are used on one floor, 197 feet on another, 173 feet on another, and 304 feet on another floor, how much remains unused? Ans. 7 ft.

(81) There are 5 small water-tanks in a building, whose capacities are respectively 137, 629, 136, 819, and 31 gallons. The board of health orders these tanks to be taken out and a large one used instead, whose capacity is 2,200 gallons. What is the increase in tank capacity?

Ans. 448 gal.

(82) The thickness in inches of 4 iron plates as indicated by the American wire gauge are respectively .46 for No. 0000 gauge, .4096 for No. 000 gauge, .3648 for No. 00 gauge, and .3248 for No. 0 gauge. Find the difference between the thickness of Nos. 0 and 0000, 00 and 000, 0 + 00 and 0000, 00 + 0 + 000 and 0000.

Ans. { .1352 in. .0448 in. .2296 in. .6392 in.

(83) (*a*) Five wipe joints, each weighing $\frac{1}{2}$ of a pound, are made from a pot containing 10 pounds of solder. If 5 pounds of solder are added to the pot, and 10 more joints wiped, how much solder is left? (*b*) If the pot contains 16 pounds of solder and 12 joints are made, then 11 pounds of solder added and 12 more joints made and 3 pounds of solder remain, how much was used in each joint, assuming them to be of the same size and weight?

Ans. { (*a*) $7\frac{1}{2}$ lb. (*b*) 1 lb.

(84) If a steam pump running with a uniform speed night and day makes 2 strokes in a second, how many strokes will it make in a week? (*b*) If it discharges 1 gallon in 2 strokes, what is the total number of gallons discharged in a week?

Ans. { (*a*) 1,209,600. (*b*) 604,800.

(85) A piece of pipe work is composed of 17 lines of

pipe. Each line is 125 feet long and has 35 branches, each branch being 79 feet long. Find the total length of the pipe. Ans. 49,130 ft.

(86) A quantity of old scrap lead weighing 3,240 pounds was bought at $2\frac{1}{2}$ cents a pound; $\frac{1}{10}$ of it was used for calking purposes, and the remainder was melted into bars and sold for $3\frac{1}{2}$ cents a pound, $\frac{1}{9}$ of this being lost in dross during the process of melting. (*a*) How much was paid for the lead? (*b*) How much was received for the bars of lead?

Ans. { (*a*) \$81. (*b*) \$90.72.

(87) A tank whose capacity is $143\frac{1}{2}$ gallons is to be filled with water by a pail whose capacity is $3\frac{1}{2}$ gallons. How many pails full are required to fill the tank? Ans. 41.

(88) A cistern whose capacity is 11,857.5 gallons is supplied with rain-water from the roof of a building. (*a*) How long is it capable of supplying a family whose daily consumption of water is $127\frac{1}{2}$ gallons? (*b*) If the cistern is $\frac{1}{2}$ filled, and a drought prevails for 39 days, how much water will be in the cistern at the end of that time?

Ans. { (*a*) 93 da. (*b*) 956.25 gal.

(89) What is the exact length of 97 lengths of cast-iron spigot and socket pipes if each length of pipe measures 5 feet $3\frac{1}{2}$ inches over all and the socket is 4 inches long?

Ans. 481 ft. $3\frac{1}{2}$ in.

(90) A cubic inch of lead weighs .41 of a pound, and a cubic inch of cast iron weighs .26 of a pound. What is the difference in weight between 97.15 cubic inches of lead and 107 cubic inches of iron? Ans. 12.0115 lb.

(91) If water be conducted from a tank by 7 lengths of gas-pipe screwed together, each length being 12 ft. 6 in. long, $\frac{3}{4}$ of an inch being allowed at each of the joints for screwing together, and $\frac{3}{4}$ of an inch for screwing into the tank, how far from the tank is the water discharged?

Ans. 87.8125 ft.

ARITHMETIC.

(ARTS. 182–339.)

EXAMINATION QUESTIONS.

(92) What is 25 per cent. of 8,428 lb.? Ans. 2,107 lb.

(93) What is 1 per cent. of $100? Ans. $1.

(94) What is ½ per cent. of $35,000? Ans. $175.

(95) What per cent. of 50 is 2? Ans. 4%.

(96) What per cent. of 10 is 10? Ans. 100%.

(97) Solve the following:

(*a*) Base = $2,522 and percentage = $176.54. What is the rate? (*b*) Percentage = 16.96 and rate = 8 per cent. What is the base? (*c*) Amount = 216.7025 and base = 213.5. What is the rate? (*d*) Difference = 201.825 and base = 207. What is the rate?

Ans. (*a*) 7%. (*b*) 212. (*c*) 1½%. (*d*) 2½%.

(98) A farmer gained 15% on his farm by selling it for $5,500. What did it cost him? Ans. $4,782.61.

(99) A man receives a salary of $950. He pays 24% of it for board, 12½% of it for clothing, and 17% of it for other expenses. How much does he save in a year?

Ans. $441.75.

(100) If 37½ per cent. of a number is 961.38, what is the number? Ans. 2,563.68.

(101) A man owns ¾ of a property. 30% of his share is worth $1,125. What is the whole property worth?

Ans. $5,000.

(102) What sum diminished by 35% of itself equals $4,810? Ans. $7,400.

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(103) A merchant's sales amounted to $197.55 on Monday, and this sum was 12½% of his sales for the week. How much were his sales for the week? Ans. $1,580.40.

(104) The distance between two stations on a certain railroad is 16.5 miles, which is 12½% of the entire length of the road. What is the length of the road? Ans. 132 miles.

(105) After paying 60% of my debts, I find that I still owe $35. What was my whole indebtedness? Ans. $87.50.

(106) Reduce 28 rd. 4 yd. 2 ft. 10 in. to inches.
Ans. 5,722 in.

(107) Reduce 5,722 in. to higher denominations.
Ans. 28 rd. 4 yd. 2 ft. 10 in.

(108) How many seconds in 5 weeks and 3.5 days?
Ans. 3,326,400 sec.

(109) How many pounds, ounces, pennyweights, and grains are contained in 13,750 gr.?
Ans. 2 lb. 4 oz. 12 pwt. 22 gr.

(110) Reduce 4,763,254 links to miles.
Ans. 595 mi. 32 ch. 54 li.

(111) Reduce 764,325 cu. in. to cu. yd.
Ans. 16 cu. yd. 10 cu. ft. 549 cu. in.

(112) What is the sum of 2 rd. 2 yd. 2 ft. 3 in.; 4 yd. 1 ft. 9 in.; 2 ft. 7 in.? Ans. 3 rd. 2 yd. 2 ft. 1 in.

(113) What is the sum of 3 gal. 3 qt. 1 pt. 3 gi.; 6 gal. 1 pt. 2 gi.; 4 gal. 1 gi.; 8 qt. 5 pt.? Ans. 16 gal. 3 qt. 2 gi.

(114) What is the sum of 240 gr. 125 pwt. 50 oz. and 3 lb.?
Ans. 7 lb. 8 oz. 15 pwt.

(115) What is the sum of 11° 16′ 12″; 13° 19′ 30″; 20° 25″; 26′ 29″; 10° 17′ 11″? Ans. 55° 19′ 47″.

(116) What is the sum of 130 rd. 5 yd. 1 ft. 6 in.; 215 rd. 2 ft. 8 in.; 304 rd. 4 yd. 11 in.? Ans. 2 mi. 10 rd. 5 yd. 7 in.

(117) What is the sum of 21 A. 67 sq. ch. 3 sq. rd. 21 sq. li.; 28 A. 78 sq. ch. 2 sq. rd. 23 sq. li.; 47 A. 6 sq. ch. 2 sq.

rd. 18 sq. li.; 56 A. 59 sq. ch. 2 sq. rd. 16 sq. li.; 25 A. 38 sq. ch. 3 sq. rd. 23 sq. li.; 46 A. 75 sq. ch. 2 sq. rd. 21 sq. li.?
Ans. 255 A. 3 sq. ch. 14 sq. rd. 122 sq. li.

(118) From 20 rd. 2 yd. 2 ft. 9 in. take 300 feet.
Ans. 2 rd. 1 yd. 2 ft. 9 in.

(119) From a farm containing 114 A. 80 sq. rd. 25 sq. yd., 75 A. 70 sq. rd. 30 sq. yd. are sold. How much remains?
Ans. 39 A. 9 sq. rd. 25¼ sq. yd.

(120) From a hogshead of molasses, 10 gal. 2 qt. 1 pt. are sold at one time, and 26 gal. 3 qt. at another time. How much remains? Ans. 25 gal. 2 qt. 1 pt.

(121) If a person were born June 19, 1850, how old would he be August 3, 1892? Ans. 42 yr. 1 mo. 14 da.

(122) A note was given August 5, 1890, and was paid June 3, 1892. What length of time did it run?
Ans. 1 yr. 9 mo. 28 da.

(123) What length of time elapsed from 16 min. past 10 o'clock A. M., July 4, 1883, to 22 min. before 8 o'clock P. M., Dec. 12, 1888? Ans. 5 yr. 5 mo. 8 da. 9 hr. 22 min.

(124) If 1 iron rail is 17 ft. 3 in. long, how long would 51 rails be, if placed end to end? Ans. 53 rd. 1½ yd. 9 in.

(125) Multiply 3 qt. 1 pt. 3 gi. by 4.7.
Ans. 4 gal. 2 qt. 1.7 gi.

(126) Multiply 3 lb. 10 oz. 13 pwt. 12 gr. by 1.5.
Ans. 5 lb. 10 oz. 6 gr.

(127) How many bushels of apples are contained in 9 bbl. if each barrel contains 2 bu. 3 pk. 6 qt.?
Ans. 26 bu. 1 pk. 6 qt.

(128) Multiply 7 T. 15 cwt. 10.5 lb. by 1.7.
Ans. 13 T. 3 cwt. 67.85 lb.

(129) Divide 358 A. 57 sq. rd. 6 sq. yd. 2 sq. ft. by 7.
Ans. 51 A. 31 sq. rd. 8 sq. ft.

(130) Divide 282 bu. 3 pk. 1 qt. 1 pt. by 12.
Ans. 23 bu. 2 pk. 2 qt. ¼ pt.

(131) How many iron rails, each 30 ft. long, are required to lay a railroad-track 23 miles long? Ans. 8,096 rails.

(132) How many boxes, each holding 1 bu. 1 pk. and 7 qt., can be filled from 356 bu. 3 pk. and 5 qt. of cranberries?

Ans. 243 boxes.

(133) If 16 square miles are equally divided into 62 farms, how much land will each contain?

Ans. 165 A. 25 sq. rd. 24 sq. yd. 3 sq. ft. 80 + sq. in.

(134) What is the square of 108? Ans. 11,664.

(135) What is the cube of 181.25? Ans. 5,954,345.703125.

(136) What is the fourth power of 27.61?

Ans. 581,119.73780641.

(137) Solve the following: (*a*) 106^2; (*b*) $(182\frac{1}{8})^2$; (*c*) $.005^2$; (*d*) $.0063^2$; (*e*) 10.06^2.

Ans. (*a*) 11,236. (*b*) 33,169.515625. (*c*) .000025. (*d*) .00003969. (*e*) 101.2036.

(138) Solve the following: (*a*) 753^3; (*b*) 987.4^3; (*c*) $.005^3$; (*d*) $.4044^3$.

Ans. (*a*) 426,957,777. (*b*) 962,674,279,624. (*c*) .000000125. (*d*) .066135317184.

(139) What is the fifth power of 2? Ans. 32.

(140) What is the fourth power of 3? Ans. 81.

(141) What are the values of: (*a*) 67.85^2; (*b*) $967,845^2$? (*c*) $(\frac{3}{8})^2$? (*d*) $(\frac{1}{4})^2$?

Ans. (*a*) 4,603.6225. (*b*) 936,723,944,025. (*c*) $\frac{9}{64}$. (*d*) $\frac{1}{16}$.

(142) What is (*a*) the tenth power of 5? (*b*) the fifth power of 9?

Ans. (*a*) 9,765,625. (*b*) 59,049.

(143) Solve the following: (*a*) 1.2^4; (*b*) 11^6; (*c*) 1^7, (*d*) $.01^4$; (*e*) $.1^5$.

Ans. (*a*) 2.0736. (*b*) 1,771,561. (*c*) 1. (*d*) .00000001. (*e*) .00001.

(144) Find the values of the following: (*a*) $.0133^3$; (*b*) 301.011^3; (*c*) $(\frac{1}{8})^3$; (*d*) $(3\frac{3}{4})^3$.

Ans. (*a*) .000002352637.
(*b*) 27,273,890.942264331.
(*c*) $\frac{1}{512}$.
(*d*) $52\frac{7}{64}$, or 52.734375.

(145) In what respect does evolution differ from involution?

NOTE.—In the answers to the following examples, a minus sign after a number indicates that the last digit is not quite as large as the number printed. Thus, 12.497 — indicates that the number really is 12.496 +, and that the 6 has been made a 7 because the next succeeding figure was 5 or greater. For example, had it been desired to use but three decimal places in example 137 (*b*), the answer would have been written 33,169.516 —.

(146) Find the square root of the following: (*a*) 3,486,784.401; (*b*) 9,000,099.4009; (*c*) .001225.

Ans. (*a*) 1,867.29 +.
(*b*) 3,000.017 —.
(*c*) .035.

(147) Extract the square root of (*a*) 10,795.21; (*b*) 73,008.04; (*c*) 90; (*d*) .09.

Ans. (*a*) 103.9.
(*b*) 270.2.
(*c*) 9.487 —.
(*d*) .3.

(148) Extract the cube root of (*a*) .32768; (*b*) 74,088; (*c*) 92,416; (*d*) .373248.

Ans. (*a*) .6894 +.
(*b*) 42.
(*c*) 45.212 —.
(*d*) .72.

(149) Extract the cube root of 2 to six decimal places.

Ans. 1.259921 +.

(150) Extract the cube root of (*a*) 1,758.416743; (*b*) 1,191,016; (*c*) $\frac{4}{343}$; (*d*) $\frac{27}{512}$.

Ans. (*a*) 12.07.
(*b*) 106.
(*c*) $\frac{1}{2}$.
(*d*) $\frac{3}{8}$.

(151) Extract the cube root of 3 to six decimal places.

Ans. 1.442250 —.

(152) Solve the following: (*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$; (*c*) $\sqrt{502,681}$; (*d*) $\sqrt{.00041209}$.

Ans. (*a*) 11.1. (*b*) 10.72 +. (*c*) 709. (*d*) .0203.

(153) Solve the following: (*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.021}$; (*c*) $\sqrt[3]{8,036,054,027}$; (*d*) $\sqrt[5]{.000004096}$; (*e*) $\sqrt[5]{17}$.

Ans. (*a*) .18663 −. (*b*) .2759 −. (*c*) 2,003. (*d*) .016. (*e*) 2.5713 −.

(154) Solve the following: (*a*) $\sqrt[4]{6,561}$; (*b*) $\sqrt[6]{117,649}$; (*c*) $\sqrt[6]{.000064}$; (*d*) $\sqrt[3]{\frac{3}{8}}$.

Ans. (*a*) 9. (*b*) 7. (*c*) .2. (*d*) .72112 +.

(155) Extract the square root of (*a*) $1\frac{2}{5}\frac{9}{7}\frac{5}{6}$; (*b*) .3364; (*c*) .1; (*d*) $25.0\frac{1}{4}$; (*e*) $.000\frac{4}{9}$.

Ans. (*a*) $\frac{3}{4}\frac{5}{4}$. (*b*) .58. (*c*) .31623 −. (*d*) 5.00749 +. (*e*) .02108 +.

(156) (*a*) Extract the fourth root of 2 to four decimal places; (*b*) extract the sixth root of 6.

Ans. (*a*) 1.1892 +. (*b*) 1.34801 −.

(157) Extract the square root of (*a*) 3.1416 and (*b*) .7854 to four decimal places.

Ans. (*a*) 1.7725 −. (*b*) .8862 +.

(158) Extract the cube root of (*a*) 3.1416 and (*b*) .5236 to four decimal places.

Ans. (*a*) 1.4646 −. (*b*) .8060 −.

(152) Solve the following: (*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$; (*c*) $\sqrt{502,681}$; (*d*) $\sqrt{.00041209}$.

Ans. { (*a*) 11.1. (*b*) 10.72 +. (*c*) 709. (*d*) .0203.

(153) Solve the following: (*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.021}$; (*c*) $\sqrt[3]{8,036,054,027}$; (*d*) $\sqrt[3]{.000004096}$; (*e*) $\sqrt[3]{17}$.

Ans. { (*a*) .18663 —. (*b*) .2759 —. (*c*) 2,003. (*d*) .016. (*e*) 2.5713 —.

(154) Solve the following: (*a*) $\sqrt[4]{6,561}$; (*b*) $\sqrt[6]{117,649}$; (*c*) $\sqrt[6]{.000064}$; (*d*) $\sqrt[3]{\frac{8}{8}}$.

Ans. { (*a*) 9. (*b*) 7. (*c*) .2. (*d*) .72112 +.

(155) Extract the square root of (*a*) $\frac{1}{6}\frac{2}{4}\frac{2}{7}\frac{5}{6}$; (*b*) .3364; (*c*) .1; (*d*) $25.0\frac{3}{4}$; (*e*) $.000\frac{4}{5}$.

Ans. { (*a*) $\frac{3}{7}\frac{5}{4}$. (*b*) .58. (*c*) .31623 —. (*d*) 5.00749 +. (*e*) .02108 +.

(156) (*a*) Extract the fourth root of 2 to four decimal places; (*b*) extract the sixth root of 6.

Ans. { (*a*) 1.1892 +. (*b*) 1.34801 —.

(157) Extract the square root of (*a*) 3.1416 and (*b*) .7854 to four decimal places.

Ans. { (*a*) 1.7725 —. (*b*) .8862 +.

(158) Extract the cube root of (*a*) 3.1416 and (*b*) .5236 to four decimal places.

Ans. { (*a*) 1.4646 —. (*b*) .8060 —.

Find the value of x in the following:

(159) $11.7 : 13 :: 20 : x$. Ans. 22.22 +.

(160) (*a*) $20 + 7 : 10 + 8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. { (*a*) 2. (*b*) 277.7 +.

(161) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. { (*a*) $x = 12$. (*b*) $x = 12$. (*c*) $x = 20$. (*d*) $x = 180$. (*e*) $x = 40$.

(162) $x : 5 :: 27 : 12.5$. Ans. $10\frac{1}{2}$.

(163) $45 : 60 :: x : 24$. Ans. 18.

(164) $x : 35 :: 4 : 7$. Ans. 20.

(165) $9 : x :: 6 : 24$. Ans. 36.

(166) $\sqrt[3]{1,000} : \sqrt[3]{1,331} = 27 : x$. Ans. 29.7.

(167) $64 : 81 = 21^2 : x^2$. Ans. 23.625.

(168) $7 + 8 : 7 = 30 : x$. Ans. 14.

(169) A man whose steps measure 2 ft. 5 in. takes 2,480 steps in walking a certain distance. How many steps of 2 ft. 7 in. will be required for the same distance?

Ans. 2,320 steps.

(170) If a horse travels 12 mi. in 1 hr. 36 min., how far will he travel at the same rate in 15 hr.? Ans. 112.5 mi.

(171) If a column of mercury 27.63 in. high weighs .76 of a pound, what will be the weight of a column of mercury having the same diameter, 29.4 inches high? Ans. .808 + lb.

(172) If 2 gal. 3 qt. 1 pt. of water will last a man 5 da., how long will 5 gal. 3 qt. last him, if he drinks at the same rate? Ans. 10 da

(173) Heat from a burning body varies inversely as the square of the distance from it. If a thermometer held 6 ft. from a stove shows a rise in temperature of 24 degrees, how

many degrees rise in temperature would it indicate if held 12 ft. from the stove? Ans. 6°.

(174) A plumber receives a salary of $950 per annum. He pays 24% of it for board, 12½% of it for clothing, and 17% of it for other expenses. How much does he save a year?
Ans. $441.75.

(175) A man owns ¾ of a plumbing establishment. 30% of his share is worth $1,125. What is the whole property worth? Ans. $5,000.

(176) The volume of water raised by a hydraulic ram per second is 18.3 cu. in., and the quantity of water going to waste is 254.5 cu. in. What percentage of the water passing through the drive pipe is the quantity raised?
Ans. 6.71%, nearly.

(177) The velocity of the water through a valve on a water-pipe when partly opened was 298 feet per minute. An increase of 1½% was made by opening the valve a little further. What was the final velocity?
Ans. 302.47 ft. per min.

(178) A cast-iron pipe is 5 ft. 3½ in. long over all. How long would 37 lengths be, if calked together, the slip of the hub being 3 in.? Ans. 186 ft. 9½ in.

(179) How many spigot and socket pipes, each 9 ft. 4 in. long (over all), are required to lay a street main 1 mi. 1,980 ft. long, the depth of each socket being 3½ in.?
Ans. 803 pipes, nearly.

(180) A line of iron pipe is composed of four lengths, each 15 ft. 5 in. long, one piece 14 ft. 8 in. long, and one piece 10 ft. 10 in. long. If there are 12 pipe hangers at an equal distance apart, one being placed a foot from each end of the line, what is the distance between the hangers?
Ans. 7 ft. 9 in., nearly.

(181) The distance around a cedar tank whose base is a circle is approximately $\frac{22}{7}$ times the distance across it. If the distance across it is 9 ft. 6½ in., find the distance around it.
Ans. 29 ft. 11⅛ in.

(182) If the length of a boiler shell is 18 ft. $11\frac{1}{4}$ in., how many rivets would there be in one of the longitudinal seams if it is a single-riveted seam, supposing the rivets to have a pitch of $2\frac{1}{4}$ in., and the two end rivets to be $1\frac{1}{8}$ in. from each end of the boiler? Ans. 101 rivets.

(183) If a pump discharging 135 gal. per minute fills a tank in 38 minutes, how long will it take a pump discharging 85 gal. per minute to fill it? Ans. $60\frac{6}{17}$ min.

(184) If a quantity of babbitt metal contains 8 lb. of copper, 8 lb. of antimony, and 80 lb. of tin, how much copper will 32 lb. of the same metal contain? Ans. $2\frac{2}{5}$ lb.

(185) Two steam pumps discharge 31.416 cu. in. and 49.087 cu. in. of water respectively, with each stroke of their pistons. How many strokes will the larger pump make in filling a tank which can be filled by the smaller pump with 520 strokes? Ans. 333 strokes, nearly.

(186) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep? Ans. $484\frac{1}{2}$ bbl.

(187) If a piece of cast-iron pipe $3\frac{1}{2}$ ft. long weighs 37.45 lb., how much would a piece $6\frac{3}{4}$ ft. long weigh? Ans. 72.225 lb.

(188) If a hydraulic ram fills a tank whose capacity is 444 gal. in 8 hr. 40 min., in what time can it fill a tank whose capacity is 1,060 gal.? Ans. 20 hr. 41.44 min.

MENSURATION AND USE OF LETTERS IN FORMULAS.

EXAMINATION QUESTIONS.

$A = 5$ $\quad h = 200$
$B = 10$ $\quad x = 12$
$i = 3.5$ $\quad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(189) $C = \dfrac{D - x}{B + i}$. Ans. $C = 8$.

(190) $Q = \dfrac{A\,h + D}{2\,x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(191) $r = \dfrac{3.246 \times B \times h}{\dfrac{A\,x + h}{A\,i - B}}$. Ans. $r = 187.269+$.

(192) $v = \sqrt{\dfrac{A\,D}{i\,B + 1.5}}$. Ans. $v = 4.05+$.

(193) $u = \sqrt[3]{\dfrac{B\,x}{.00018\,h\,(A^2 - x)}}$. Ans. $u = 6.35+$.

(194) $f = \dfrac{10\,(h - D)^2}{\sqrt[3]{D + A}}$. Ans. $f = 12{,}800$.

(195) $g = \dfrac{(B - A)^2 - \sqrt[3]{D + A}}{A^3 - (1 + D)}$. Ans. $g = 5$.

(196) $k = \sqrt{\dfrac{A\,B^2}{\sqrt[3]{A\,h}}}$. Ans. $k = 7.071+$.

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(197) $T = \sqrt{\dfrac{A^2\left[490 + \dfrac{(hx)^2}{D^2}\right]}{h + \dfrac{x}{D}(A^2 - B)^2}}$. Ans. $T = 10$.

(198) If one of the angles formed by one straight line meeting another straight line equals 152° 3′, what is the other angle equal to? Ans. 27° 57′.

(199) How many seconds are there in 140° 17′ 10″?
Ans. 505,030″.

(200) Write a definition for a degree, as applied to the measurement of angles.

(201) (*a*) How many degrees are there in 240 minutes? (*b*) How many seconds? Ans. { (*a*) 4°. (*b*) 14,400″.

(202) Draw an obtuse angle, a right angle, and an acute angle. State the name of each angle by using letters to designate them.

(203) Draw a rhombus and then draw a rectangle having the same area.

(204) Can a quadrilateral be formed with lines whose lengths are 20 inches, 9 inches, 4 inches, and 7 inches? Give reasons.

(205) A piece of sheet lead measures 11½ inches by 2½ feet. How many square inches does it contain? Ans. 345 sq. in.

(206) A roll of sheet lead is 25.7 ft. by 6.75 ft., and weighs 6.5 lb. per sq. ft. (*a*) How many square feet in the roll? (*b*) What is its weight? (*c*) What will it cost at 6¼ cents per pound? Ans. { (*a*) 173.475 sq. ft. (*b*) 11 cwt. 27.5875 lb. (*c*) $70.47.

(207) A wooden tank is 6 ft. 3 in. long, 4 ft. 6 in. wide and 3 ft. 8 in. deep on the inside, and is lined on the sides with sheet lead weighing 5½ lb. per square foot, and on the bottom with sheet lead weighing 6½ lb. per square foot. The lining is turned over the top 1 inch for nailing. (*a*) How

many square feet of sheet lead in the tank? (*b*) What is the weight of the lead? (*c*) What is the capacity of the tank in gallons when filled to within 4 in. of the top, there being 7.48 gallons in a cubic foot?

Ans. { (*a*) 108.75 sq. ft.
(*b*) { 443.4375 lb. on sides.
182.8125 lb. on bottom.
(*c*) 701.25 gal.

(208) A piece of sheet iron is cut in the form of a trapezoid. The lengths of its two parallel sides are 4.32 ft., and 5.48 ft., respectively, and the perpendicular distance between them is 2.18 ft. What is the area of its surface?

Ans. 10,682 sq. ft.

(209) A tube is rectangular in cross-section, the lengths of its sides being 9½ in. and 5½ in., inside measure, and the thickness of the metal is ½ inch. (*a*) What is the sectional area of the inside of the tube? (*b*) What is the sectional area of the metal? (*c*) If the metal weighs .24 of a pound per cubic inch, what is the weight of 9 ft. 3 in. of the tube?

Ans. { (*a*) 52.25 sq. in.
(*b*) 16 sq. in.
(*c*) 426.24 lb.

(210) A wooden tank in the shape of a cube, whose side is 4 ft. long, inside measure, is lined with sheet lead. What will it cost to line the tank at 56 cents per square foot of lining? Ans. $44.80.

(211) A bath room 20 ft. long by 15 ft. wide, has a rectangular bath 5 ft. by 12 ft. sunk flush with the floor. How many tiles, each 6 inches square, are required to cover the floor and 5 ft. up the walls, allowing for a door 3 feet wide and for a window 4 ft. wide and 2 ft. above the floor?

Ans. 2,252 tiles.

(212) (*a*) If a piece of lead 3 ft. long, 2 ft. wide, and 1 in. thick is rolled into a sheet 20 ft. long and 6 ft. wide, how thick will it be? (*b*) If melted and run into a bar 6 in.

wide and 3 in. thick, how long will it be? (*c*) What will be its weight, 1 cu. in. of lead weighing .41 lb. ?

Ans. { (*a*) $\frac{1}{20}$ of an inch. (*b*) 4 ft. (*c*) 3 cwt. 54.24 lb.

(213) A sheet of metal 2 ft. square is formed into a rectangular tube of which 8 in. is the length of the longest side. (*a*) Find the sectional area of the inside of the tube. (*b*) If the perimeter be doubled, and the width of one side of the tube is 10 in., what is the sectional area ?

Ans. { (*a*) 32 sq. in. (*b*) 140 sq. in.

(214) A triangle has three equal angles; what is it called ?

(215) If a triangle has two equal angles, what kind of a triangle is it ?

(216) Can a triangle be formed with three lines whose lengths are 12 inches, 7 inches, and 4 inches ? Give reasons for your opinion.

(217) (*a*) What is the altitude of an equilateral triangle whose sides are each 6 feet ? (*b*) What is its area ?

Ans. { (*a*) 5.196 ft. (*b*) 15.588 sq. ft.

(218) In a triangle $A\ B\ C$, angle $A = 23°$, and $B = 32°\ 32'$; what does angle C equal ? Ans. $C = 124°\ 28'$.

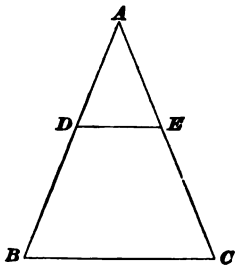

FIG. 1.

(219) In Fig. 1 if $A\ D = 10$ inches. $A\ B = 24$ inches, and $B\ C = 13\frac{1}{2}$ inches, how long is $D\ E$, $D\ E$ being parallel to $B\ C$? Ans. $D\ E = 5.625$ in.

(220) A pumping engine is located at one corner of a square field, the side of which is 300 yards long, and a tank which the pump has to supply is located at the opposite corner. The plumber estimated upon running the delivery pipe from pump straight through the field to the tank, but

was compelled to run it around the corner of the field instead. How much additional piping is required?

Ans. 175 yd. 2 ft. 2 in. +.

(221) A ladder 24 feet long rests against a house with its upper end 8 feet from the ground. How far on the ground is the lower end of the ladder from the house?

Ans. 22 ft. $7\frac{1}{2}$ + in.

(222) (*a*) Show why it is that the area of a triangle equals one-half the product of the base by the altitude. (*b*) Does it make any difference which side is taken as the base?

(223) (*a*) The area of an isosceles triangle is 200 square inches. If its altitude is 20 inches, how long is its base? (*b*) What is the length of one of its equal sides?

Ans. { (*a*) 20 in. (*b*) 22.36 in.

(224) In an equilateral heptagon one of the sides equals 3 inches; what is the length of the perimeter? Ans. 21 in.

(225) The perimeter of a regular decagon is 40 inches; what is the length of the side? Ans. 4 in.

(226) What is one angle of a regular dodecagon equal to? Ans. 150°.

(227) The area of a regular pentagon is 43 square inches. If one side is 5 inches long, what is the perpendicular distance from the center to one side? Ans. 3.44 in.

(228) It is required to make a miter-box, in which to cut molding to fit around an octagon post. At what angle with the side of the box should the saw run?

Ans. $67\frac{1}{2}$°.

(229) Calculate the area of the following irregular polygon, Fig. 2. The dimensions are to be obtained by measuring.

Ans. 1.78 + sq. in.

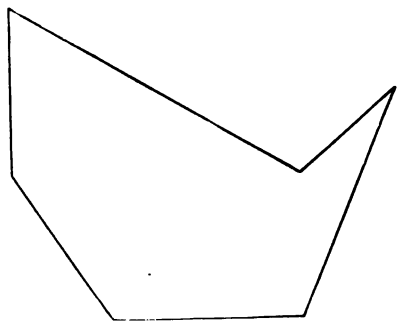

FIG. 2.

(230) An angle inscribed in a circle intercepts one-fourth the circumference. How many degrees are there in the angle? Ans. 45°.

(231) If the distance between two opposite corners of a hexagon nut is 2 inches, what is the distance between two opposite sides? Ans. 1.732 + in.

(232) In Fig. 3, if the distance BI is 6 inches and HK 18 inches, what is the diameter of the circle? Ans. 19.5 in.

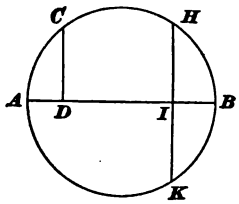

FIG. 3.

(233) In the same figure, if the diameter $AB = 32\frac{1}{2}$ feet and the distance $IB = 8$ feet, what is the length of the chord HK? Ans. 28 ft.

(234) A kitchen boiler measures 4 ft. 2.2656 in. around it; what is its diameter? Ans. 1 ft. 4 in.

(235) (*a*) How many times can 20 feet of wire be coiled around a pipe whose outside diameter is 3 in., neglecting the thickness of the wire? (*b*) If the wire is $\frac{1}{8}$ of an inch thick, how many times will it go around the pipe?

Ans. { (*a*) $25\frac{1}{2}$ times, nearly. (*b*) $24\frac{1}{2}$ times, nearly.

(236) If the diameter of a pump cylinder is $3\frac{1}{4}$ inches, (*a*) what is its area? (*b*) What is the circumference of the plunger?

Ans. { (*a*) 8.2957 sq. in. (*b*) 10.2102 in.

(237) The actual inside diameter of a 4-in. wrought-iron pipe is 4.026 in. and the outside circumference is 14.137 in. (*a*) What is the thickness of the metal? (*b*) How many feet of the same pipe are required to give an external surface of 100 sq. ft.?

Ans. { (*a*) .237 in., nearly. (*b*) 84.9 ft., nearly.

(238) How long must the arc of a circle be to contain 12 degrees, supposing the radius of the circle to be 6 inches.?

Ans. 1.25664 in.

(239) Calculate the area of a flat circular ring whose outside diameter is 22 inches, and whose inside diameter is 21 inches. Ans. 33.7722 sq. in.

(240) (*a*) What is the difference in the heating surface of coils made from 1-in. and 2-in. pipes, assuming the thickness of the metal in each case to be $\frac{1}{8}$ of an inch and the pipe 100 ft. long? [NOTE.—*The heating surface is the external surface of the pipe.*] (*b*) How many lineal feet of the 1-in. pipe are required to have a heating surface equal to 200 ft. of the 2-in. pipe?

Ans. { (*a*) 26.18 sq. ft.
(*b*) 360 ft. of 1-in. pipe.

(241) A steam-heating coil or radiator is composed of 175 ft. of 1-in. pipes. The external diameter of a 1-in. pipe being 1.315 in., (*a*) how many square feet of heating surface are in the coil, allowing 2.25 sq. ft. for elbows and manifolds? (*b*) If the length of the coil is 25 ft., how many rows of tubes are there? (*c*) How many square feet of heating surface in the coil if $1\frac{1}{4}$-in. tubes are used instead of 1 in., allowing .75 of a square foot additional for larger fitting? [A $1\frac{1}{4}$-in. pipe has an outside circumference of 5.215 sq. in.]

Ans. { (*a*) 62.5 sq. ft., nearly.
(*b*) 7 rows.
(*c*) 79.05 sq. ft.

(242) Find the area of a segment of a circle whose diameter is 56 inches, the height of the segment being 5 inches. Ans. 108.5 sq. in.

(243) What are the dimensions of the end of the largest square bar that can be planed from an iron bar 2 inches in diameter? Ans. 1.4142 in. square.

(244) What is the area of the sector of a circle 15 inches in diameter, the angle between the two radii forming the sector being $12\frac{1}{2}°$? Ans. 6.1359 sq. in.

(245) What would be the length of the side of a square metal plate having an area of 103.8691 square inches? (*b*) What would be the diameter of a round plate having this area? (*c*) How much shorter is the circumference of the round plate than the perimeter of the square plate?

Ans. { (*a*) 10.1916 in.
(*b*) $11\frac{1}{2}$ in.
(*c*) 4.638 in.

(246) A plate 46 inches long is to be rolled into a shell; what will be the diameter of the shell if 5 inches are allowed for lap? Ans. 13.05 in.

(247) What is the convex area in square feet of a section of a smokestack 26 inches in diameter and $10\frac{1}{2}$ feet long?
Ans. 71.4714 sq. ft.

(248) Find the area in square feet of the entire surface of a hexagonal column 12 feet long, each edge of the ends of the column being 4 inches long. Ans. 24.5774 sq. ft.

(249) Find the cubical contents of the above column in cubic inches. Ans. 5,985.9648 cu. in.

(250) Compute the weight per lineal foot of an iron boiler tube 4 inches outside diameter and 3.73 inches inside diameter, the weight of the iron being taken at .28 pound per cubic inch. Ans. $5\frac{1}{2}$ lb.

(251) The dimensions of a return tubular boiler are as follows: Diameter, 60 inches; length between heads, 16 feet; outside diameter of tubes, $3\frac{1}{2}$ inches; number of tubes, 64; distance of mean water-line from top of boiler, 18 inches; (*a*) compute the steam space of the boiler in cubic feet. (*b*) Determine the number of gallons of water that will be required to fill the boiler up to the mean water level.

Ans. { (*a*) 79.2 cu. ft.
(*b*) 1,246 gallons, nearly.

(252) A steam pump having a 4-in. piston and a 10-in. stroke, and giving 120 strokes per minute, delivers water to 2 tanks; one cylindrical in shape, is 10 ft. deep and has a diameter of 9 ft., and the other, rectangular, is 6 ft. by 5 ft. by 3 ft. deep. (*a*) How long will it take to fill them? (*b*) How long will it take to fill the large tank alone if the percentage of the loss is 10? (*c*) If the pump piston moves at the rate of 75 ft. per minute, what is the diameter of the piston necessary to fill the small tank in half an hour? Allow nothing for leakage.

Ans. { (*a*) 1 hr. 23.213 min.
(*b*) 1 hr. 21 min.
(*c*) $2\frac{11}{16}$ in., nearly.

(253) Find the volume of a triangular pyramid 10 inches high; the base is an equilateral triangle, and one edge measures 10 inches. Ans. 144.336 cu. in.

(254) The slant height of a square pyramid is 25 inches, and one edge of its base is 16 inches. Find its altitude by the principle of the right-angled triangle. Ans. 23.6854 in.

(255) The circumference of the base of a cone is 18.8496 inches, and its slant height is 10 inches. Find the area of the entire surface of the cone. Ans. 122.5224 sq. in.

(256) If the altitude of the above cone were 9 inches, what would be its volume? Ans. 84.8232 cu. in.

(257) The conical top of a ventilator has a base 16 in. in diameter, and an altitude of 6 in. Find the area of the convex surface of the cone. Ans. 251.328 sq. in.

(258) A square vat is 11 feet deep, 15 feet square at the top and 12 feet square at the bottom. How many gallons will it hold? Ans. 15,058.29 gal.

(259) How many pails of water would be required to fill the vat, the pail having the following dimensions: Depth, 11 inches; diameter at the top, 12 inches; diameter at the bottom, 9 inches? Ans. 3,627.28.

(260) Required, the area of the convex surface of the frustum of a hexagonal pyramid, whose slant height is 32 inches, the perimeter of the lower base measuring 48 inches and the upper base 36 inches. Ans. 1,344 sq. in.

(261) It is desired to find the number of gallons that a certain tank will hold. By measuring with a tape measure it is found to be 190 inches in circumference at the bottom, and 170½ inches in circumference at the top. The depth of the tank is 7 feet and the thickness of the sides 1¼ inches. Make the calculation in round numbers. Ans. 861 gal.

NOTE.—To obtain the area of the upper and lower bases, first find the outside diameters, and then deduct the thickness of the walls, or $2 \times 1\frac{1}{4}$ inches.

(262) Find (*a*) the area of the surface, and (*b*) the cubical contents of a ball $22\frac{1}{2}$ inches in diameter.

Ans. { (*a*) 1,590 435 sq. in.
(*b*) 5,964.1313 cu. in.

(263) What is the volume of a ball whose surface has an area of 201.0624 square inches? Ans. 268.0832 cu. in.

(264) The diameter of a ball float is 8 in.; (*a*) what is its volume? (*b*) If the ball is submerged in a vessel 10 in. by 12 in. by 20 in. deep, which is half full of water, how high will the water rise in the vessel? (*c*) If the volume of the ball is 179.5948 cu. in., find its diameter.

Ans. { (*a*) 268.0832 cu. in.
(*b*) 2.234 in.
(*c*) 7 in.

(265) A copper ball 10 in. inside diameter, weighs 2 lb. 8 oz. How much will it weigh when half full of water, if 1 cu. in. of water weighs .03517 lb.? Ans. 12 lb., nearly.

(266) A steam kettle in the form of a cylinder with a hemispherical bottom is 24 in. in diameter and 24 in. deep. How many gallons in it, if the water in the kettle is 16 in. deep? Ans. 23.5008 gal.

(267) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches, and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight.

Ans. Weight = 21 lb.

NOTE.—The weight of cast iron is .261 lb. per cubic inch.

MECHANICS.

(ARTS. 433–565.)

EXAMINATION QUESTIONS.

(268) What do you understand by (*a*) a solid? (*b*) a liquid? (*c*) a gaseous body?

(269) What is a vapor?

(270) What is velocity, and how is it measured?

(271) What is a force?

(272) Explain what you understand by inertia.

(273) What do you understand by the center of gravity of a body?

(274) What should be the length of the power arm of a lever to raise 975 pounds by the application of a force of 75 pounds, if the weight arm be 4 inches long?

Ans. 4 ft. 4 in.

(275) In Fig. 76, Art. **489**, suppose that the distances $F\ b$ and $F\ c$ were, respectively, 4 inches and 15 inches; what force must be applied in the direction of the arrow to raise a weight W of 57 pounds? Ans. 15.2 lb.

(276) (*a*) If the distance between the center line of the handle and axis of the drum in Fig. 79, Art. **491**, is 12 inches, and the diameter of the drum is 5 inches, what force will be required to raise a roll of sheet lead 25 feet by 6 feet, weighing 6 pounds per square foot? (*b*) What distance will the handle P travel in raising the roll 10 feet?

Ans. { (*a*) 187.5 lb.
(*b*) 48 ft., nearly.

(277) (*a*) A block and tackle is rigged to raise a fly-wheel weighing 500 pounds. The top block is secured to a beam over the engine; the bottom or movable block is composed of two pulley wheels; what pull is necessary on

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the free end of the rope to raise the fly-wheel? (*b*) If there were three movable pulleys, and the same force were applied to the free end of the rope, what weight could be raised?

Ans. { (*a*) 125 lb.
(*b*) 750 lb.

(278) (*a*) What is friction? (*b*) If wrought iron rubs against wrought iron, and lead against lead, in which is the coefficient of friction the highest?

(279) A cube of lead has the same weight as a lead ball. If both are impelled forward upon a plane surface with the same force, which will come to rest first, and why?

(280) What is work? How is it measured?

(281) What horsepower is exerted in raising 10,200 gallons of water in 10 minutes from the bottom of a well 125 feet deep to a tank 75 feet above the ground? (A gallon of water weighs 8.355 pounds.)

Ans. 51.65 H. P., nearly.

(282) What is meant by the specific gravity of a body?

(283) What do you understand by hydrostatic pressure?

(284) A house tank is in the form of a cube, one side of which measures 5 feet. (*a*) Find the pressure upon its bottom when the tank is filled. (*b*) What is the pressure per square inch upon its bottom when the tank is $\frac{2}{3}$ full?

Ans. { (*a*) 7,812.5 lb.
(*b*) 1.447 lb. per sq. in.

(285) If, in Fig. 94, Art. **539,** the vessel *a* is 12 inches in diameter, and *b* is 2 inches in diameter at the top and 12 inches at the base, the other dimensions remaining unchanged, (*a*) what is the pressure upon the bottom of each? (*b*) What is the weight of the water in each vessel? (*c*) What is the upward pressure on the surface *c*?

Ans. { (*a*) 98.2 lb., nearly.
(*b*) { Vessel *a*, 98.2 lb.
Vessel *b*, 42.5 lb.
(*c*) 55.7 lb.

(286) A vertical line of soil pipe, 80 feet high, is plugged at its base and filled with water. (*a*) If a pressure gauge be applied at a point 10 feet above the base, what pressure should it indicate? (*b*) If the plug at the bottom of the pipe is 4½ inches in diameter, what force tends to blow it out?

Ans. { (*a*) 30.38 lb. per sq. in., nearly.
(*b*) 552.1 lb.

(287) Explain why water seeks its level.

(288) (*a*) If the piston *a*, in Fig. 100, Art. **550,** is 1 inch, and *b* is 20 inches in diameter, and a pressure of 10 pounds is applied to the piston *a*, what weight can be raised by the piston *b*? (*b*) How high will piston *b* be raised if *a* falls 10 inches?

Ans. { (*a*) 4,000 lb.
(*b*) .025 in.

(289) A hydraulic elevator has a piston 9 inches in diameter. (*a*) What pressure of water is necessary to produce enough force upon the piston to raise a weight of 3 tons? (*b*) How much water should be used in raising the piston 10 feet?

Ans. { (*a*) 94.3 lb. per sq. in.
(*b*) 33 gal., nearly.

(290) A reservoir in the form of a frustum of a cone has sloping sides which form an angle of 45° with its base. (*a*) How would you find the pressure upon any point of the sloping sides? (*b*) If the reservoir is 16 feet deep, what is the pressure per square inch at a point ¾ down the sloping sides?

Ans. (*b*) 5.21 lb.

(291) A cylindrical boiler 30 inches in diameter has elliptical ends riveted to the boiler shell, as shown in Fig. 105, Art. **557;** the rivets being 2 inches apart from center to center, what force tends to tear off each rivet if the boiler is subjected to a pressure of 80 pounds per square inch?

Ans. 1,203.17 lb.

(292) Explain what is meant by the buoyant effects of water.

(293) Why does a piece of iron sink if placed in water, and why will it float if placed in melted lead?

(294) What force is required to submerge in water a cube whose side measures 6 inches, and whose weight is 3 pounds? Ans. 4.81 lb.

(295) What is capillary attraction?

(296) If the drivers of a locomotive are 6 feet in diameter and make 220 revolutions per minute, what is the speed of the train in miles per hour? Ans. 47.124 mi. per hr.

(297) Show how a force may be represented by a line.

(298) In Fig. 65, Art. **479,** let the weights of W_1, W_2, W_3, and W_4 be, respectively, 20, 16, 32, and 12 pounds, and the distances between their centers as follows: Between W_1 and W_2, 24 inches; between W_2 and W_3, 28 inches; between W_3 and W_4, 18 inches, and between W_4 and W_1, 32 inches. Find the center of gravity C of the balls. Measure with a scale any distance not given, and consider the line joining W_4 and W_3 to be at right angles to the line joining W_4 and W_1. Ans. 11.92″ from line W_1 W_4.

(299) A triangle has sides whose lengths are 4, 5, and 6 inches long; what is the distance of the center of gravity from the longest side? Ans. $1\frac{3}{32}$″.

(300) Take a piece of flat cardboard paper of irregular shape and find the center of gravity by the method described in Art. **483.** Make a sketch of the outline of the cardboard and describe the operation.

(301) What force is required to draw a weight of 1,720 pounds along a horizontal plane when the coefficient of friction is .21? Ans. 361.2 lb.

(302) In Fig. 89, Art. **503,** suppose that the distance $F P$ is 9 inches, and that the screw has 3 threads per inch. If the efficiency is 41%, what force should be applied at P to raise a load of 6,780 pounds? Ans. 97.5 lb., nearly.

(303) In the last example (*a*) what is the theoretical force required to raise the load? (*b*) What is the velocity ratio?

Ans. { (*a*) 40 lb., nearly.
(*b*) 169.5, nearly.

(304) A lead ball, 4 inches in diameter, is attached to a cord and revolved in a circle 36 inches in diameter, at the rate of 96 revolutions per minute; what is the pull in the cord? Ans. 64.72 lb.

(305) How much will a copper cylinder, open at both ends, having an outside diameter of 6 inches, an inside diameter of 5 inches, and a length of 12 inches, weigh when entirely submerged in water? Ans 29.22 lb.

(306) How many horsepower are required to overcome the friction of the cross-head of an engine when the average pressure on the guides is 11,426 pounds, the coefficient of friction .08, and the speed of the cross-head 580 feet per minute? Ans. 16.07 H. P.

(307) Explain what you understand by (*a*) potential energy; (*b*) kinetic energy.

(308) How many foot-pounds of work are done in raising a weight of 120 pounds through a height of 15 feet by means of a block and tackle having three fixed and three movable pulleys? (Neglect friction, and give reasons for your answer.)

(309) A certain body weighing 400 pounds has a velocity of a mile a minute; what is its kinetic energy in foot-pounds per second? Ans. 48,159.2 ft.-lb.

(310) What is the principle of the conservation of energy? Give an illustration.

(311) How much will 1,000 bricks weigh if they are $8'' \times 4'' \times 2''$? Ans. 4,370 lb.

(312) What is the weight of a cubic yard of sand?
Ans. 3,051 lb.

(313) If a force of 100 pounds be applied to the free end of the rope of the combination of pulleys shown in Fig. 84, Art. **498**, (*a*) how great a weight can be raised, the efficiency of the arrangement being 82%? (*b*) What is the velocity ratio?
Ans. { (*a*) 492 lb.
(*b*) 6.

(314) (*a*) If in Figs. 71 and 72, Art. **484,** the lengths of both levers were 43 inches, and the distances $F\ b$ both $5\frac{1}{4}$ inches, what weights can forces of 28 pounds applied at the points c raise? (*b*) What is the cause of the difference in the weights lifted in each case?

Ans. (*a*) $\begin{cases} 201\frac{1}{3} \text{ lb.} \\ 229\frac{1}{3} \text{ lb.} \end{cases}$

(315) In Fig. 80, Art. **493,** what force is required to be applied at P in order to raise a load of 900 pounds when the diameters of A and F are, respectively, 21 and 6 inches, and the number of teeth in B, C, D, and E are 14, 70, 20 and 96, respectively? Take the efficiency of the combination as 85%. Ans. 12.6 lb.

(316) Suppose the length of the base $a\ c$ of the inclined plane shown in Fig. 85, Art. **500,** to be 108 feet and the height $b\ c$ 18 feet; what force exerted parallel to $a\ b$ will pull a weight of 275 pounds up the plane? Ans. 45.21 lb.

(317) A solid cast-iron ball 6 inches in diameter is sunk in mercury to a depth of 28 inches; what force is necessary to keep the ball from rising to the surface? Ans. 26.13 lb.

MECHANICS.

(ARTS. 566–652.)

EXAMINATION QUESTIONS.

(318) When water flows through a pipe its velocity in the center of the pipe is greater than that at the sides; (*a*) what is the cause of this? (*b*) What do you understand by mean velocity?

(319) Water flows through a $1\frac{1}{4}$-inch pipe with a mean velocity of 8 feet per second; how long will it take to fill a tank 8 feet by 6 feet by 3 feet deep? Ans. $35\frac{1}{3}$ min.

(320) What is the velocity of efflux?

(321) What do you understand by the contracted vein, and how does it affect the velocity of efflux?

(322) A $\frac{1}{2}$-inch pipe and a 2-inch pipe are 500 feet long and convey water from the same tank to the same outlet point; (*a*) in which pipe has the water the greater velocity, and why? (*b*) If the head is 50 feet and the coefficient of friction is .025, what is the velocity of efflux in each pipe?

Ans. (*b*) $\begin{cases} \frac{1}{2}\text{-inch pipe} = 3.274 \text{ ft. per sec.} \\ 2\text{-inch pipe} = 6.483 \text{ ft. per sec.} \end{cases}$

(323) A $\frac{3}{4}$-inch service pipe is taken from a point in the street main where the head is 190 feet, and led to a building 300 feet away; how many gallons will it discharge in a minute? Work the example as though the whole length of the pipe were 300 feet. Ans. 15.44 gal.

(324) What do you understand by the tension of gases?

(325) What is a vacuum?

(326) If a vacuum occupies a space of 10 cubic inches, and .31 grain of air be admitted, (*a*) what is the space then called? (*b*) Will the air rise to the top or fall to the bottom of the space, or what will become of it?

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(327) (*a*) What is atmospheric pressure? (*b*) Give an illustration of the effect of atmospheric pressure.

(328) Give an illustration of how the volume of a gas may be changed.

(329) If a gas at atmospheric pressure be diminished to $\frac{1}{3}$ of its original volume, to what pressure will it be subjected, assuming that the temperature is constant?

Ans. 44.1 lb. per sq. in.

(330) An air chamber whose volume is 1,000 cubic inches is placed upon a water pipe containing water under a total pressure of 80 pounds per square inch; (*a*) what will be the volume of air in the chamber when the water is admitted at full pressure? (*b*) If the total force exerted when a faucet is suddenly closed be 200 pounds per square inch, what will be the volume of the air in the chamber during the shock?

Ans. { (*a*) 183.75 cu. in.
(*b*) 73.5 cu. in.

(331) What is the effect of heat on the expansion of gases?

(332) A gas chandelier has a ball-and-socket joint near the ceiling; if this joint leaks how is it that the smell of gas is distinctly noticed down near the floor, when it is known that the specific gravity of the gas is about .5?

(333) Can a perfect vacuum be created by means of an air pump? If not, why?

(334) The diameter of the piston in Fig. 125, Art. **599,** is 10 inches, the weight of the piston and the rod is 30 pounds. If the plug be unscrewed and the piston pushed in until no air remains between the cylinder end and the face of the piston, and the plug be then screwed in its place air-tight, what force will be required to pull the piston out of the cylinder? Ans. 1,124.54 lb.

(335) If a small hole be made at the point *C* in Fig. 127, Art. **603,** what will result? Explain the cause.

(336) Explain the action of the suction pump.

(337) If the vertical distance between the valve V of the pump in Fig. 130, Art. **608**, and the water in the well is 34 feet, can water be pumped out at the spout A? If not, why?

(338) If the pump in Fig. 132, Art. **611**, discharges into a tank above it, P being connected to the bottom of the tank, (*a*) what will be the result if sand is raised with the water and gets between the valves V, V', and their seats? (*b*) If a pump, after standing unused for a length of time, loses its water, why is it necessary to prime it with water?

(339) What are the chief objections to the single-acting lift or force pump?

(340) (*a*) What is a double-acting pump? (*b*) Explain its action and advantages over the single-acting pump.

(341) What is an air chamber, and is it generally placed upon the delivery pipe of a pump?

(342) Is there an air chamber shown on the pump in Fig. 133, Art. **612**? If so, locate it.

(343) Explain the action of a hydraulic ram.

(344) What is the necessary horsepower of a steam pump if the vertical distance to the level of the point of discharge is 100 feet; the diameter of the pump piston, 4 inches; the stroke, 9 inches, and the number of strokes per minute, 120? Ans. 1.786 H. P.

(345) (*a*) What is meant by tensile strength? (*b*) by breaking stress in pounds per square inch? (*c*) by working stress in pounds per square inch?

(346) (*a*) What is the greatest weight that can be hung from a wrought-iron rod $\frac{3}{8}$ inch in diameter? (*b*) What would you consider the greatest safe steady load for such a rod? Ans. (*a*) 5,520 lb.

(347) What pressure per square inch is required to burst a lead pipe 2 inches in diameter and $\frac{1}{8}$ inch thick? Ans. 275 lb. per sq. in.

(348) A wrought-iron cylindrical kitchen boiler 14 inches in diameter and $\frac{1}{8}$ inch thick is supplied with water from a

tank 75 feet above it (vertically); what is the stress in pounds per square inch of the metal forming the sides of the boiler? Ans. 1,823 lb. per sq. in., nearly.

(349) What is the probable discharge of a double-acting steam pump whose plunger is $7\frac{1}{2}$ inches in diameter; stroke, 18 inches, and which makes 60 strokes per minute?

Ans. 165.24 gal. per min.

(350) What should be the size of the steam cylinder and the number of strokes per minute of a double-acting steam pump which is required to deliver 150 gallons of water per minute against a head of 175 feet, the piston speed to be 120 feet per minute, and the steam pressure 60 pounds? Height of suction is 18 feet. Give diameter of cylinder to the nearest $\frac{1}{8}$ inch, and take the stroke as $1\frac{1}{2}$ times the diameter.

Ans. { (*a*) Diameter, 8″. (*b*) Stroke, 12″.

(351) A 3-inch pipe discharges 120 gallons of water per minute; what is the mean velocity of efflux?

Ans. 5.447 ft. per sec.

(352) A 6-inch pipe discharges 39,600 gallons of water per hour; if the length of the pipe is 3,500 feet, what is the head? Ans. 127.7 ft.

(353) Find the pressure in pounds per square inch which can be sustained by a wrought-iron cylinder 20 inches in diameter, whose plates are $\frac{3}{16}$ inch thick. Take the safe tensile strength of the plates as 8,000 pounds per square inch. Ans. 150 lb. per sq. in.

(354) What should be the greatest diameter of a lead pipe $\frac{1}{8}$ inch in thickness to safely resist a steady pressure of 50 pounds per square inch? Ans. 2 in.

(355) A close link wrought-iron chain is made from $\frac{3}{8}$″ iron; what is the greatest safe load that it will carry?

Ans. 1,687.5 lb.

(356) What is the allowable working load for a steel wire rope $5\frac{1}{4}$″ in circumference? Ans. 27,562.5 lb.

(357) What steady force is required to shear a steel crank-pin which is 6″ in diameter? Ans. 1,696,464 lb.

(358) (*a*) What is a barometer? (*b*) What is the essential difference between a mercurial and an aneroid barometer?

(359) What load may be safely carried by a hemp rope 4″ in circumference? Ans. 1,600 lb.

(360) What load can be safely sustained by a round wooden pillar 8″ in diameter and 10 feet long, having both ends flat? Ans. 13⅓ tons.

(361) What force is required to punch a 1″ hole through a wrought-iron plate $\frac{7}{16}$″ thick? Ans. 54,978 lb.

(362) A vessel contains 42 cubic feet of coal gas having a tension of 20¼ pounds per square inch; what will be the new tension when allowed to communicate with a perfectly empty vessel whose volume is 14 cubic feet?

Ans. 15.19 lb. per sq. in., nearly.

(363) What safe steady load can be sustained by a 1½″ round wrought-iron bar, the load producing a tensile stress?

Ans. 17,671 lb.

(364) What load can a hemp rope 6″ in circumference carry with safety? Ans. 3,600 lb.

(365) What load will a hollow cast-iron pillar support with safety if the pillar is 20 feet long; outside diameter, 14″; inside diameter, 11½″, and both ends are fixed?

Ans. 219.24 tons.

(366) What force is required to punch a hole 1½″ in diameter through a ¾″ steel plate? Ans. 212,058 lb.

(367) A bar of steel having a cross-section of 1¾″ × 3″ is subjected to a tensile stress; if the stress is suddenly applied, what is the greatest load that it will safely carry?

Ans. 31,500 lb.

(368) A load of 2,400 pounds is to be raised by means of a hemp rope; what should be the circumference of the rope? Ans. 4.9″.

(369) The volume of steam in an engine cylinder at the beginning of compression is 1.11 cubic feet, and its pressure is 18 pounds per square inch. At the end of compression, the volume is .3 cubic foot; what is the final pressure?

Ans. 66.6 lb. per sq. in.

(370) What should be the least cross-sectional area of the studs for a 17″ cylinder, the steam pressure being 160 lb. per sq. in., and the number of studs 16? Assume that they are subject to shocks. Ans. .454 sq. in., say $\frac{7}{8}$ studs.

(371) What should be the circumference of a hemp rope to safely sustain a load of 4,200 lb.? Ans. $6\frac{1}{2}$″.

(372) A vessel contains 25 cubic feet of air having a pressure of 45 pounds per square inch. When allowed to communicate with a second vessel which is entirely empty, the pressure falls to 15 pounds per square inch. What is the volume of the second vessel? Ans. 50 cu. ft.

(373) What should be the least diameter of a wrought-iron bolt that is to resist a sudden pull of 12,000 pounds?

Ans. 1.95″+.

(374) A steel wire rope is $4\frac{3}{4}$″ in circumference; what load will it safely sustain? Ans. 22,562.5 lb.

(375) A white pine beam supported at both ends has a rectangular cross-section 4″ wide by 10″ deep; if the span is 28 feet long, what total uniform load will it support in safety? Ans. 3,428$\frac{4}{7}$ lb.

(376) The vacuum gauge of a condensing engine indicates 20″ of vacuum; what is the pressure in the condenser?

Ans. 4.9 lb. per sq. in.

(377) It is desired to handle loads up to 14,000 pounds by means of an iron wire rope; what should be its circumference? Ans. 4.83″, nearly.

(378) What is the greatest load that a bar of wrought iron 2″ in diameter and 6 feet span can safely sustain in the middle? The bar is merely supported at its ends.

Ans. 480 lb.

(379) The volume of the receiver of an air compressor is 300 cubic feet, and the pressure of the air which is contained

in it is 52 pounds per square inch; if 120 cubic feet of air, having a pressure of 52 pounds per square inch, are removed from the receiver, what will be the pressure of the air which remains? Ans. 31.2 lb. per sq. in.

(380) (*a*) What must be the height of the mercury column to indicate a vacuum of 12″? (*b*) of 18″?

(381) (*a*) What is a stress? (*b*) a strain? (*c*) a unit stress?

(382) The links in a stud link wrought-iron chain are made from iron $\frac{13}{16}$″ in diameter; what is the greatest safe load that the chain can handle? Ans. 11,883 lb.

(383) What uniform load can be safely sustained by a steel beam 20 feet span, 2″ wide, and 6″ deep?

Ans. 4,608 lb.

(384) What safe load may be carried by a close link wrought-iron chain whose links are made from $\frac{5}{8}$″ iron?

Ans. 4,687.5 lb.

(385) What is the allowable working load for an iron wire rope 6″ in circumference? Ans. 21,600 lb.

(386) What force is required to shear a wrought-iron strip 4 feet long and $\frac{1}{2}$″ thick? Ans. 960,000 lb.

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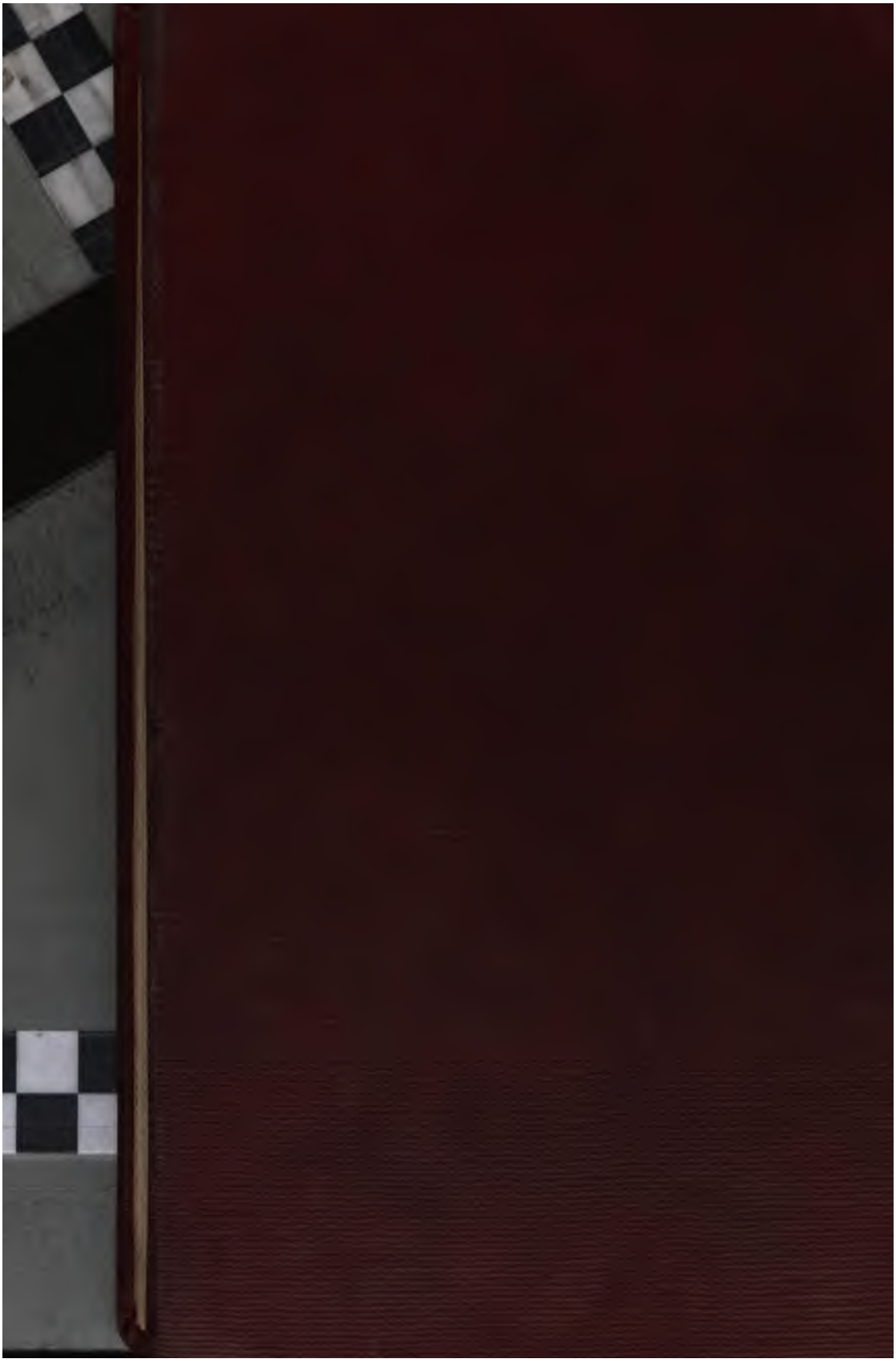
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